The University of British Columbia
CPSC 414 – Midterm A Solution
Friday, Oct 26, 2001
Time: 60 min
Instructor: Wolfgang Heidrich

Student’s name:_________________________________________________________

Student number:________________________________________________________

Signature:________________________________________________________________

This examination consists of 13 pages, including this cover page. Check to ensure that this exam is complete.

This is a closed book examination. The weight of each question is given in parentheses. The total number of marks is 100. Start with the questions you think are easiest, and then go back and do the harder ones. Show all your work. Good Luck!

1. Each candidate should be prepared to produce his/her student ID on request.

2. No candidate shall be permitted to enter the room after the expiration of one half hour, or to leave during the first half hour of the examination.

3. No candidate shall be permitted to ask questions of the invigilators, except in the cases of supposed errors or ambiguities in the examination questions.

4. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
   - Having at the place of writing communication devices, any books, papers or memoranda, calculators, audio or visual cassette players, or other memory aid devices other than that specifically approved by the instructor.
   - Speaking or communicating with other candidates.
   - Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

<table>
<thead>
<tr>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
<th>Q6</th>
<th>Q7</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>13</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>15</td>
<td>15</td>
<td>100</td>
</tr>
</tbody>
</table>
1 2D Coordinate Systems (12 Points)

Describe the point $P$ in the following figure in terms of the coordinate systems given by the vectors $(x_i, y_i)$ and origins $Q_i$, for $i = 1 \ldots 4$. That is, determine $a_i$ and $b_i$ such that

$$P = a_i \cdot x_i + b_i \cdot y_i + Q_i,$$

By visual inspection (see figure above):

$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{pmatrix} 3/2 \\ -1 \end{pmatrix}, \quad \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = \begin{pmatrix} -1 \\ -6 \end{pmatrix}, \quad \begin{pmatrix} a_3 \\ b_3 \end{pmatrix} = \begin{pmatrix} -6 \\ 1 \end{pmatrix}$$

Coordinate system 4:

$$\begin{pmatrix} 5 \\ 8 \end{pmatrix} = b_4 \cdot \begin{pmatrix} -3 \\ 2 \end{pmatrix} + a_4 \cdot \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 7 \\ 1 \end{pmatrix}$$

Thus

$$3b_4 - 3a_4 = 2$$
$$2b_4 + a_4 = 7$$

$$\Rightarrow \begin{pmatrix} a_4 \\ b_4 \end{pmatrix} = \begin{pmatrix} 17/9 \\ 23/9 \end{pmatrix}$$
2 2D Affine Transformations (13 Points)

Derive the $3 \times 3$ homogeneous matrix that corresponds to the 2D affine transformation depicted in the following figure. **Show all your work!**

**Note:** remember that you are transforming homogeneous points, not vectors! Also, remember the special structure of a homogeneous matrix.

Before

![Before Image](image1)

After

![After Image](image2)

Need mapping for 3 points. E.g:

\[
\begin{pmatrix}
  m_{11} & m_{12} & m_{13} \\
  m_{21} & m_{22} & m_{23} \\
  0 & 0 & 1
\end{pmatrix}
\cdot
\begin{pmatrix}
  1 & 0 & 2 \\
  0 & 2 & 2 \\
  1 & 1 & 1
\end{pmatrix}
=
\begin{pmatrix}
  0 & 0 & 2 \\
  0 & 2 & 2 \\
  1 & 1 & 1
\end{pmatrix}.
\]

This yields 6 equations:

\[
\begin{align*}
  m_{11} + m_{13} &= 0 \\
  m_{21} + m_{23} &= 0 \\
  2m_{12} + m_{13} &= 0 \\
  2m_{22} + m_{23} &= 2 \\
  2m_{11} + 2m_{12} + m_{13} &= 2 \\
  2m_{21} + 2m_{22} + m_{23} &= 2
\end{align*}
\]

Solving yields

\[
M = \begin{pmatrix}
  1 & 1/2 & -1 \\
  0 & 1 & 0 \\
  0 & 0 & 1
\end{pmatrix}.
\]
3 Composite Transformations (12 Points)

Use the figures A-F of the L-shape for the questions below. Assume that the function `drawL()` renders the L-shape as depicted in Figure A. Suppose we have the following display function in our program:

```c
void redraw() {
    glClear( GL_COLOR_BUFFER_BIT );

    glMatrixMode( GL_MODELVIEW );
    glLoadIdentity();
    gluLookAt( 0.0, 0.0, 3.0, 0.0, 0.0, 0.0, 0.0, 1.0, 0.0 );

    drawStuff();

    glutSwapBuffers();
}
```

For each of the following definitions of the body of `drawStuff()`, indicate the figure that will result. For example:

- `drawL();`
  **Matching figure:** A
Here are the ones that you are supposed to find out. Use the word “none” to indicate that none of the figures A-F matches the code.

