CPSC 340:
Machine Learning and Data Mining

Data Exploration

This lecture roughly follows:
http://www-users.cs.umn.edu/~kumar/dmbook/dmslides/chap2_data.pdf

Admin

- Assignment 1 is due next Friday: start early, work often
- Tutorials & office hours start next week (times on course webpage)
- Make sure to scan the FAQ
  - E.g. for Auditing requirements

Data Mining: Bird's Eye View

1) Collect data.

2) Data mining!

3) Profit?

Unfortunately, it's often more complicated…

Data Mining: Some Typical Steps

1) Learn about the application.
2) Identify data mining task.
3) Collect data.
4) Clean and preprocess the data.
5) Transform data or select useful subsets.
6) Choose data mining algorithm.
7) Data mining!
8) Evaluate, visualize, and interpret results.
9) Use results for profit or other goals.

(often, you'll go through cycles of the above)

Why Are So Many Data Scientists Quitting Their Jobs?

Data Mining: Some Typical Steps

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2) Identify data mining task.

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9) Use results for profit or other goals.

   (often, you'll go through cycles of the above)

Why Are So Many Data Scientists Quitting Their Jobs?

What is Data?

- We'll define data as a collection of examples, and their features.

Age	Job?	City	Rating	Income
23	Yes	Van	A	22,000.00
23	Yes	Bur	BBB	21,000.00
22	No	Van	CC	0.00
25	Yes	Sur	AAA	57,000.00
19	No	Bur	BB	13,500.00
22	Yes	Van	A	20,000.00
21	Yes	Ric	A	18,000.00

"feature"

"example"

- Each row is an "example", each column is a "feature".
  - Examples are also sometimes called "samples" or data points

Types of Data

- Categorical features come from an unordered set:
  - Binary: job? {yes, no} or {1,0}
  - Nominal: city. {Vancouver, Burnaby, Surrey}

- Numerical features come from ordered sets:
  - Counts like age in {0, 1, 2, 3,...}
  - Ordinal like ratings in {best (1), good (2) neutral (3), bad (4), worst (5)}
  - Continuous/real-valued like height in {173.5, 162.4, 190.2,...}

- Q: How could we convert categorical into numerical features?

Converting to Numerical Features

- Often want a real-valued example representation:

Age	City	Income
23	Van	22,000.00
23	Bur	21,000.00
22	Van	0.00
25	Sur	57,000.00
19	Bur	13,500.00
22	Van	20,000.00

$\longrightarrow$

Age	Van	Bur	Sur	Income
23	1	0	0	22,000.00
23	0	1	0	21,000.00
22	1	0	0	0.00
25	0	0	1	57,000.00
19	0	1	0	13,500.00
22	1	0	0	20,000.00

- This is called a "1 of k" or "one hot" encoding.
- We can now interpret examples as points in space:
  – E.g., first example is at (23,1,0,0,22000).

Data "Space"

Age	Van	Bur	Sur	Income
23	1	0	0	22,000.00
23	0	1	0	21,000.00
22	1	0	0	0.00
25	0	0	1	57,000.00
19	0	1	0	13,500.00
22	1	0	0	20,000.00

Examples or "datapoints"
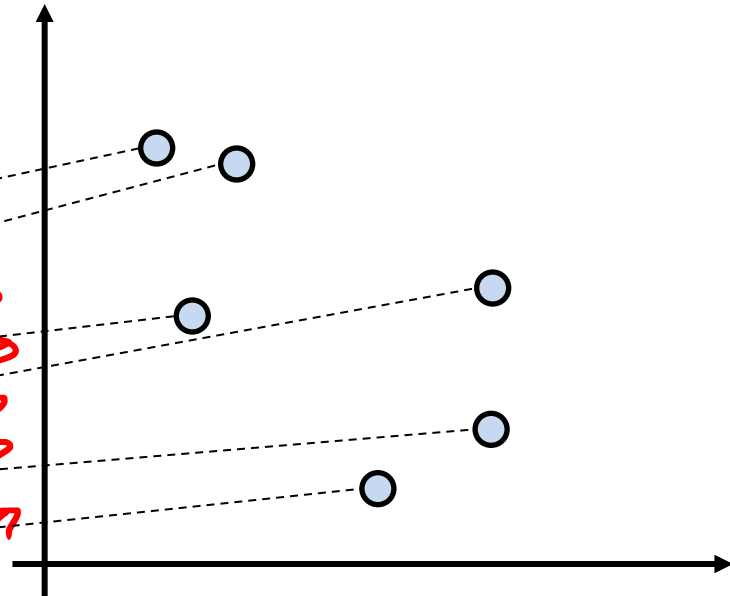
"feature space"

- You can compute a "distance" between examples in feature space.
  - "Are these examples close to each other?"

Approximating Text with Numerical Features

- Bag of words replaces document by word counts:

> The **International Conference on Machine Learning** (ICML) is the leading international academic conference in machine learning

ICML	International	Conference	Machine	Learning	Leading	Academic
1	2	2	2	2	1	1

- Ignores order, but often captures general theme.
- You can compute a "distance" between documents.
  - To find similar documents, or decide if two documents are similar.

Approximating Images and Graphs

- We can think of other data types in this way:
  - Images:

graycale intensity →

(1,1)	(2,1)	(3,1)	...	(m,1)	...	(m,n)
45	44	43	...	12	...	35

  - Graphs:
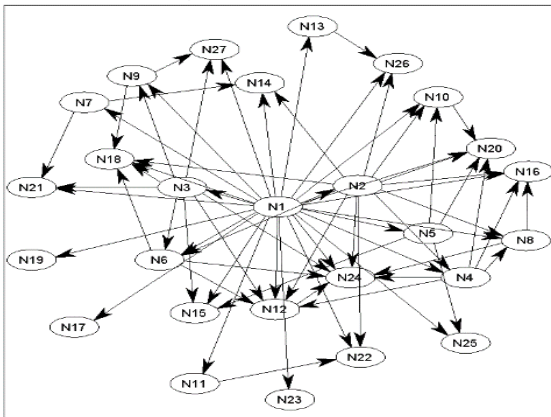
adjacency matrix →

N1	N2	N3	N4	N5	N6	N7
0	1	1	1	1	1	1
0	0	0	1	0	1	0
0	0	0	0	0	1	0
0	0	0	0	0	0	0

Data Cleaning

- ML+DM typically assume 'clean' data.
- Ways that data might not be 'clean':
  - Noise (e.g., distortion on phone).
  - Outliers (e.g., data entry or instrument error).
  - Missing values (no value available or not applicable)
  - Duplicated data (repetitions, or different storage formats).
- Any of these can lead to problems in analyses.
  - Want to fix these issues, if possible.
  - Some ML methods are robust to these.
  - Often, ML is the best way to detect/fix these.

How much data do we need?

- A difficult if not impossible question to answer.

- My usual answer: "more is better".
  - With the warning: "as long as the quality doesn't suffer".

- Another popular answer: "ten times the number of features".

A Simple Setting: Coupon Collecting

- Assume we have a categorical variable with 50 possible values:
  - {Alabama, Alaska, Arizona, Arkansas,...}.
- Assume each category has probability of 1/50 of being chosen:
  - How many examples do we need to see before we expect to see them all?
- Expected value is ~225.
- Coupon collector problem: O(n log n) in general.
  - Gotta Catch'em all!

- Obvious sanity check: need more samples than categories:
  - Situation is worse if they don't have equal probabilities.
  - Typically want to see categories more than once to learn anything.
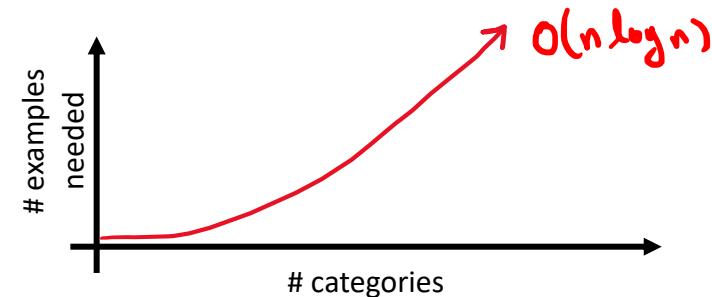
Feature Aggregation

- **Feature aggregation**:
  - Combine features to form new features:

Van	Bur	Sur	Edm	Cal
1	0	0	0	0
0	1	0	0	0
1	0	0	0	0
0	0	0	1	0
0	0	0	0	1
0	0	1	0	0

BC	AB
1	0
1	0
1	0
0	1
0	1
1	0

- Fewer province "coupons" to collect than city "coupons".

Feature Transformation

- Mathematical transformations:
  - Discretization (binning): turn numerical data into categorical.

Age
23
23
22
25
19
22

< 20	>= 20, < 25	>= 25
0	1	0
0	1	0
0	1	0
0	0	1
1	0	0
0	1	0

- Only need to collect 3 coupons.
  - We will see many more transformations (addressing other problems).

Feature Selection

- Feature Selection:
  - Remove features that are not relevant to the task.

SID:	Age	Job?	City	Rating	Income
3457	23	Yes	Van	A	22,000.00
1247	23	Yes	Bur	BBB	21,000.00
6421	22	No	Van	CC	0.00
1235	25	Yes	Sur	AAA	57,000.00
8976	19	No	Bur	BB	13,500.00
2345	22	Yes	Van	A	20,000.00

  - Student ID is probably not relevant (do not need to collect these coupons).

Next Topic: Supervised Learning

Motivating Example: Food Allergies

- You frequently start getting an upset stomach
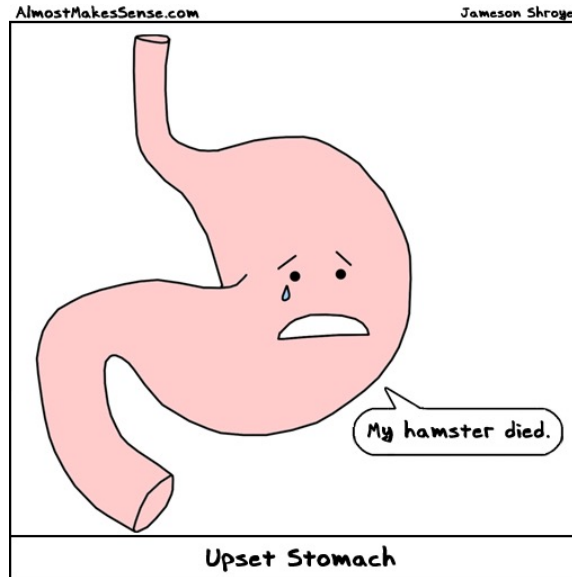
- You suspect an adult-onset food allergy.
- Q: How can I find out whether my meal will make me sick?

Motivating Example: Food Allergies

- To solve the mystery, you start a food journal:
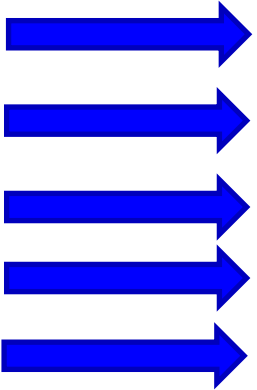
Egg	Milk	Fish	Wheat	Shellfish	Peanuts	...	Sick?
0	0.7	0	0.3	0	0		1
0.3	0.7	0	0.6	0	0.01		1
0	0	0	0.8	0	0		0
0.3	0.7	1.2	0	0.10	0.01		1
0.3	0	1.2	0.3	0.10	0.01		1

- But it's hard to find the pattern:
  - You can't isolate and only eat one food at a time.
  - You may be allergic to more than one food.
  - The quantity matters: a small amount may be ok.
  - You may be allergic to specific interactions.

