Reduce

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Objectives

- Understand how reduce combines values using a tree.
- Describe the performance issues for reduce: trade-offs of time for computation and time for communication
- Describe 2 or 3 examples or reduce.

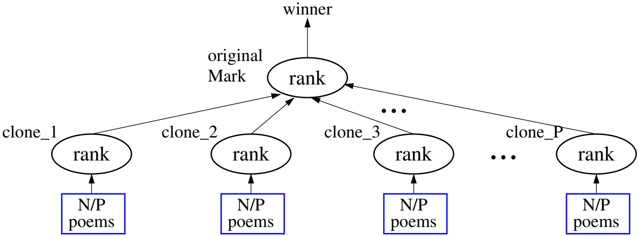
CPSC 418 Poetry Competition

- The competition:
  - Everyone writes a poem.
  - Everyone submits to poem to Mark (the contest judge).
  - Mark reads all of the poems, compares them, selects the best poem.
  - The winner receives an original manuscript of the complete poems of Li Bai, signed by the author.

Sequential Time for the Poetry Competition

- N students in the class.
- t_{rank} to read and rank two poems.
- Total time $(N - 1)t_{rank}$.
- Works fine until N becomes so large that we can't judge all the poems in a reasonable amount of time.

Parallel poetry: the procedure



- Clone P copies of Mark.
- Each Mark-clone reads and ranks N/P poems and sends the best poem to the original Mark.
- The original Mark receives P candidates for the best poem, and selects the best one.
- The winner receives the prize.

Parallel poetry: time

- The time for each of the P clones to select the best poem out of N/P:

  ____

- The time from when all P clones start until all P clones finish:

  ____

- The time for the original Mark to rank the P finalists:

  ____

- Simplify this to get ________________________.

- $SpeedUp = \frac{T_{\mathrm{seq}}}{T_{\mathrm{par}}} =$ ________________________.

Bureaucratic Overhead

- To satisfy UBC privacy policies, the messages between the Mark-clones and the original mark must be sent in special envelopes.
- There's lots of special procedure for handling these envelopes, takes time λ to send or receive a message.
- The original Mark receives P messages from the P clones. This takes time λP.

- The total time is now: ____________________.
- $\mathit{SpeedUp} = \frac{T_{\text{seq}}}{T_{\text{par}}} =$ ____________________.

Can we do better?

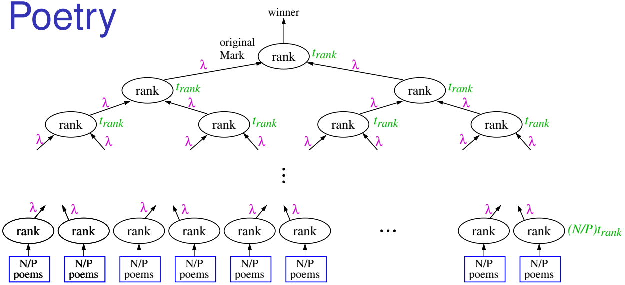
Revenge of the Clones

- While Mark is working through the pile of envelopes, some of the clones realize that they could pair up and combine their results.
  - This costs λ time, the clones have to follow the rules as well.
  - The original Mark ends up with half as many envelopes to handle.

- What is the total time? ____________________.

- What is the speed-up? ____________________.

Up a Tree with Poetry



- The optimization on the previous slide worked great.
  - We're computer scientists, let's apply the optimization recursively.
  - Viewed from another angle, this is an example of divide-and-conquer.
- Combine the results in a tree.
  - How many levels in the tree?
  - How much time at each level?
- What is the total time? ____________________.
- What is the speed-up? ____________________.

Is there more to life than poetry?

- Find the largest element in a list.
- Find the sum of the elements in a list.
- Count the number of 3s in a list.
- What to these all have in common?
- We'll look at more examples on Friday

The Reduce Pattern

- We have a problem that takes $T_{\text{seq}}(N)$ sequential time, where N is the "size" of the problem instance.
- We can divide this into P tasks with "perfect" speed-up:
  - Each task takes time $T_{\text{seq}}(N)/P$ time.
  - Combining the results takes $\lceil \log_2(P) \rceil \lambda$ time.
- $\mathit{SpeedUp} = \frac{T_{\text{seq}}(N)}{T_{\text{seq}}(N)/P + \lceil \log_2(P) \rceil \lambda}$
- What happens to *SpeedUp* as P goes large (for fixed N)?
- What happens to *SpeedUp* as N goes large (for fixed P)?
  - Assume $T(N)$ grows faster than $\log N$.

When can we use reduce?

- We have N values, V_1, V_2, V_3, $\ldots$ V_N.
- We want to compute them with some operator, $\circ$. I.e. we want:
  $$Total_{seq} = V_1 \circ V_2 \circ V_3 \circ \cdots \circ V_N$$
- I'll assume the sequential computation is left-to-right, so
  $$Total_{seq} = (\cdots((V_1 \circ V_2) \circ V_3) \circ \cdots) \circ V_N$$
- Using reduce, we do these operations in clusters of N/P, and then combine the results:

$$\begin{array}{l} Total_{reduce} \hfill = \\ \quad ((\cdots((V_1 \circ V_2) \circ V_3) \circ \cdots) \circ V_{N/P}) \\ \circ\, ((\cdots((V_{(N/P)+1} \circ V_{(N/P)+2}) \circ V_{(N/P)+3}) \circ \cdots) \circ V_{2N/P}) \\ \circ\, ((\cdots((V_{(2N/P)+1} \circ V_{(2N/P)+2}) \circ V_{(2N/P)+3}) \circ \cdots) \circ V_{3N/P}) \\ \ldots \\ \circ\, ((\cdots((V_{((P-1)N/P)+1} \circ V_{((P-1)N/P)+2}) \circ V_{((P-1)N/P)+3}) \circ \cdots) \circ V_N) \end{array}$$

- A sufficient condition is: $(X \circ Y) \circ Z = X \circ (Y \circ Z)$.
- What term describes an operator with this property?

