In this lecture we:

- Discussed THE COURSE SYLLABUS;
- TOOK A SHORT QUIZ (not graded, solutions posted to Piazza);
- AND started studying CONVEX HULLS - Jarvis March

Handouts (posted on webpage):

- CS 420+500 Syllabus (aka the webpage)
- quiz

Reading: NO ASSIGNED READING(S) THIS WEEK.

1 CONVEX HULLS

Suppose we’re in charge of making the salad dressing. We have bottles of oil and vinegar in the following ratios:

Table 1: Ingredients in unsatisfactory premixed salad dressing

<table>
<thead>
<tr>
<th>bottle</th>
<th>oil</th>
<th>vinegar</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>15%</td>
<td>36%</td>
</tr>
<tr>
<td>B</td>
<td>9%</td>
<td>21%</td>
</tr>
</tbody>
</table>

Q: Can we mix bottles A and B to get 13% oil and 31% vinegar?
A: Yes. We use 2 parts A and 1 part B.

Table 2: New salad dressing

<table>
<thead>
<tr>
<th>bottle</th>
<th>oil</th>
<th>vinegar</th>
<th>proportion</th>
<th>oil'</th>
<th>vinegar'</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>15%</td>
<td>36%</td>
<td>2/3</td>
<td>10%</td>
<td>24%</td>
</tr>
<tr>
<td>B</td>
<td>9%</td>
<td>21%</td>
<td>1/3</td>
<td>3%</td>
<td>7%</td>
</tr>
<tr>
<td>A + B</td>
<td>1</td>
<td>13%</td>
<td>1</td>
<td>13%</td>
<td>31%</td>
</tr>
</tbody>
</table>

Q: Can we create a mixture of 12% oil and 30% vinegar?
A: No.

How can we tell which mixtures are achievable?
We can make any dressing with ratios lying on the line connecting \(AB\).

If we have a new bottle \(C\), then we can make any ratio within the area of the connected shape \(\triangle ABC\).

A mixture is a convex combination of points \(P = \{p_1, p_2, \ldots, p_n\}\) representing the contents of bottles or \(\sum_{i=1}^{n} \alpha_i p_i\), where \(\sum_{i=1}^{n} \alpha_i = 1\), \(\alpha_i \geq 0\) for all \(i\).

[def] The convex hull of \(P\) or \(\text{CH}(P)\) is the smallest convex set containing \(P\).

[def] A set \(T\) is convex if for all \(a, b \in T\), \(\overline{ab}\) is in \(T\).

[def] A supporting line is a line going through a boundary point \(b \in T\) such that all points in \(T\) fall on one side of that line.

**Problem:** Input set of points \(P = \{p_1, p_2, \ldots, p_n\}\)  
Output convex hull of \(P\)

### 1.1 Jarvix March (Gift wrapping) 1973

**ALGORITHM:** Imagine hammering nails (representing \(P\)) into a board, then wrapping a rubber band around the outer nails.

1. Find point \(p_0\) that is guaranteed to be in the convex hull of \(P\) such as the point with the minimum y-coordinate.
2. Set \(h = 0\)
3. Repeat
   - Pick \(q \in P \setminus \{p_h\}\)
   - For each \(p \in P\)
     - If rightturn\((p_h, q, p)\)
       - \(q = p\)
     - \(p_{h+1} = q\)
     - \(h = h + 1\)
   - Until
     - \(p_h = p_0\)
RUNTIME Now we consider the running time of Jarvis March.

- Step(1) \( \in O(n) \)
- Step(2) \( \in O(1) \)
- Step(3) \( \in \Theta(n^2) \)

[THINK] worst case: each right turn check \( \in O(n) \) and we must do this for each point \( n \) times \( \therefore O(n^2) \) but since we must go through all elements, run time is also \( \Omega(n^2) \) thus overall, we have \( \Theta(n^2) \)

The CON of Jarvis March is that we end up checking the same points multiple times, repeating similar work. What if we do a sort by angle first?

1.2 NEXT LECTURE: Graham’s Scan 1972