CpSc 418 **Homework 5** Due: Dec. 14, 2012, 11:59pm

Extra credit: 65 points

Please submit your solution using the handin program as: `cs418 hw5`
Your submission should consist of the following files:

`hw5.c` – C source (ASCII text). All functions requested in this assignment must be exported by this module.

`hw5.txt` – plain, ASCII text, or `hw5.pdf` – PDF.

1. **Count 3's with pthreads (40 points)**
   Write an implementation of count3's using pthreads on gambier. Illustrate each of the issues described in Lin and Snyder:
   (a) A race that results in the wrong answer. (**10 points**)
   (b) Excessive locking that results in poor performance. (**10 points**)
   (c) False sharing that results in poor performance. (**10 points**)
   (d) A good implementation that results in good speed-up. (**10 points**)

   Draw a plot of speed-up versus number of processors for the last three versions.

2. **Correctness of bitonic sort (25 points)**
   Let $x_0, x_1, \dots x_{n-1}$ be a bitonic sequence. Let $y_0, y_1, \dots y_{n-1}$ be defined by:

$$\begin{aligned} y_i &= \min(x_i, x_{i+\lceil n/2 \rceil}), && \text{if } i < (n-1)/2 \\ &= x_i, && \text{if } i = (n-1)/2 \\ &= \max(x_i, x_{i-\lceil n/2 \rceil}), && \text{if } i > (n-1)/2 \end{aligned}$$

   (a) (**5 points**) Show that the sequence $y_0, \dots y_{\lfloor (n-1)/2 \rfloor}$ is bitonic.
   (b) (**5 points**) Show that the sequence $y_{\lceil (n-1)/2 \rceil} \dots y_{n-1}$ is bitonic.
   (c) (**5 points**) Show that for every $i \in 0 \dots \lfloor (n-1)/2 \rfloor$ and every $j \in \lceil (n-1)/2 \rceil \dots (n-1)$, $y_i \leq y_j$.
   (d) (**5 points**) Given an intuitive explanation for how the results in the previous three sub-problems can be used to show that a bitonic merge produces a sorted output sequence from two input sequences that are sorted in opposite directions.
   (e) (**5 points**) Now that you've argued that bitonic merge is correct, explain why this makes the entire bitonic sorting algorithm correct.

   A few hints:

   - Use the 0-1 principle: you only need to consider sequences that consist only of 0's and 1's.
   - Consider two cases:
     - The total number of 1's is less than or equal to $n/2$.
     - The total number of 1's is greater than $n/2$.
   - All of the floor and ceiling stuff is to show that bitonic merge works whether n is even or odd. I also think it's kind of cool what it shows about the element in the middle when n is odd. On the other hand, if the floors and ceilings make you dizzy, then assume n is even, and then each of first three sub-parts will be worth 4 points instead of 5. In this case, you'll get:

$$\begin{aligned} y_i &= \min(x_i, x_{i+(n/2)}), && \text{if } i < n/2 \\ &= \max(x_i, x_{i-(n/2)}), && \text{if } i \geq n/2 \end{aligned}$$

     and you'll need to show:

(a) (**4 points**) the sequence $y_0, \dots y_{(n/2)-1}$ is bitonic.

(b) (**4 points**) the sequence $y_{n/2} \dots y_{n-1}$ is bitonic.

(c) (**4 points**) for every $i \in 0 \dots (n/2) - 1$ and every $j \in n/2 \dots (n-1)$, $y_i \leq y_j$.

(d) (**5 points**) same as above.

(e) (**5 points**) same as above.