Reduce is a Higher Order Function

- `reduce(WorkerTree, Leaf, Combine) -> Result`
  - `WorkerTree`: a collection of worker processes, organized as a tree.
    - "Organized" means each process knows who its parent and children processes are so it can send and receive the messages needed for reduce.
  - `Leaf`: a function – what to do at each leaf to produce a result to combine using reduce.
    - In our poetry example, `Leaf` finds the best poem in its subset of all poems submitted to the contest.
  - `Combine(Left, Right)`: a function – the operation to be applied at each node.
  - `Result`: the value computed at the root of the tree.

wtree:reduce

- Reduce as implemented in the CpSc 418 Erlang library.
- Example, `count3s` using `wtree:reduce`

```
count3s(WorkerTree, Key) ->
  wtree:reduce(WorkerTree,
    fun(ProcState) -> count3s_leaf(ProcState, Key) end,
    fun(Left, Right) -> count3s_combine(Left, Right) end
  ).

count3s_leaf(ProcState, Key) ->
  MyList = workers:get(ProcState, Key),
  length([E || E <- MyList, E =:= 3]).
count3s_combine(Left, Right) -> Left+Right.
```

- The code is available at reduce_intro.erl.

`count3s` notes (1/2)

- `WorkerTree`: a tree of workers.
  - To create a tree of `Nworker` workers, call `wtree:create(Nworkers)`.
  - When your done, you can clean up by calling `wtree:reap(WorkerTree)`.
- `ProcState`: Erlang is functional, how do workers remember anything?
  - Each worker executes a tail-recursive “get-a-task” function that is called with `ProcState` as a parameter.
  - This function does a `receive` to get a new `Task` (blocking if no task is ready).
    - ★ `NewProcState` = Task(ProcState).
    - ★ `Task` is called with `ProcState` as a parameter.
    - ★ `Task` returns an updated process state, `NewProcState`.
  - The worker recursively calls its get-a-task function, with `NewProcState` as the process state parameter.
  - `ProcState` is an Erlang key-list (i.e. a dictionary).

`count3s` notes (2/2)

- `workers:get(ProcState, Key)`: fetch the value for `Key` from `ProcState`. Called by a worker process.
- `NewProcState = workers:put(ProcState, Key, Value)`: create a new process-state where `Key` maps to `Value`, and all other mappings are the same as in `ProcState`. Called by a worker process.
- `workers:update(Workers, Key, ValueList)`: Called by the top-level process. `ValueList` should be a list with one element per worker. The elements of `ValueList` are stored in the `ProcState` of the corresponding workers with the key `Key`.
- `workers:retrieve(Workers, Key) -> ValueList`: fetch the value associate with `Key` for each worker in `Workers`.
  - Frequently, we have a list distributed across the workers. In this case, `workers:retrieve(Workers, Key)` returns a list of the form `[List1, List2, ...ListP]` where `P` is the number of workers, and `ListI` is the segement of the list held by worker `I`.
  - To merge these segements into one list:

    `lists:append(workers:retrieve(Workers, Key))`

Testing `count3s`

```
count3s_test(N_workers, N_values)
  when is_integer(N_workers), N_workers >= 0,
       is_integer(N_values), N_values >= 0 ->
  WorkerTree = wtree:create(N_workers),
  % create a random list of N_values integers chosen in [1, 10], distribute
  % it across the workers of WorkerTree and associate it with the key 'data'.
  workers:rlist(WorkerTree, N_values, 10, data),
  Par3s = count3s(WorkerTree, data),
  Data = lists:append(workers:retrieve(WorkerTree, data)),
  Seq3s = length([E || E <- Data, E == 3]),

  case Par3s =:= Seq3s of
    true ->
      io:format("passed:  N_values = ~b, Par3s = ~b~n",
                [N_values, Par3s]),
      ok;
    false ->
      io:format("failed:  N_values = ~b, Par3s = ~b, Seq3s = ~b~n",
                [N_values, Par3s, Seq3s]),
      fail
  end.
```

Preview

September 14: Reduce – The Pattern
Reading: Lin & Snyder, chapter 5, pp. 112–125

September 17: Scan
Homework: Homework 1 deadline for early-bird bonus (11:59pm)
Homework 2 goes out (due Oct. 1) – Reduce and Scan

September 19: Reduce & Scan Examples
Homework: Homework 1 due **11:59pm**

September 21 – 26: Parallel Architecture

Sept. 28 – Oct. 5: Performance Analysis

October 8 – 15: Sorting

October 17: Intro. to CUDA

October 19: Midterm review

October 22: Midterm

Oct. 24 – Nov. 30: Data Parallel Computing, GPUs, and CUDA