- `glRotatef( -90.0, 0.0, 0.0, 1.0 );
  drawL();
  Matching figure: NONE

- `glTranslatef( 2.0, 1.0, 0.0 );
  glScalef( 2.0, 2.0, 1.0 );
  drawL();
  Matching figure: E

- `glTranslatef( 3.0, 0.0, 0.0 );
  glRotatef( 45.0, 0.0, 0.0, 1.0 );
  drawL();
  Matching figure: D

- `glScalef( 2.0, 2.0, 1.0 );
  glTranslatef( 2.0, 1.0, 0.0 );
  drawL();
  Matching figure: F

- `glRotatef( 45.0, 0.0, 0.0, 1.0 );
  glTranslatef( 3.0, 0.0, 0.0 );
  drawL();
  Matching figure: C

- `glTranslatef( 2.0, 1.0, 0.0 );
  glScalef( 2.0, 2.0, 0.0 );
  drawL();
  Matching figure: NONE or E

- `glRotatef( 90.0, 0.0, 0.0, 1.0 );
  drawL();
  Matching figure: B
4 Hierarchical Transformations (15 Points)

Give an appropriate sequence of the following commands for drawing the scene depicted in the figure below. Assume that the current transformation matrix is initialized to the identity.

Commands (all transformation matrices are multiplied to current transformation matrix from the right):

- `drawSquare()`: draw a 1×1 square, bottom left corner at origin
- `translate( x, y )`: translation by x, y
- `scale( x, y )`: scaling in x and y direction
- `rotateZ( theta )`: rotation around z axis (i.e. in x-y plane)
- `pushMatrix()`: duplicates top entry of matrix stack
- `popMatrix()`: deletes top entry of matrix stack

More space is on the next page.
pushMatrix();
scale(3,2);
drawSquare();
popMatrix();

translate(3,2);
rotateZ(alpha);
translate(0,-2);
pushMatrix();
scale(5,2);
drawSquare();
popMatrix();

translate(5,2);
rotateZ(beta);
translate(0,-2);
pushMatrix();
scale(4,2);
drawSquare();
popMatrix();
5 Directed Edge Data Structure and Triangle Strips (18 Points)

a) For the triangle mesh in the following figure, draw the triangle strip starting at the half-edge indicated by the arrow, and ending at the mesh boundary. Make sure you consider the orientation of the triangles!

b) Assume that a triangle mesh like the one depicted above is given in terms of the directed edge data structure. Write a function `outputTriStrip( Edge *e )` that outputs (in sequence) all the vertices that belong to the triangle strip starting at `e`. The output should terminate when a boundary edge of the mesh is reached. You do not have to handle closed meshes and meshes of a topology that would cause the triangle strip to intersect itself.

You will need the following member functions of the `Edge` class:

- `Vertex *getStart()`
- `Vertex *getEnd()`
- `Edge *getPrev()`
- `Edge *getNext()`
- `Edge *getDual()`
- `Triangle *getFace()`

Also, you can use the function `outputVertex( Vertex *v )` for outputting a vertex in the triangle strip.
outputStrip( Edge *e ) {
  bool CCW = true;

  outputVertex( e->getStart() );
  outputVertex( e->getEnd() );

  while( e->getFace() != NULL ) {
    if( CCW )
      e = e->getDual()->getPrev()->getDual();
    else
      e = e->getNext();

    outputVertex( e->getEnd() );
    CCW = !CCW;
  }
}
For the following questions on the CIE diagram, use the figure above (with D as the “white point”). Attempt to find the values requested as accurately and precisely as possible. Work neatly! Marks will be deducted for sloppy work (even if you have the correct idea). **Show all your work.**
6.1 (3 marks)

Find the dominant wavelength of the three colors F, G, and H, with respect to D. If the color does not have a dominant wavelength, indicate this, and find the dominant wavelength of its complementary color. Write the wavelength values (to the nearest nanometer) in the spaces below.

