In this lecture we:

- Continue the Pennant Race Problem
- Introduce the Open Pit Mining Problem

Miscellaneous:

- Midterms have been marked and can be viewed via Handback
- Midterm Average: 68.6%
- Additional office hours with Will: Tuesday (Feb 14) 2pm-3pm

1 Pennant Race Problem

- \( w = \#A’s \) wins (assuming A wins all remaining games)
- \( w_i = \#T_i’s \) wins (assuming A wins all remaining games)
- \( \{(T_i,T_j)\} \) = games remaining to be played

Assume \( w_i \leq w \) for all \( i \)

![Flow graph of the Pennant Race example given on Feb 6](image)

Figure 1: Flow graph of the Pennant Race example given on Feb 6

Edges

- \((s,T_iT_j)\) with capacity 1
- \((T_iT_j,T_i)(T_iT_j,T_j)\) with capacity 1
- \((T_i,t)\) with capacity \( w - w_i \)

If max flow = \# games to play then A still has hope.
2 Open Pit Mining

What is Open Pit Mining? A mining technique where you attempt to dig to a location that would give a profit, but before you may do so you must remove a certain amount of dirt that lays above the location. Removing dirt has some cost associated with it. The goal is to achieve the maximum profit.

Input: Directed Acyclic Graph

- $G = (V, E)$ where $V$ = set of tasks
- $E = \{(u, v) \mid u \text{ must be done before } v \}$
- A function $w(v)$ that specifies the profit from doing the task

![Figure 2: An example of a Directed Acyclic Graph](image)

In this example:

- Both D and E must be done before H
- A must be done before D
- B must be done before E

**Definition 1.** An initial set is a set of vertices that has no edge coming into it from the outside

In example above:

- \{D, G\} is not an initial set
- \{A,D,G\} is an initial set

Convert the problem to a network flow problem so that

1. Any finite capacity cut corresponds to an initial set
2. A minimum capacity cut corresponds to max profit initial set
2.1 Conversion Part 1: Finite Capacity Cut

Claim 2. In this "network", any finite capacity cut \((S, T)\) defines an initial set \(T = \{t\}\)

Proof. If cut \((S, T)\) has finite capacity then no original edge is directed into \(T\) from \(S\) thus \(T - \{t\}\) is an initial set. If set \(U\) is an initial set then \(T = U \cup \{t\}\), \(S = V - T\) is a cut with no original edge entering \(T\) thus it has finite capacity.

2.2 Conversion Part 2: Minimum Capacity Cut

Given a directed acyclic graph, we want to connect the vertices so that:

- If \(w(u)\) is positive, then \(\underbrace{u \rightarrow t}_{w(u)}\)
- If \(w(v)\) is negative, then \(\underbrace{s \rightarrow v}_{-w(v)}\)

Note that in Figure 4 we get maxflow = mincut = 1. Then the min cut gives us initial set \(\{A, B\}\). But the max flow value does not correspond to the total profit from the task.