Supervised Learning

- We can formulate this as supervised learning:

Egg	Milk	Fish	Wheat	Shellfish	Peanuts	...		Sick?
0	0.7	0	0.3	0	0		⟹	1
0.3	0.7	0	0.6	0	0.01		⟹	1
0	0	0	0.8	0	0		⟹	0
0.3	0.7	1.2	0	0.10	0.01		⟹	1
0.3	0	1.2	0.3	0.10	0.01		⟹	1
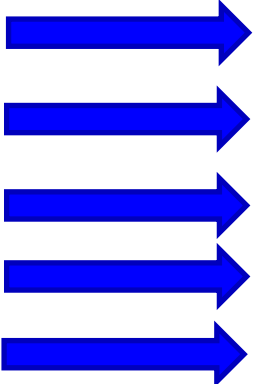
- Input for an example (day of the week) is a set of features (quantities of food).
- Output is a desired class label (whether or not we got sick).
- Goal of supervised learning:
  - Use data to find a model that outputs the right label based on the features.
  - Model predicts whether foods will make you sick (even with new combinations).

Supervised Learning

- General supervised learning problem:
  - Take features of examples and corresponding labels as inputs.
  - Find a model that can accurately **predict the labels of new examples**.

- This is the most successful machine learning technique:
  - Spam filtering, optical character recognition, speech recognition, object detection, classifying tumours, machine translation, and so on.

- We'll first focus on categorical labels, which is called "classification".
  - The model is a called a "classifier".

Naïve Supervised Learning: "Predict Mode"

Egg	Milk	Fish	Wheat	Shellfish	Peanuts	...
0	0.7	0	0.3	0	0	
0.3	0.7	0	0.6	0	0.01	
0	0	0	0.8	0	0	
0.3	0.7	1.2	0	0.10	0.01	
0.3	0	1.2	0.3	0.10	0.01	

Sick?
1
1
0
1
1

- A very naïve supervised learning method:
  - Count how many times each label occurred in the data (4 vs. 1 above).
  - Always predict the most common label, the "mode" ("sick" above).
    - This model is 80% accurate on the dataset above.
- But it ignores the features, so is only accurate if we only have 1 label.
- We want to use the features, and there are MANY ways to do this.
  - Next time we will consider a classic way known as decision tree learning.

Next Topic: Summary Statistics

Exploratory Data Analysis

- You should always "look" at the data first.

- But how do you "look" at features and high-dimensional examples?
  - Summary statistics.
  - Visualizations.
  - ML + DM (later in course).

Categorical Summary Statistics

- Summary statistics for a categorical feature:
  - Frequencies of different classes.
  - Mode: category that occurs most often.
  - Quantiles: categories that occur more than t times.

Population by year, by province and territory (Number)

	2014
Canada	35,540.4
Newfoundland and Labrador	527.0
Prince Edward Island	146.3
Nova Scotia	942.7
New Brunswick	753.9
Quebec	8,214.7
Ontario	13,678.7
Manitoba	1,282.0
Saskatchewan	1,125.4
Alberta	4,121.7
British Columbia	4,631.3
Yukon	36.5
Northwest Territories	43.6
Nunavut	36.6

Frequency: 13.3% of Canadian residents live in BC.
Mode: Ontario has largest number of residents (38.5%)
Quantile: 6 provinces have more than 1 million people.

Continuous Summary Statistics

- Measures of location for continuous features:
  - Mean: average value.
  - Median: value such that half points are larger/smaller.
  - Quantiles: value such that 'k' fraction of points are larger.

- Measures of spread for continuous features:
  - Range: minimum and maximum values.
  - Variance: measure of how far values are from mean.
    - Square root of variance is "standard deviation".
  - Intequantile ranges: difference between quantiles.

Continuous Summary Statistics

- Data: [0 1 2 3 3 5 7 8 9 10 14 15 17 200]
- Measures of location:
  - Mean(Data) = 21
  - Mode(Data) = 3
  - Median(Data) = 7.5
  - Quantile(Data,0.5) = 7.5
  - Quantile(Data,0.25) = 3
  - Quantile(Data,0.75) = 14
- Measures of spread:
  - Range(Data) = [0 200].
  - Std(Data) = 51.79
  - IQR(Data,.25,.75) = 11
- Notice that mean and std are more sensitive to extreme values ("outliers").

"outlier"

Entropy as Measure of Randomness

- Another common summary statistic is entropy.
  - Entropy measures "randomness" of a set of variables.
    - Roughly, another measure of the "spread" of values.
    - Formally, "how many bits of information are encoded in the average example".

  - For a categorical variable that can take 'k' values, entropy is defined by:
$$\text{entropy} = -\sum_{c=1}^{k} p_c \log p_c$$
    where p_c is the proportion of times you have value 'c'.

  - Minimum value is 0 (no randomness).
    - We use the convention that $0 \log 0 = 0$.
  - Maximum value is log(k).

Entropy as Measure of Randomness

Low entropy means "very predictable"   High entropy means "very random"
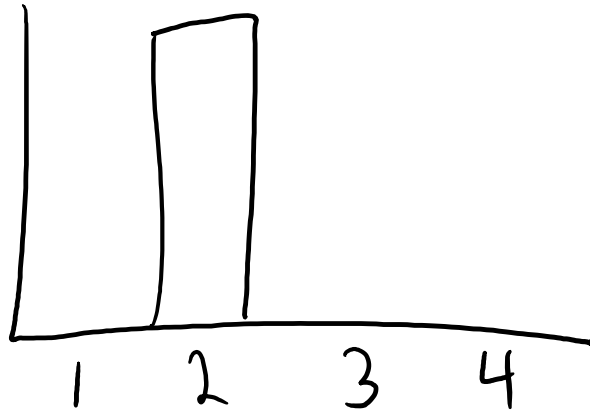
- For categorical features: uniform distribution has highest entropy.
- For continuous densities with fixed mean and variance:
  - Normal distribution has highest entropy (not obvious).
- Entropy and Dr. Seuss (words like "snunkoople" increase entropy).

Distances and Similarities

- There are also summary statistics between features 'x' and 'y'.
  - Hamming distance:
    - Number of elements in the vectors that aren't equal.
  - Euclidean distance:
    - How far apart are the vectors?
  - Correlation:
    - Does one increase/decrease linearly as the other increases?
    - Between -1 and 1.

x	y
0	0
0	0
1	0
0	1
0	1
1	1
0	0
0	1
0	1

Distances and Similarities

- There are also summary statistics between features 'x' and 'y'.
  - Rank correlation:
    - Does one increase/decrease as the other increases?
      - Not necessarily in a linear way.

- Distances/similarities between other types of data:
  - Jaccard coefficient (distance between sets):
    - (size of intersection of sets) / (size of union of sets)
  - Edit distance (distance between strings):
    - How many characters do we need to change to go from x to y?
    - Computed using dynamic programming (CPSC 320).

x	y
0	0
0	0
1	0
0	1
0	1
1	1
0	0
0	1
0	1

Next Topic: Visualizing Data

Limitations of Summary Statistics

- On their own summary statistic can be misleading.

- [Why not to trust statistics](#)

- Anscombe's quartet:
  - Almost same means.
  - Almost same variances.
  - Almost same correlations.
  - Look completely different.

- [Datasaurus dozen](#).
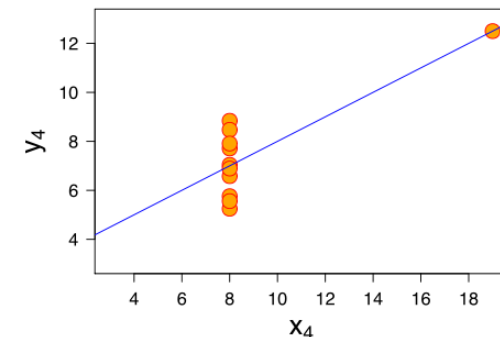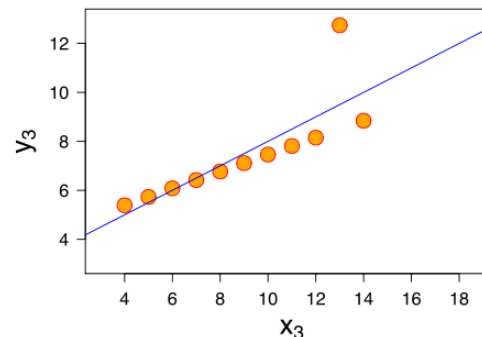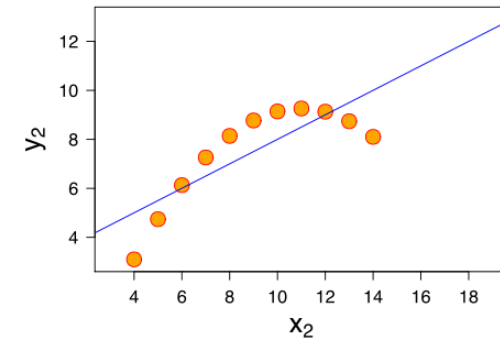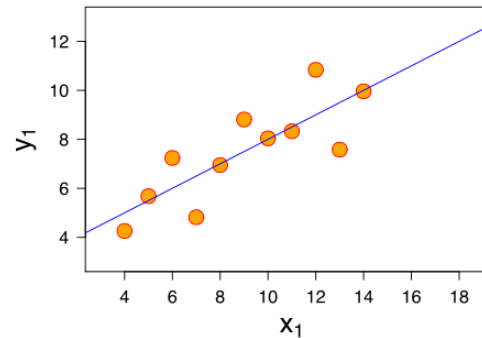
Links to figure sources will be here.

Visualization

- You can learn a lot from 2D plots of the data:
  – Patterns, trends, outliers, unusual patterns.

Lat	Long	Temp
0	0	30.1
0	1	29.8
0	2	29.9
0	3	30.1
0	4	29.9
…	…	…

vs.

Annual Mean Temperature
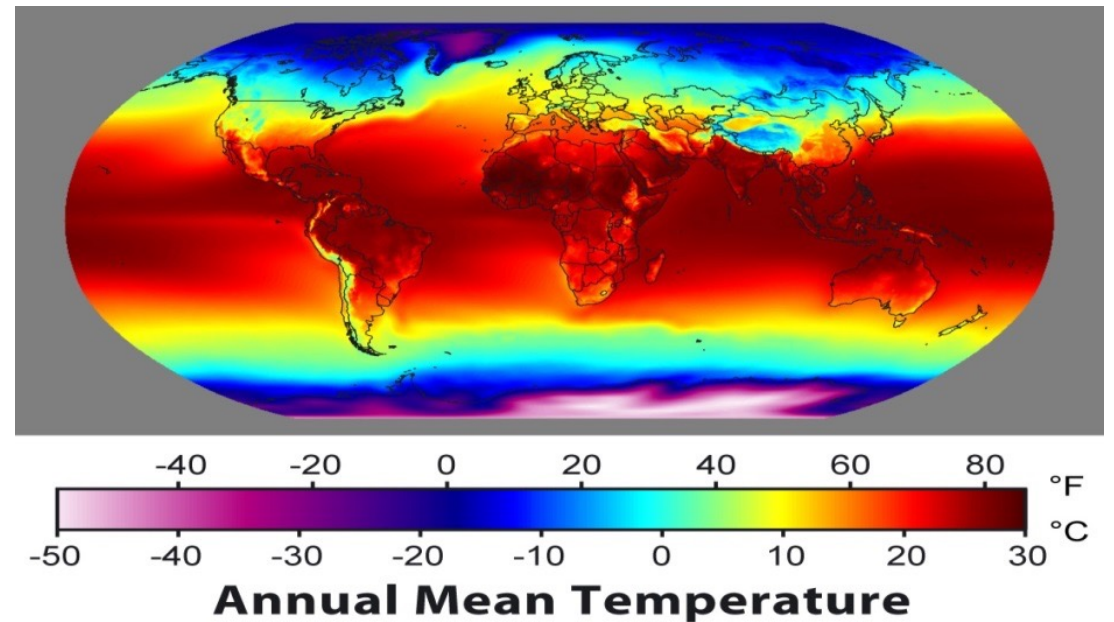
Basic Plots: Lines, Histograms, Box Plots, Heatmaps