<table>
<thead>
<tr>
<th></th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>554.3 nm</td>
<td>594.4 nm</td>
<td>537.4 nm</td>
</tr>
</tbody>
</table>

(no dominant wavelength)

6.2 (3 marks)

What is the chromaticity of the color J that is complementary to G (with respect to D), and has the same saturation (or purity) as G?

To two significant figures, excitation purity of G is 0.5, since it lies midway between points D and M on the diagram. Therefore J should lie at the midpoint between D and K (K is the point at which the line extended from G thru D hits the curved part of the horseshoe). To two significant figures, J lies at the chromaticity coordinates (0.18, 0.30).

The fact that J lies so close to the gamut boundary of Q, S, R, and H is a coincidence.

6.3 (4 marks)

Suppose you had a computer monitor with four phosphors, with the following characteristics:

- phosphor Q, monochromatic light at 470nm
- phosphor R, with a x-component of chromaticity equal to 0.4, and complementary to Q
- phosphor H, with chromaticity coordinates indicated on the CIE diagram
- phosphor S, maximally saturated color complementary to a pure color at 494nm

In the above, complementary means with respect to D.

Given these phosphors, draw the points corresponding to their chromaticities on the diagram given (H is already done for you). Also indicate the gamut of this display device.

See the diagram for the points Q, R, and S.

Gamut is indicated by the shaded region on the diagram. Note that point R lies entirely within the gamut of the other three colours. Therefore the gamut would be the same if phosphor R were not used.

6.4 (5 marks)

Suppose you have a light source, L, with chromaticity \((\frac{1}{3}, \frac{1}{2})\) and luminance\(^1\) 100. What are the \((X, Y, Z)\) values of a light source that, together with L, will give a combined light source with chromaticity \((\frac{2}{5}, \frac{2}{5})\) and luminance 200?

---

\(^1\)By luminance it is meant the luminous energy of the light source.
Let $W$ be the new light source that needs to be combined with $L$.

Recall that $Y$ corresponds to the luminance of a light source, and that the chromaticity is defined to be $(x, y, z) = (X, Y, Z)/(X + Y + Z)$ so that $x + y + z = 1$.

$L$ has chromaticity $(0.1, 0.5, 1.0 - 0.1 - 0.5) = (0.1, 0.5, 0.4)$ and a luminance of 100, so $(X, Y, Z)_L = (20, 100, 80)$.

The combined lights should have a chromaticity of $(0.4, 0.4, 1.0 - 0.4 - 0.4) = (0.4, 0.4, 0.2)$ and a luminance of 200, so $(X, Y, Z)_{L+W} = (200, 200, 100)$.

Therefore $(X, Y, Z)_W = (X, Y, Z)_{L+W} - (X, Y, Z)_L = (200, 200, 100) - (20, 100, 80) = (180, 100, 20)$. 

7 Line Clipping (15 Points)

a) Briefly describe the meaning of outcodes and of window-edge coordinates (WEC) in line clipping.

- Outcodes: 4 bits or flags for every one of the two vertices of a line. Each represents the relative position of the vertex with respect to one of the borders: 0 for in, 1 for out.
- WEC: the signed distance of a point with respect to a boundary edge. Positive: inside, negative: outside.

b) Outline the general principle of the $\alpha$-clipping algorithm for clipping lines to convex polygons.

- express the line as $L(\alpha) = v_1 + \alpha(v_2 - v_1)$
- maintain parameter value $\alpha_1$ for start point of line segment (initialized to 0)
- maintain parameter value $\alpha_2$ for end point of line segment (initialized to 1)
- adjust $\alpha_1, \alpha_2$ by clipping, in turn, every vertex against every boundary.
- if $\alpha_1 > \alpha_2$ then nothing is visible. Otherwise, $L(\alpha_1), L(\alpha_2)$ represents the clipped line segment.

c) Given two lines from $(-2, 3)$ to $(2, -2)$ and from $(-2, 3)$ to $(2, 2)$, write down the steps and the results of the $\alpha$-clipping algorithm for clipping these lines to the square $(-1, -1, 1, 1)$.

- $(-2, 3)$ to $(2, -2)$:
  - $\alpha_1$ set to intersection with top boundary
  - $\alpha_2$ set to intersection with right boundary
  - $\alpha_1 < \alpha_2 \Rightarrow L(\alpha_1), L(\alpha_2)$ defines clipped line segment.

- $(-2, 3)$ to $(2, 2)$:
  - $OC(v_1) \& OC(v_2) \neq 0 \Rightarrow$ trivial reject (top boundary).