THE UNIVERSITY OF BRITISH COLUMBIA
Midterm Examination – 14 March 2001

Computer Science 414 – Section 201
Introduction to Computer Graphics

Time: 50 minutes

Student’s Name: ________________________________
(Please print in BLOCK letters, SURNAME first.)

Student Number: ______________________________

Signature: ______________________________

Instructor’s Name: Rob Scharein

This examination consists of 8 pages, including this cover page.
Check to ensure that this exam is complete.

This is a closed book examination.
The weight of each question is given in parentheses. The total number of marks is 50. Start with
the questions you think are easiest, and then go back and do the harder ones. Show all your work.
Good Luck!

1. Each candidate should be prepared to produce, upon request, his/her library card.

2. READ AND OBSERVE THE FOLLOWING RULES:

   • No candidate shall be permitted to ask questions of the invigilators, except in the cases
     of supposed errors or ambiguities in examination questions.

   • CAUTION – Candidates suspected of any of the following, or similar, dishonest prac-
     tices shall be immediately dismissed from the examination and shall be liable to disci-
     plinary action.

     (a) Having at the place of writing communication devices, any books, papers or mem-
         oranda, audio or video players, FAX machines, mobile radio telephones, electronic
         computational devices, microfiche readers, or other memory aid devices other than
         that specifically approved by the instructor.

     (b) Speaking or communicating with other candidates.

     (c) Purposely exposing written papers to the view of other candidates. The plea of
         accident or forgetfulness shall not be received.

   • The burning of any material is not permitted during examinations.

<table>
<thead>
<tr>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
<th>Q6</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>5</td>
<td>5</td>
<td>10</td>
<td>10</td>
<td>50</td>
</tr>
</tbody>
</table>

Page 1 of 8
Question 1  (10 marks)

Given the OpenGL fragment shown on the right, together with the notation

\[
S(a, b, c) = \begin{pmatrix}
a & 0 & 0 & 0 \\
0 & b & 0 & 0 \\
0 & 0 & c & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}, \quad T(r, s, t) = \begin{pmatrix}
1 & 0 & 0 & r \\
0 & 1 & 0 & s \\
0 & 0 & 1 & t \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\]

for scalings and translations, and

\[
R_x(\beta) = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \beta & -\sin \beta & 0 \\
0 & \sin \beta & \cos \beta & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\]

for rotations about the x-axis (and similar equations for \(R_y(\beta)\) and \(R_z(\beta)\), the rotations about the other coordinate axes), write down the value for the matrix \(A\) in terms of products of the matrices \(B, S, T, R_x, R_y,\) and \(R_z\) (you don’t have to work out the matrix products).
Question 2  (10 marks)

Given a conical helix defined by the parametric equations

\[ \mathbf{C}(t) = (t \cos t, t \sin t, t), \]

compute the Frenet frame \((\mathbf{T}, \mathbf{N}, \mathbf{B})\) along the curve at the parameter value \(t = 0\). Is the second derivative of the path \(\mathbf{C}''(t)\) perpendicular to \(\mathbf{T}\) at \(t = 0\)? If not, what is the angle between the two vectors?
Question 3  (5 marks)

Recall that two methods were used in Assignment #2 to create extruded surfaces, one method used the Frenet frame, and the other method used the parallel transport frame. Discuss the advantages and disadvantages of each method.

Question 4  (5 marks)

What does affine invariance of a Bézier curve mean? Explain why Bézier curves (of any order) are affine invariant.
Question 5  (10 marks)

Given four points $P_0$, $P_1$, $P_2$, and $P_3$, the cubic Bézier curve that interpolates $P_0$ and $P_3$ can be written as

$$P(t) = (1 - t)^3 P_0 + 3(1 - t)^2 t P_1 + 3(1 - t) t^2 P_2 + t^3 P_3, \text{ for } 0 \leq t \leq 1.$$ 

Choose any assignment of $P_0$, $P_1$, $P_2$, and $P_3$ to the four points shown below (as long as the assignment is 1-to-1) and show how you would construct $P(1/3)$ using the de Casteljau algorithm. Label the points clearly and show all your work. In addition to finding $P(1/3)$ precisely, sketch an approximation to the complete Bézier curve.
Question 6  (10 marks)

Consider the paths
\[ \gamma(t) = (t, t^2) \quad \text{and} \quad \eta(t) = (2t + 1, t^3 + 4t + 1), \]
both defined on the interval \(0 \leq t \leq 1\). The curves join, since \(\gamma(1) = (1, 1) = \eta(0)\). Show that they meet with \(G^1\) continuity, but not with \(C^1\) continuity.
Extra space in which to work
More extra space in which to work