Line plots visualize one variable as a function of another.
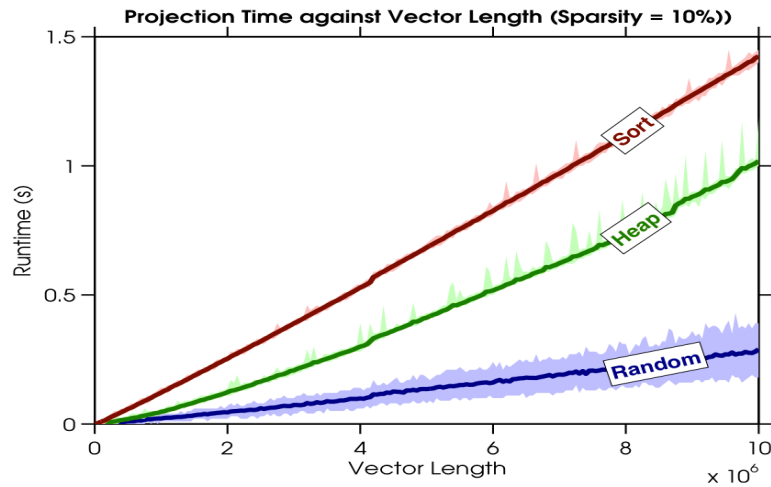
Histograms display counts of a variable, split into "bins".
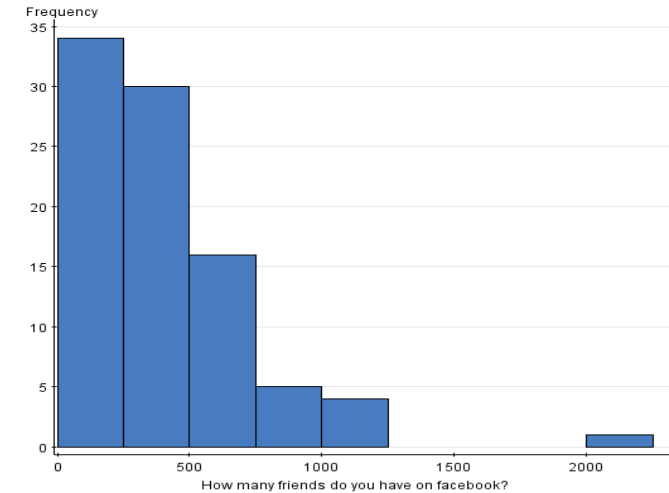
Box plots visualize spread of continuous variables.
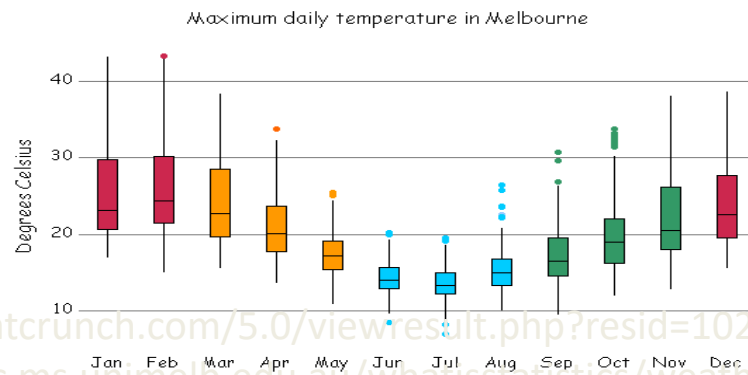
Heatmaps visualize table as an image (sometimes show trends):
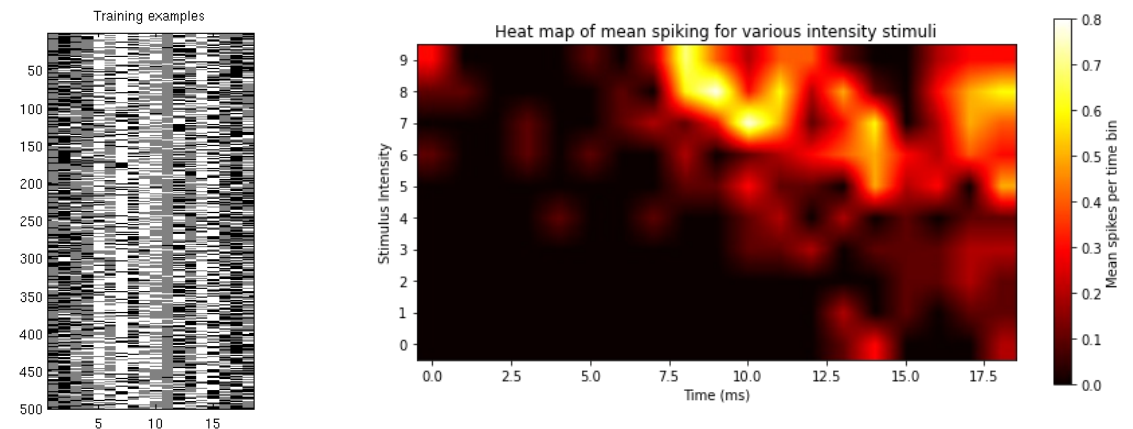
Histogram

- "[Four Basic Data Science Lessons Illustrated by COVID-19 Data](#)"
  - First two lessons come from just plotting:
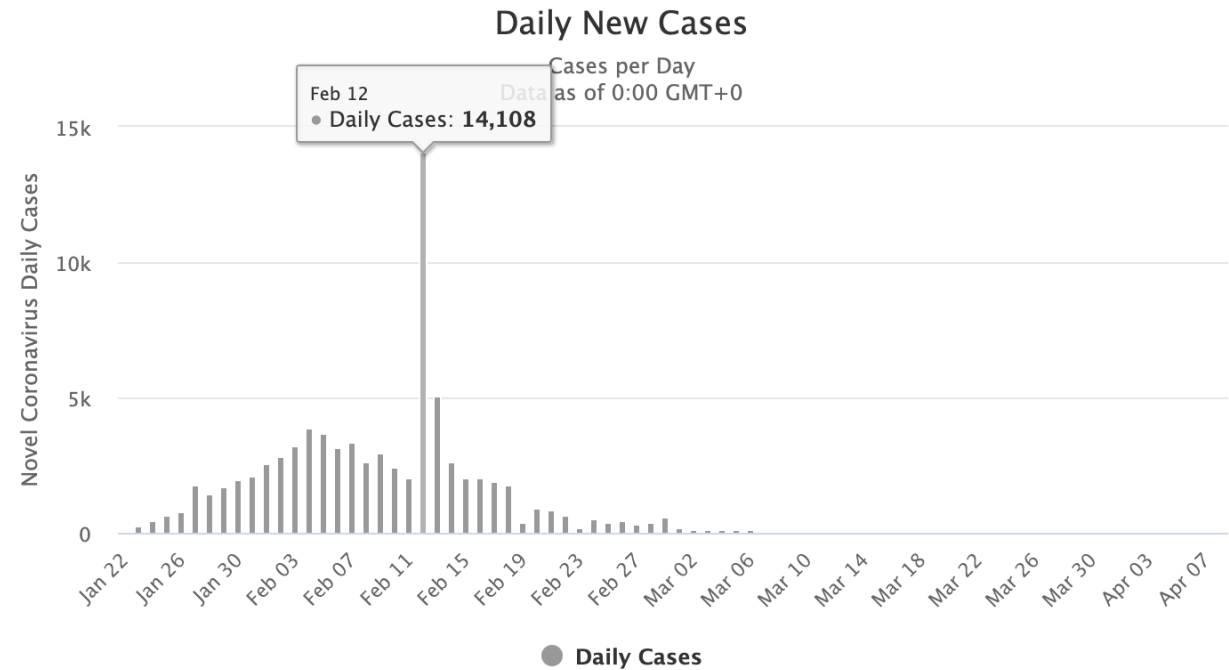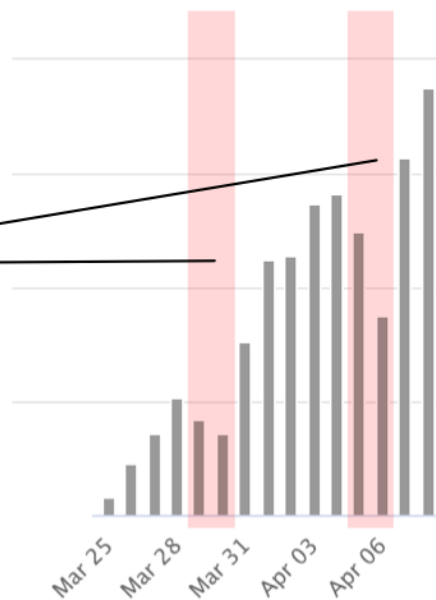
  - Plots easily show these oddities (had to do with how data was recorded.)

Box Plot

- Photo from CTV Olympic coverage in 2010 to show spread:

Measuring Similarites using Heatmaps

- Often use heatmaps to visualize feature similarities (or differences):
  - Visualize all similarity among all pairs:

	BTC	ETH	XRP	XEM	ETC	LTC	DASH	XMR
BTC	1.00	0.61	0.36	0.51	0.60	0.56	0.55	0.66
ETH	0.61	1.00	0.28	0.49	0.68	0.43	0.70	0.64
XRP	0.36	0.28	1.00	0.48	0.08	0.35	0.40	0.44
XEM	0.51	0.49	0.48	1.00	0.40	0.43	0.47	0.52
ETC	0.60	0.68	0.08	0.40	1.00	0.47	0.56	0.53
LTC	0.56	0.43	0.35	0.43	0.47	1.00	0.59	0.67
DASH	0.55	0.70	0.40	0.47	0.56	0.59	1.00	0.74
XMR	0.66	0.64	0.44	0.52	0.53	0.67	0.74	1.00

