In this lecture we discussed:

- Witness for SAT;
- NP vs co-NP;
- Approximation Algorithms.

1 NP-Completeness Wrap-up

Witness example: SAT

\[ \phi = (x \lor y \lor z) \land (\overline{x} \lor \overline{y}) \land (x \lor \overline{z}) \]

\[ \phi \in SAT \equiv \phi \text{ has a satisfying truth assignment} \]
A witness in this case is a truth assignment

**Note**: It may be impossible to find a small, easily verifiable witness for some problems.
Ex: \( \phi \in \text{co-SAT} \equiv \phi \text{ has no satisfying assignment iff } \overline{\phi} \in \text{SAT} \)
A witness for co-SAT problem would have to show that \( \phi \) is not satisfied by any assignment.

NP vs co-NP:

**Example**: True Quantified Boolean Formula

\[ \forall x \exists y \forall x (x \lor y \lor z) \land (\overline{x} \lor \overline{y}) \]

NP \( \equiv \exists \text{ witness } \equiv \Sigma_1 \)
co-NP \( \equiv \forall \text{ witness } \equiv \Pi_1 \)

**Note**:
\( \Sigma_2 \equiv \exists x \forall y \) (verifiable in polynomial time)
\( \Pi_2 \equiv \forall x \exists y \) (verifiable in polynomial time)
2 Approximation Algorithms

What to do when need to solve NP-hard optimization problem?

- **Heuristic**. Disadvantage: bad adaptability to new input.

- **Approximation**. Advantage: can prove how close is the approximated solution to the optimal solution for any input.

**Definition**: An algorithm A is a $\rho$-approximation if for every input I with optimal solution value OPT(I):

$$A(I) \approx OPT(I) \equiv \begin{cases} A(I) \leq \rho OPT(I) & \text{(minimization problem)} \\ A(I) \geq \frac{OPT(I)}{\rho} & \text{(maximization problem)} \end{cases}$$