THE UNIVERSITY OF BRITISH COLUMBIA
Midterm Examination – 8 Feb 1999

Computer Science 414 – Section 201
Introduction to Computer Graphics
Time: 45 minutes

Student’s Name: ______________________________________
(Please print in BLOCK letters, SURNAME first.)

Student Number: ____________________________

Signature: ____________________________

Instructor’s Name: Rob Scharein

This examination consists of 10 pages, including this cover page.
Check to ensure that this exam is complete.

This is a closed book examination.
The weight of each question is given in parentheses. The total number of marks is 100. Start with the questions you think are easiest, and then go back and do the harder ones. Show all your work. Good Luck!

1. Each candidate should be prepared to produce, upon request, his/her library card.

2. READ AND OBSERVE THE FOLLOWING RULES:

   • No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

   • No candidate shall be permitted to ask questions of the invigilators, except in the cases of supposed errors or ambiguities in examination questions.

   • CAUTION – Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

     (a) Having at the place of writing communication devices, any books, papers or memoranda, calculators, audio or visual cassette players, or other memory aid devices other than that specifically approved by the instructor.

     (b) Speaking or communicating with other candidates.

     (c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

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Question 1  (20 marks)

(a) Write the $4 \times 4$ three-dimensional transformation matrices for each of the following transformations. Assume a right-handed coordinate system with all of the usual conventions. Express all matrix elements as numeric values.

$R_x(90)$ — a rotation of $90^\circ$ about the $x$-axis.

$R_y(45)$ — a rotation of $45^\circ$ about the $y$-axis.

$R_z(60)$ — a rotation of $60^\circ$ about the $z$-axis.

$S_x(2)$ — a scaling factor of two along the $x$-axis and no change along the $y$- and $z$-axes.

$S_y\left(\frac{1}{2}\right)$ — a scaling factor of one half along the $y$-axis and no change along the $x$- and $z$-axes.

$T_x(-1)$ — a translation of negative one unit along the $x$-axis and no change along the $y$- and $z$-axes.

$T_{yz}(+1)$ — a translation of positive one unit along both the $y$- and $z$-axes and no change along the $x$-axis.
(b) Fill in the following table with “Yes” or “No” in each entry to indicate whether multiplying two $4 \times 4$ three-dimensional transformation matrices $A$ and $B$ results in two products in which $AB = BA$ if the matrices are of the prescribed types.

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For example, the entry in the second row and third column refers to whether the product of a three-dimensional translation matrix $A$ multiplied by a three-dimensional scaling matrix $B$ is commutative. Do not assume that the scaling is uniform in all directions.

(c) For each entry that you filled in with “No” in the second row of the table, provide an example of two matrices of the prescribed types chosen from the matrices in Part (a) of this question having the property that they do not commute. Demonstrate this by writing both of the products $AB$ and $BA$ of the two matrices you choose.

(d) How would your answer for part (b) of this question change if the matrices were all $3 \times 3$ two-dimensional transformation matrices instead of $4 \times 4$ three-dimensional transformation matrices?
Question 2  (5 marks)

Shade the following diagram according to the odd-parity rule.

(Note: just place an “x” in all the regions to be shaded if you’re short on time)

Question 3  (10 marks)

Suppose we have two vectors \( \mathbf{a} \) and \( \mathbf{b} \). Find the single rotation that transforms \( \mathbf{a} \) into \( \mathbf{b} \) (i.e. find the axis and angle of rotation). Draw a diagram.
Question 4  (10 marks)

This question tests your knowledge of the CIE chromaticity diagram shown below.

(a) Explain why it is not possible to reproduce all visible colours with only three primaries. The CIE diagram shows a typical gamut for a display device with three primaries.

(b) Indicate how the gamut shown in the CIE diagram above would be altered if a fourth primary were added that consisted of monochromatic light of 500nm wavelength.

(c) What is the small circle inside the triangle shown on the CIE diagram above and what does it represent?
Question 5  (40 marks — 2 marks each)

Explain the following terms as they have been used so far in our course on computer graphics. Where appropriate, include explanatory diagrams or formulae in your answers.

(a) aspect ratio (include a formula)

(b) `gluLookAt()`

(c) synthetic camera (include a diagram)

(d) quaternion

(e) homogeneous coordinates

(f) dominant wavelength
(g) `glVertex3f()`

(h) polling

(i) Snell’s Law (include a diagram and a formula)

(j) special orthogonal

(k) YIQ colour model

(l) one-point perspective

(m) Ivan Sutherland
(n) tristimulus theory

(o) non-spectral colour

(p) GLUT

(q) non-commutativity

(r) z-buffer

(s) viewport

(t) callback
Question 6  (15 marks)

In two dimensions, suppose that the line $L$ passes through the origin $(x, y) = (0, 0)$ and makes an angle $\theta$ with respect to the positive $x$-axis. Now suppose we apply a sequence of two reflections. The first reflection is about the $x$-axis, and this is followed by a second reflection about the line $L$. Prove that this sequence of two reflections is equivalent to a rotation about the origin by an angle $2\theta$ (include a diagram).
Extra space in which to work.