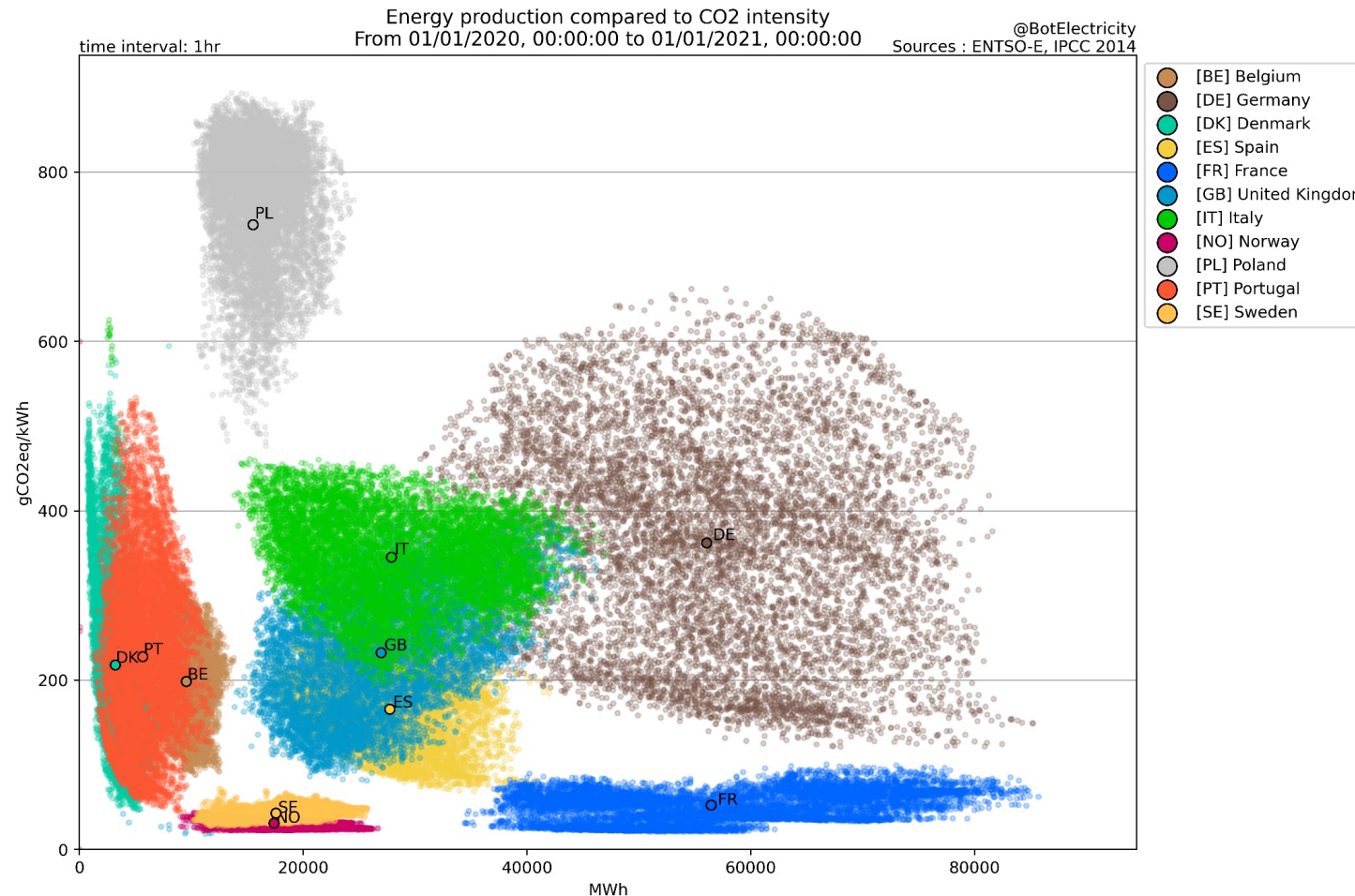"Correlation plot"

Scatterplot

- Look at distribution of two features:
  - Feature 1 on x-axis.
  - Feature 2 on y-axis.
  - Basically a "plot without lines" between the points.
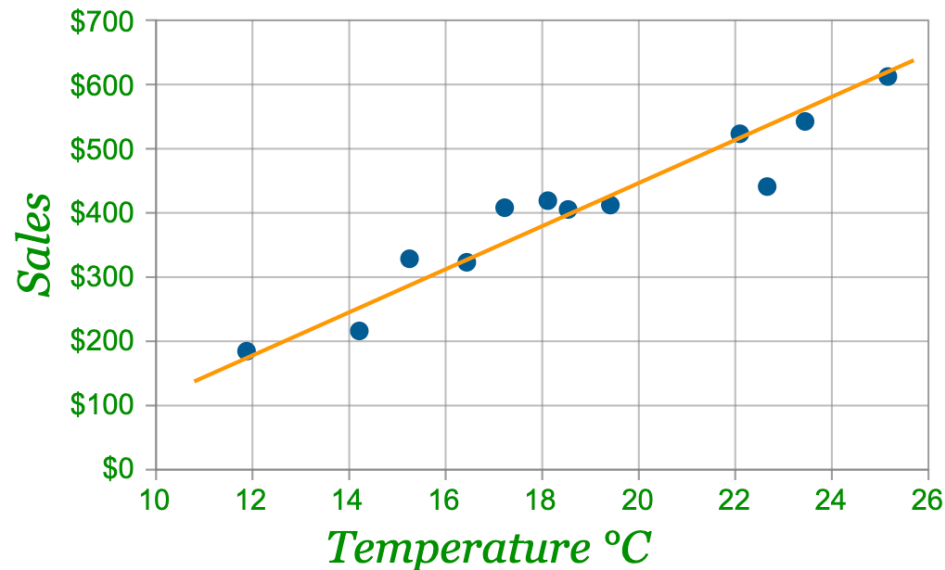
- Shows correlation between "personality" score and "looks" score.

Scatterplot

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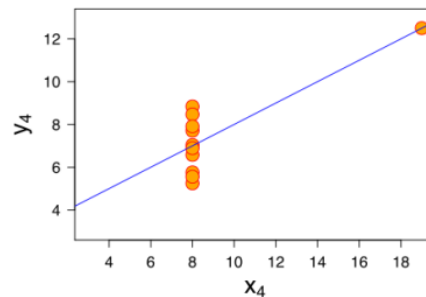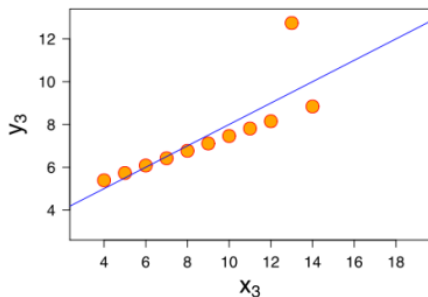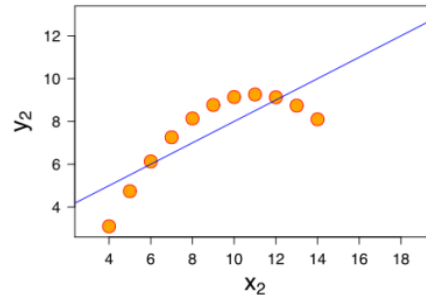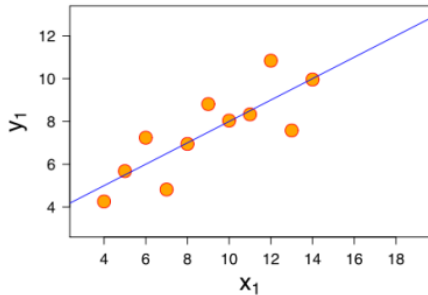
- Shows correlation between "personality" score and "looks" score.
- But scatterplots let you see more complicated patterns.
  - These four plots all have the ~same mean (x and y), variance (x and y), correlation, R^2, etc.

Coloured Scatterplot

- You can add colour to display a 3rd variable:

Energy production compared to CO2 intensity
From 01/01/2020, 00:00:00 to 01/01/2021, 00:00:00

time interval: 1hr

@BotElectricity
Sources : ENTSO-E, IPCC 2014

Legend:
- [BE] Belgium
- [DE] Germany
- [DK] Denmark
- [ES] Spain
- [FR] France
- [GB] United Kingdom
- [IT] Italy
- [NO] Norway
- [PL] Poland
- [PT] Portugal
- [SE] Sweden

Scatterplot Arrays

- For multiple variables, you can use scatterplot arrays.

https://en.wikipedia.org/wiki/Iris_flower_data_set
http://www.ats.ucla.edu/stat/r/pages/layout.htm

Scatterplot Arrays

- For multiple variables, you can use scatterplot arrays.

Scatterplot Arrays

- For multiple variables, you can use scatterplot arrays.

"Why Not to Trust Plots"

- We've seen how summary statistics can be mis-leading.

- Note that plots can also be mis-leading, or can be used to mis-lead.

- Next slide: first example from UW's excellent course:
  - "Calling Bullshit in the Age of Big Data".
    - A course on how to recognize when people are trying to mis-lead you with data.
  - I recommend watching all the videos here:
    - https://www.youtube.com/watch?v=A2OtU5vlR0k&list=PLPnZfvKID1Sje5jWxt-4CSZD7bUI4gSPS

  - Recognizing BS not only useful for data analysis, but for daily life.

Mis-Leading Axes

- This plot seems to show amazing recent growth:

Explosive growth at callingbullshit.org
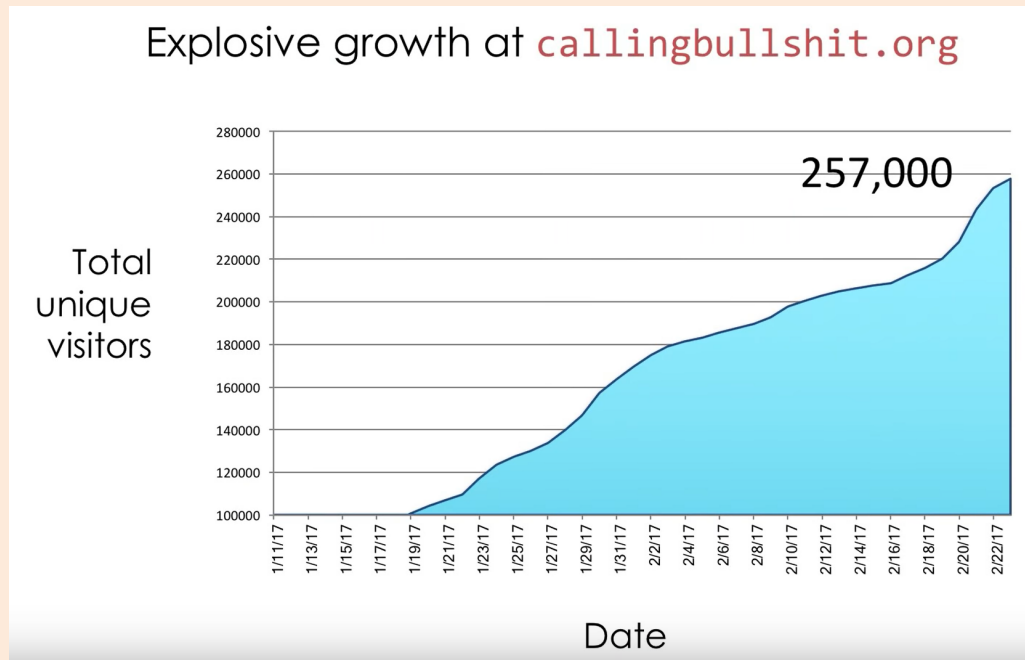
- But notice y-axis starts at 100,000 (so ~40% of growth was earlier).
- And it plots "total" users (which necessarily goes up).

Mis-Leading Axes

- Plot of actual daily users (starting from 0) looks totally different:

Explosive growth at callingbullshit.org ?

Unique daily visitors / Date

start of other plot's data

Cumulative iPhone sales

- People can mis-lead to push agendas/stories:

Mis-Leading Axes

Average Female Height per country
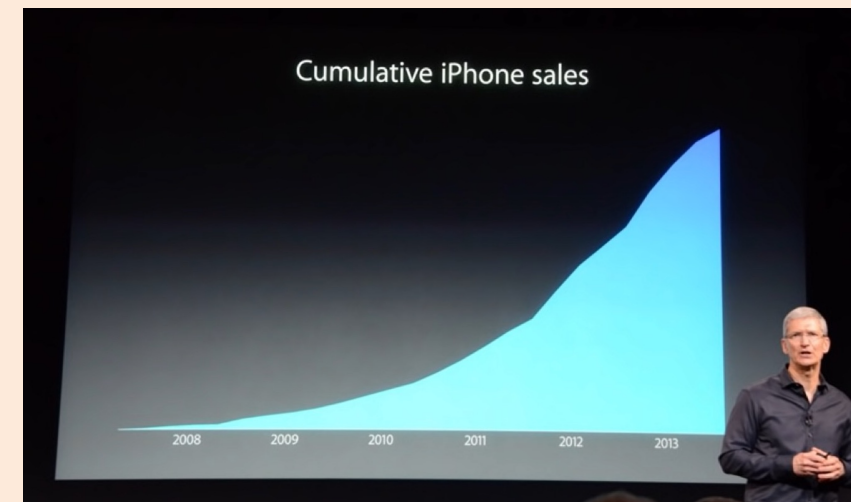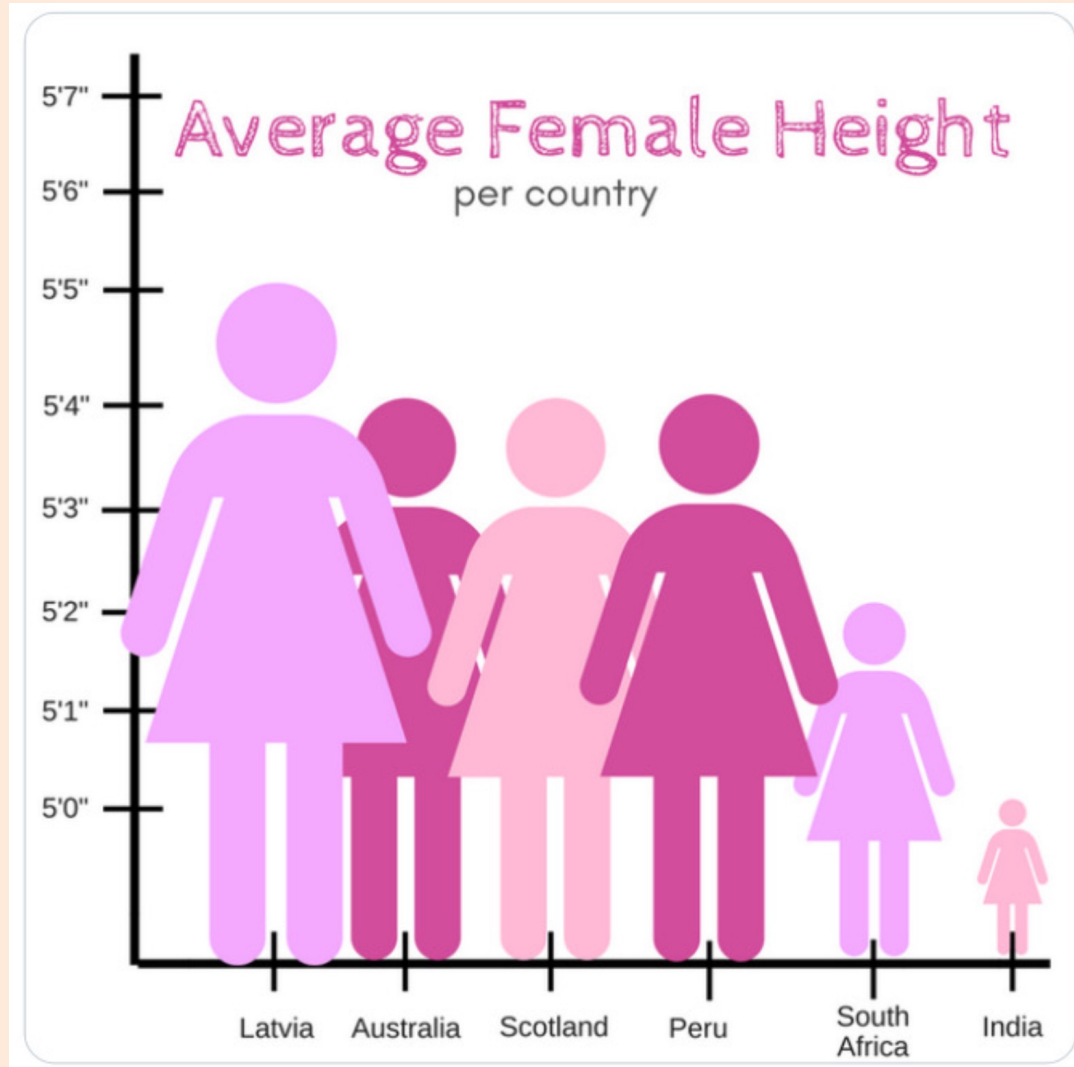
Mis-Leading Axes

- We see "lack of appropriate axes" ALL THE TIME in the news:
  - "British research revealed that patients taking ibuprofen to treat arthritis face a 24% increased risk of suffering a heart attack"
    - What is probability of heart attack if I you don't take it? Is that big or small?
    - Actual numbers: less than 1 in 1000 "extra" heart attacks vs. baseline frequency.
      - There is a risk, but "24%" is an exaggeration.
    - "Health-scare stories often arise because their authors simply don't understand numbers."
      - Or it could be that they do understand, but media wants to "sensationalize" mundane news.

  - Bonus slides: more "Calling Bullshit" examples on "political" issues:
    - Global warming, vaccines, gun violence, taxes.

Summary

- Typical data mining steps:
  - Involves data collection, preprocessing, analysis, and evaluation.
- Example-feature representation and categorical/numerical features.
  - Transforming non-vector examples to vector representations.
- Feature transformations:
  - To address coupon collecting or simplify relationships between variables.
- Exploring data:
  - Summary statistics and data visualization.
- Supervised learning:
  - Using data to write a program based on input/output examples.

- Post-lecture bonus slides: other visualizations, parallel/distributed calculations.

- Next class: let's start some machine learning…

# The last slide was the end of the lecture. (The lectures end on a "Summary" slide.)

The slides after the "Summary" slide are typically more "bonus" material related to the topics of the lecture.

Data Cleaning and the Duke Cancer Scandal

- See the Duke cancer scandal:
  - http://www.nytimes.com/2011/07/08/health/research/08genes.html?_r=2&hp

- Basic sanity checks for data cleanliness show problems in these (and many other) studies:
  - E.g., flipped labels, off-by-one mistakes, switched columns etc.
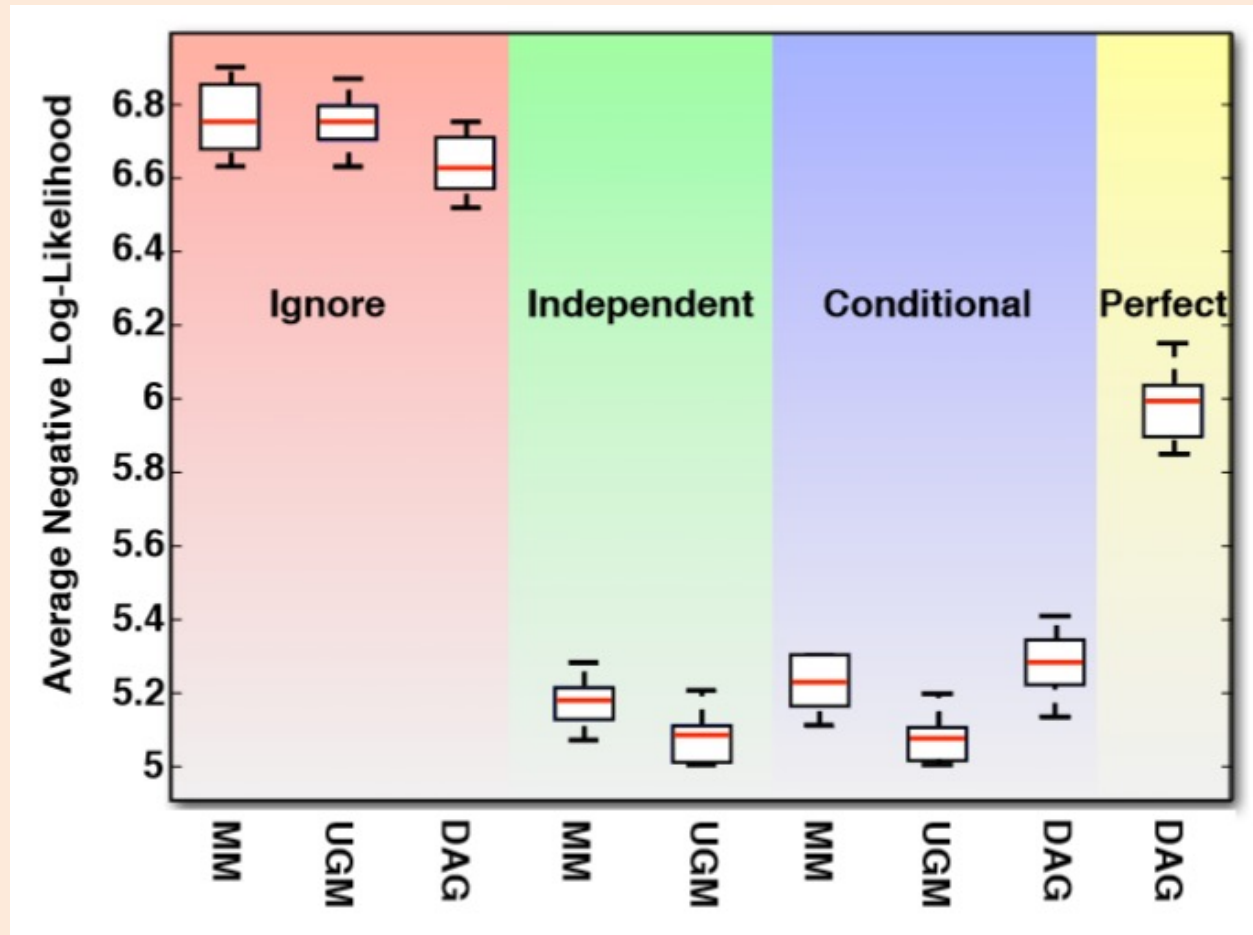 - https://arxiv.org/pdf/1010.1092.pdf
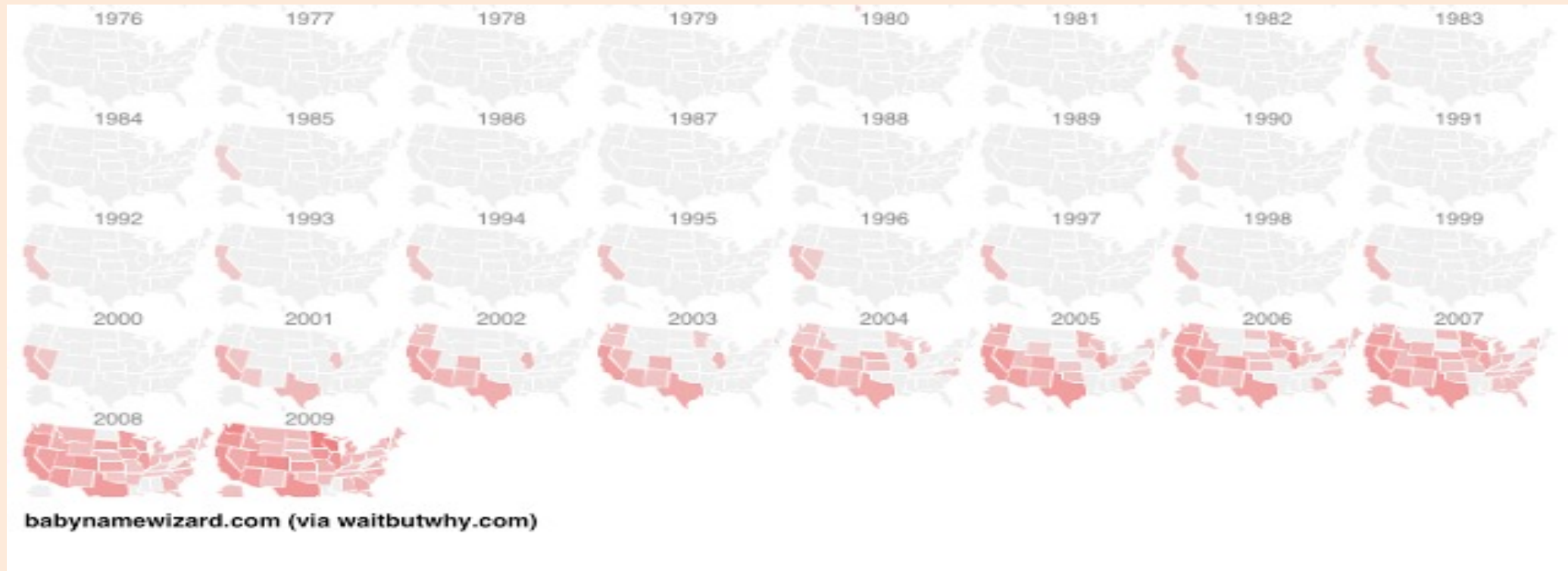
Histogram

- Histogram with grouping:
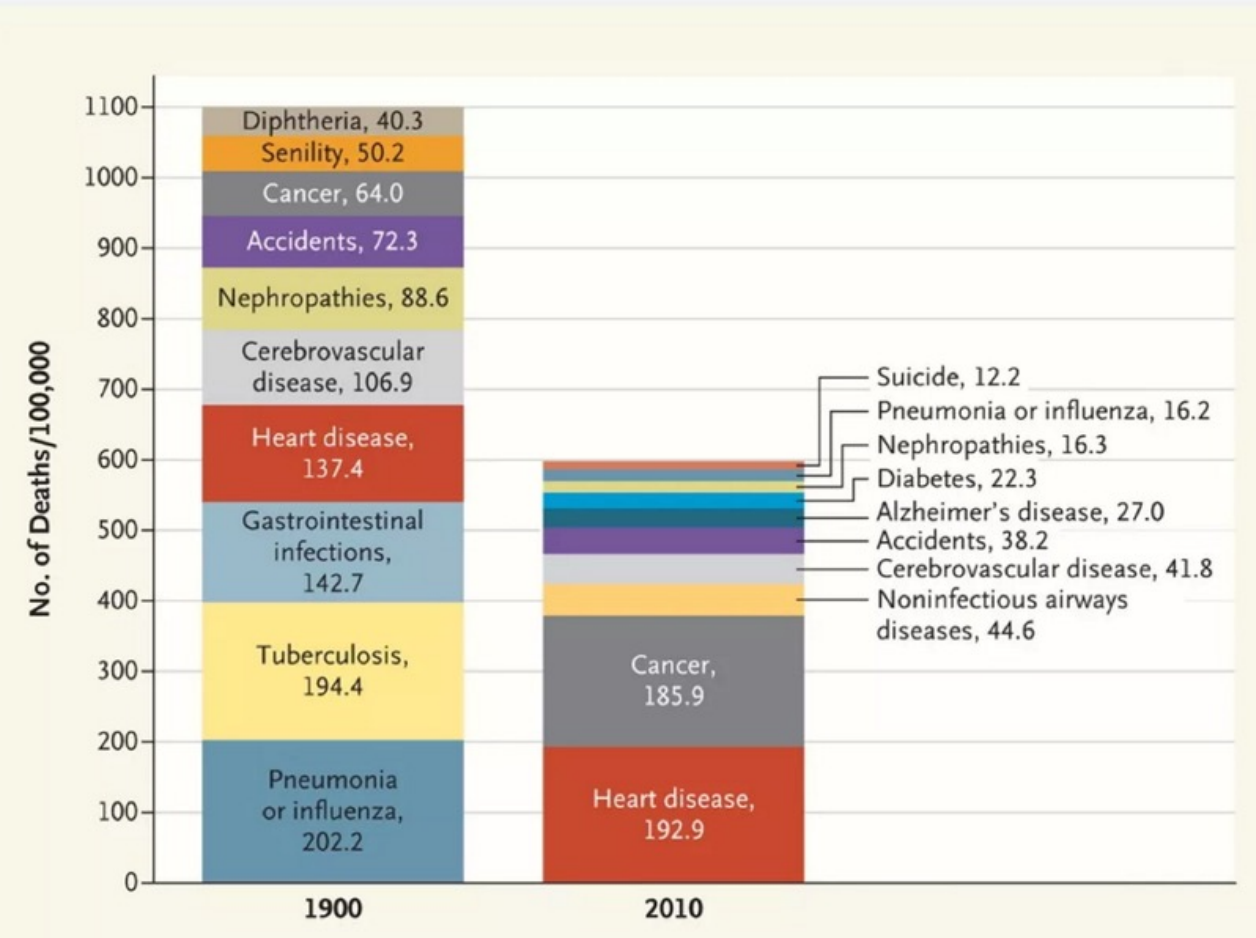
Box Plots

- Box plot with grouping:

Map Coloring

- Color/intensity can represent feature of region.

Popularity of naming baby "Evelyn" over time:

babynamewizard.com (via waitbutwhy.com)

But not very good if some regions are very small.

Canadian Income Mobility

http://waitbutwhy.com/2013/12/how-to-name-baby.html

Map Coloring

- Variation just uses fixed-size blocks and tries to arrange geographically:
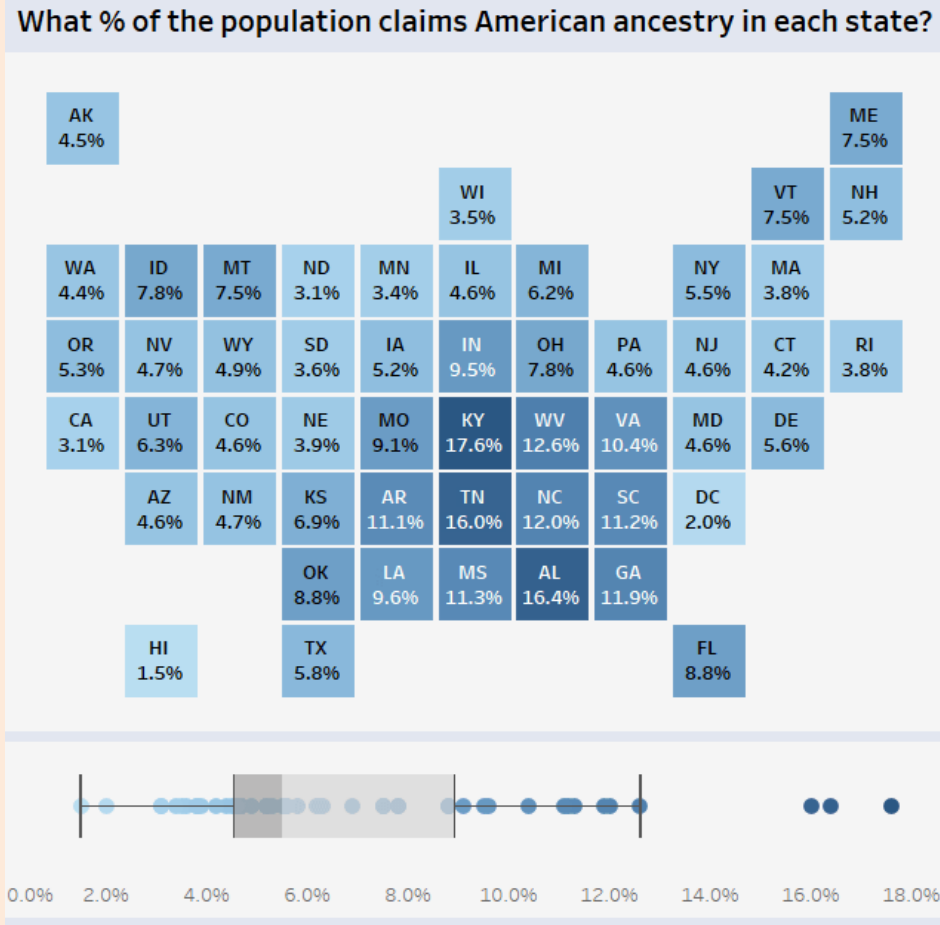
What % of the population claims American ancestry in each state?

http://www.visualcapitalist.com/america-immigration-1820/
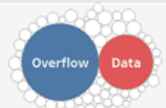
Contour Plot

- Colour visualizes 'z' as we vary 'x' and 'y'.
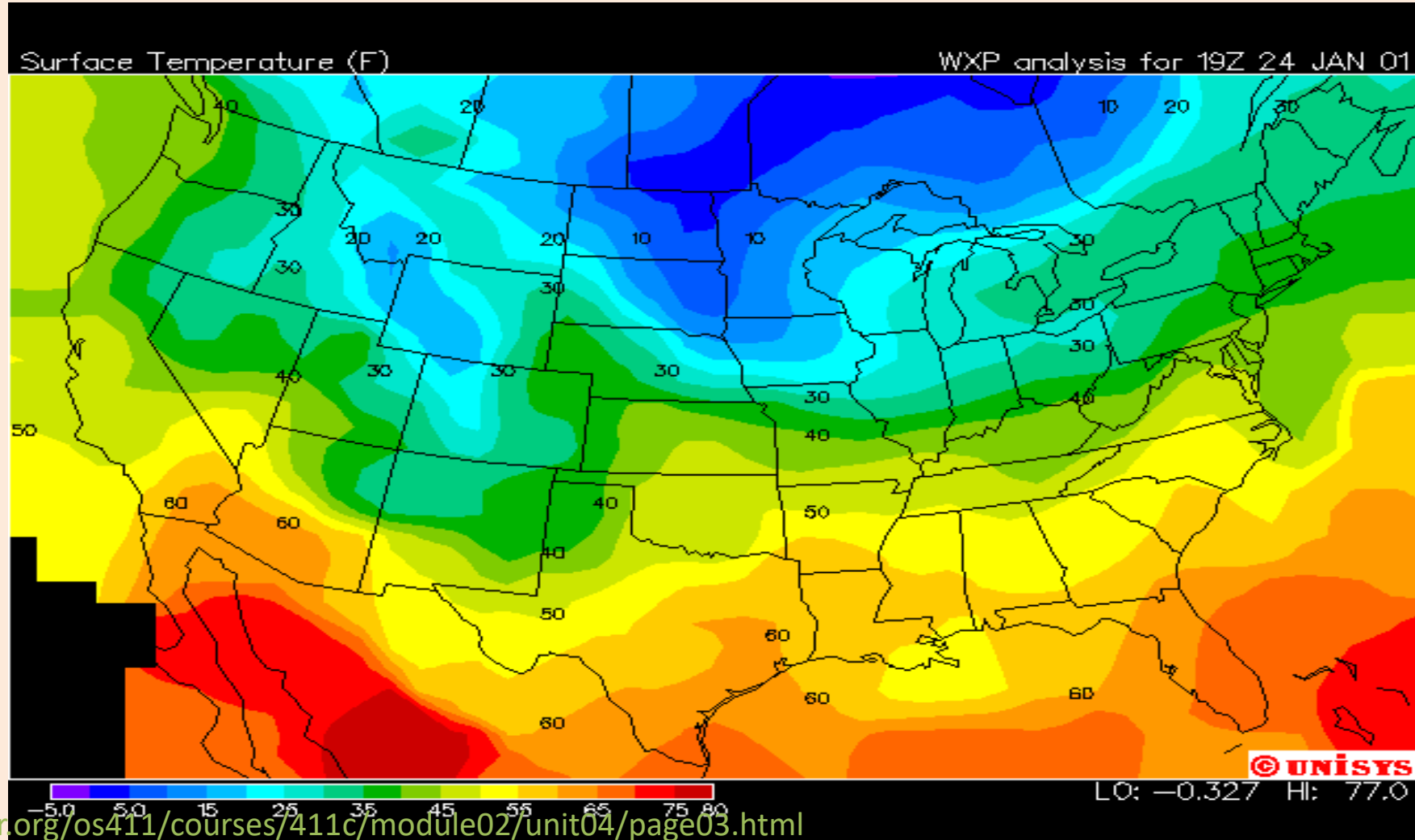
Treemaps

- Area represents attribute value:
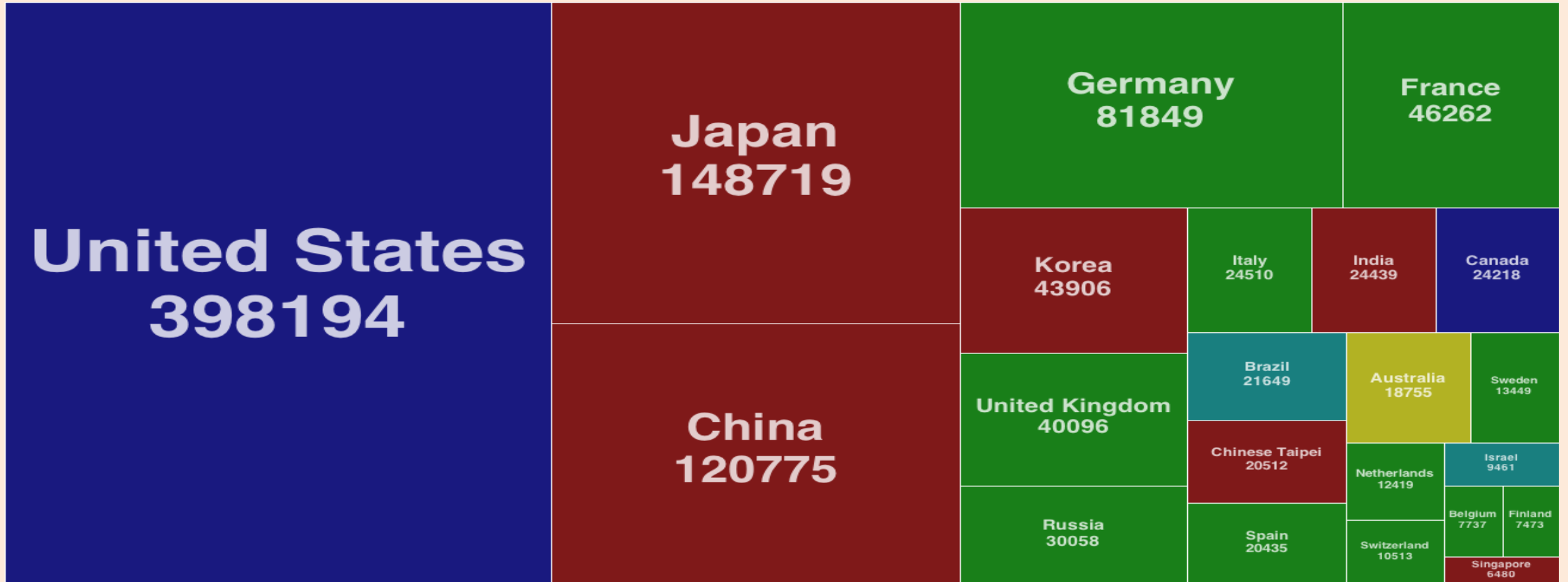
Cartogram

- Fancier version of treemaps:

Stream Graph
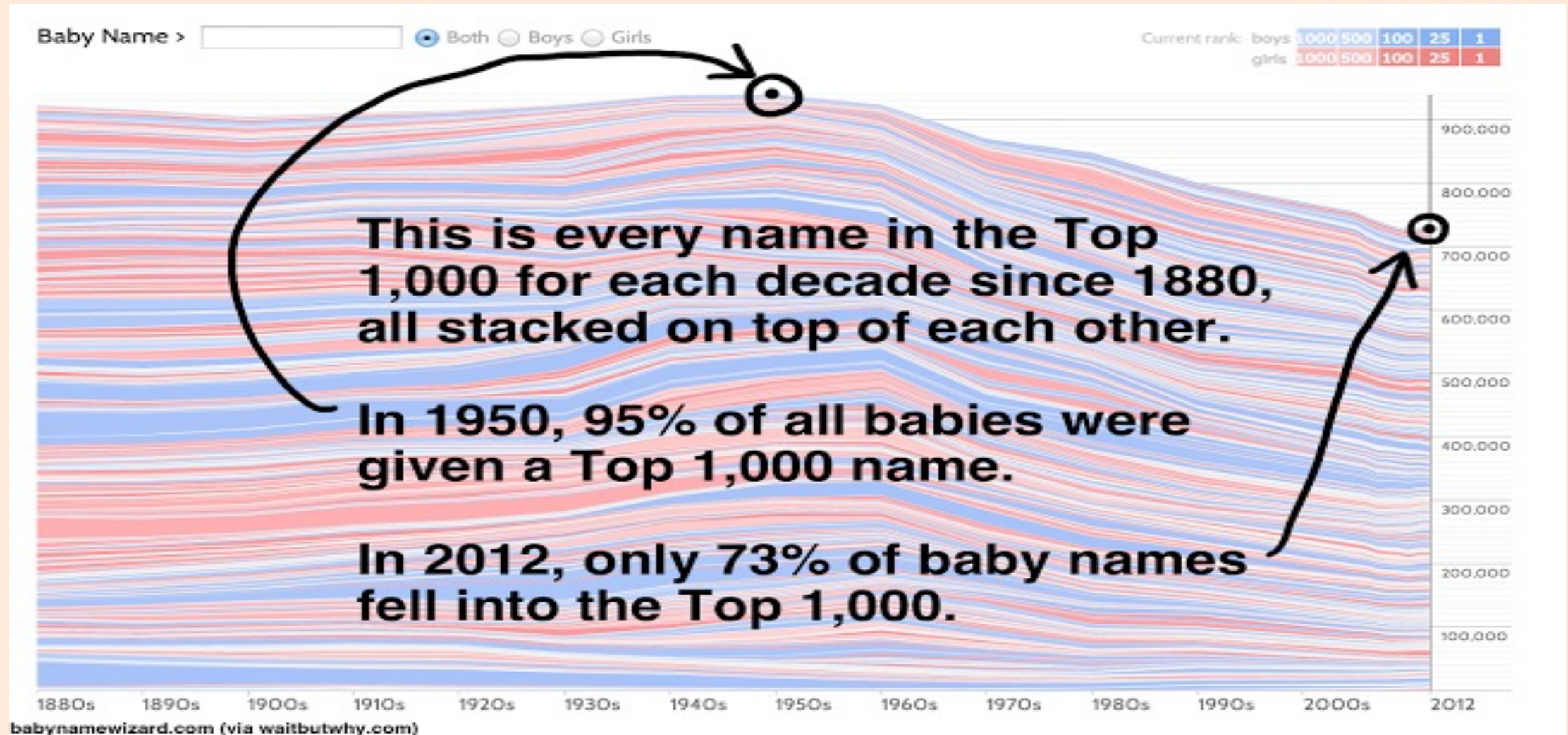
Stream Graph
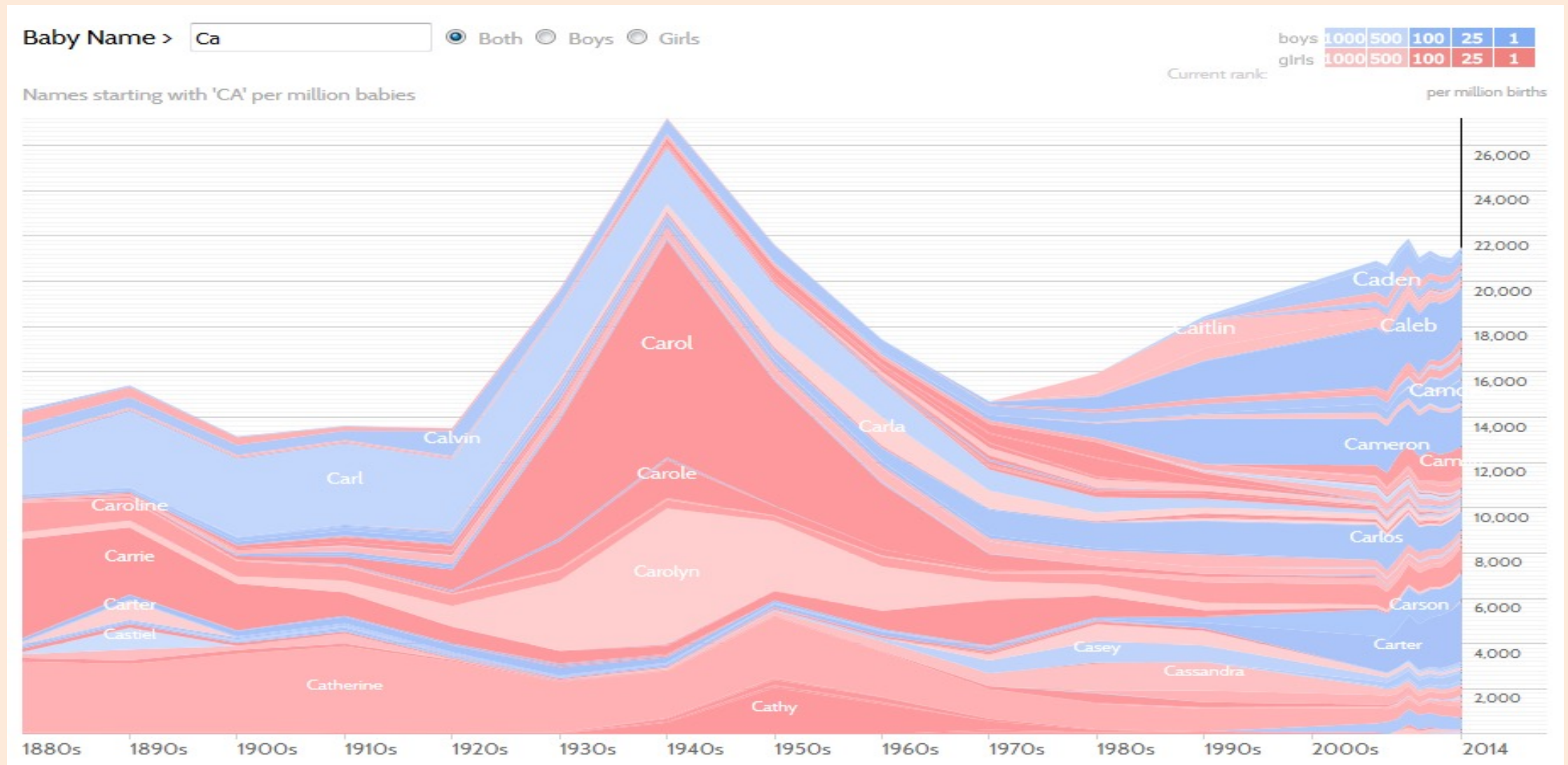
Stream Graph
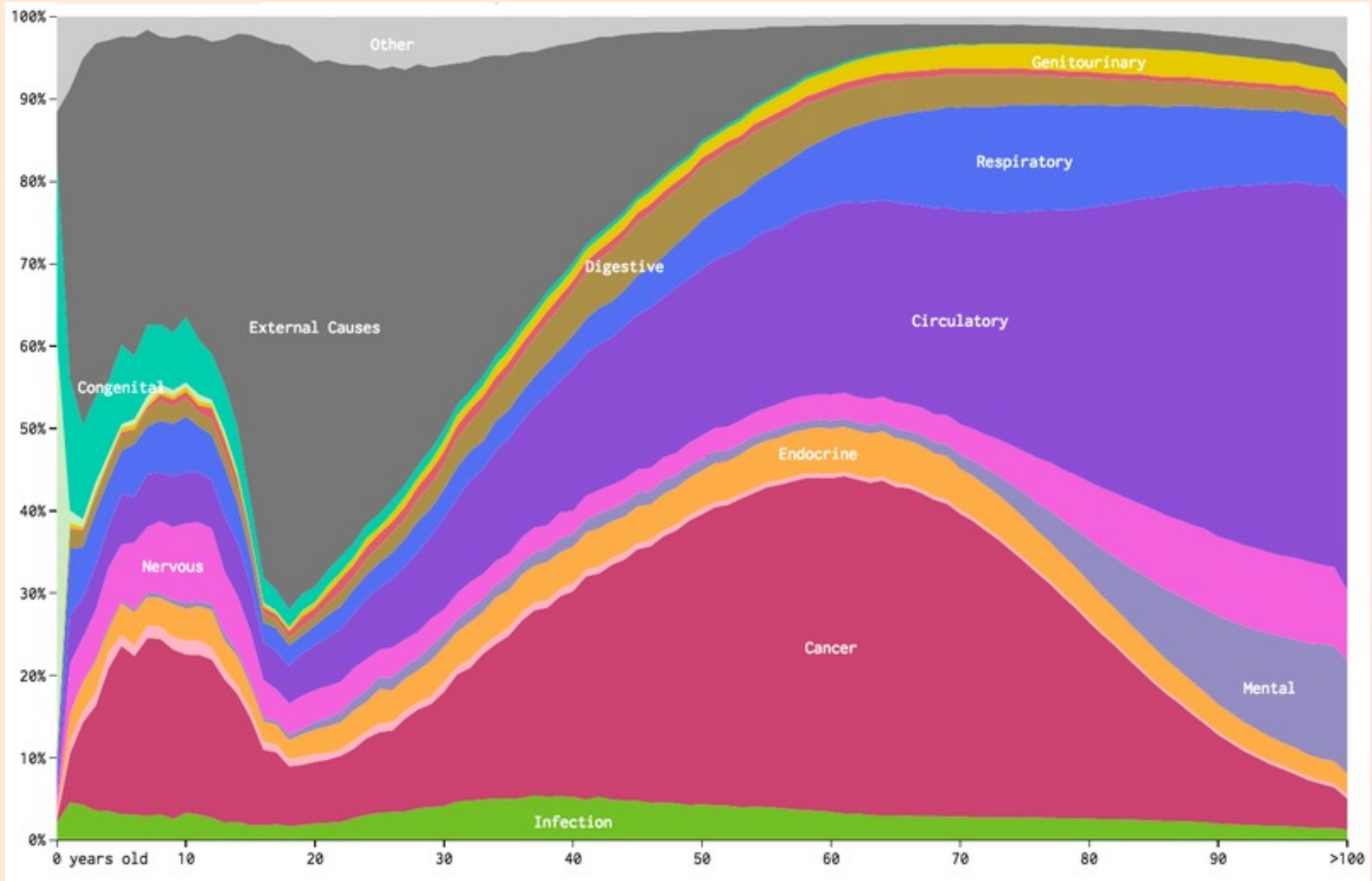
Videos and Interactive Visualizations

- For data recorded over time, videos can be useful:
  - [Map colouring over time](#)
  - [Climate Spiral](#)

- There are also lots of neat interactive visualization methods:
  - [Sale date for most expensive paintings](#).
  - [Global map of wind, weather, and oceans](#).
  - [Many examples here](#).

More Mis-Leading Axes from "Calling Bullshit"

"THE ONLY GLOBAL WARMING CHART YOU NEED FROM NOW ON"

Average Annual Global Temperature in Fahrenheit 1880-2015
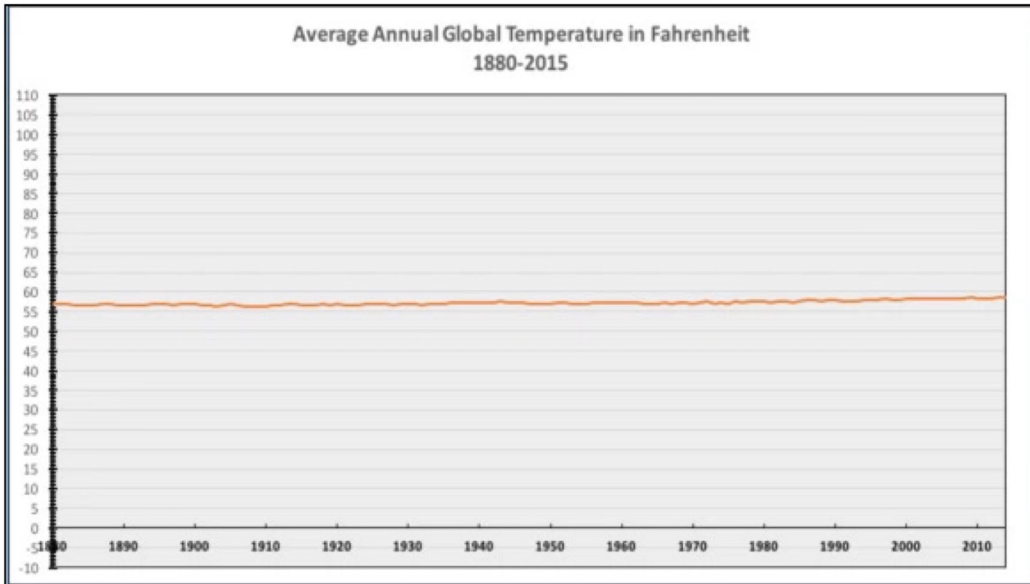
Powerline blog

Average global temperature by year

Data from NASA/GISS.
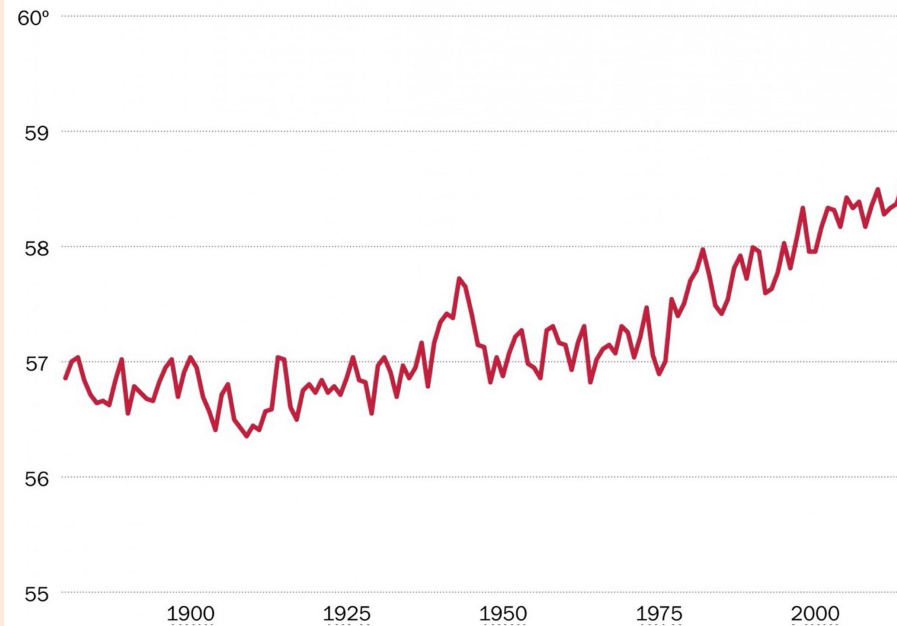
Philip Bump for the *Washington Post*

https://www.youtube.com/watch?v=9pNWVMxaFuM

More Mis-Leading Axes from "Calling Bullshit"

Averaged AD/ASD prevalence and MMR coverage in UK and Scandinavian countries
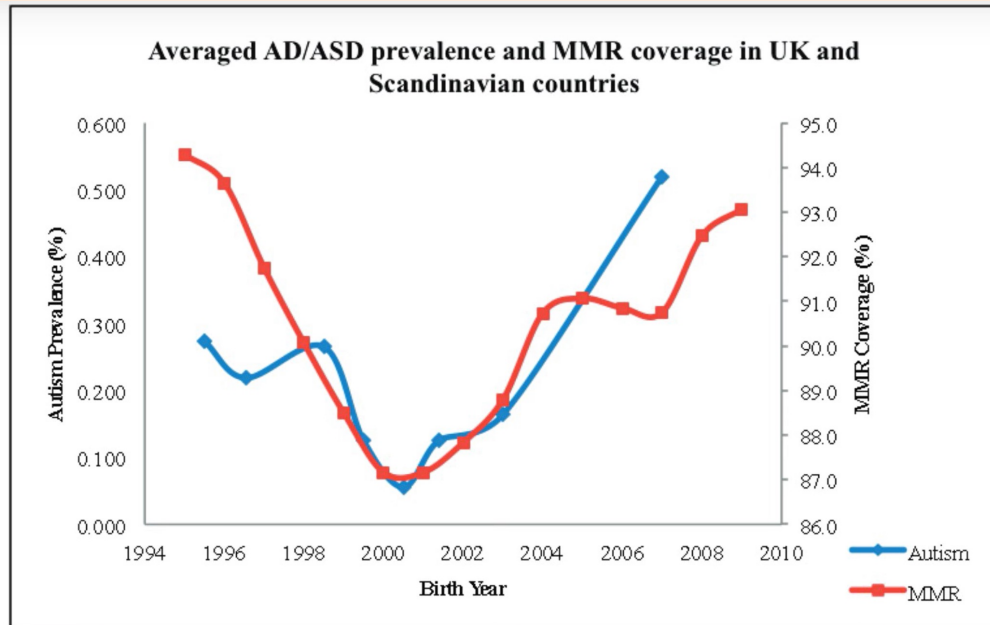
Figure 1-Averaged AD/ASD prevalence and MMR coverage in UK, Norway and Sweden. Both MMR and AD/ASD data are normalized to the maximum coverage/prevalence during the time period of this analysis.

Diesher *et al.* 2015 *Issues in Law and Medicine*
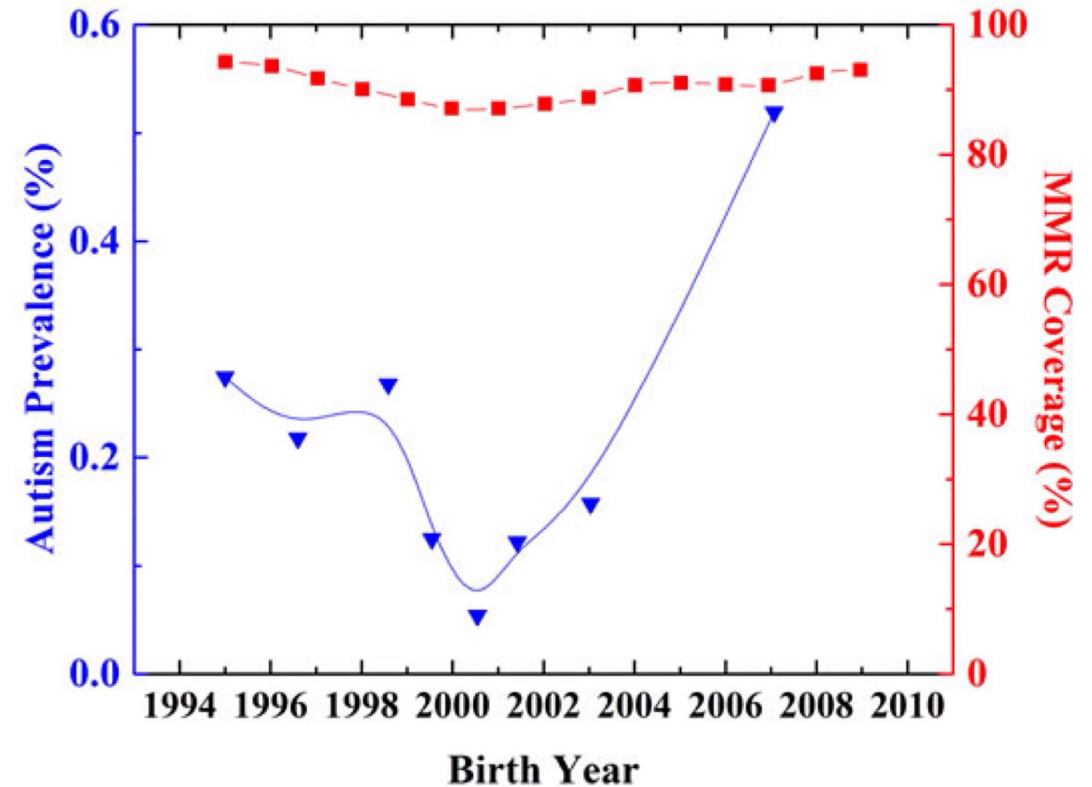
Matt Carey via sciencebasedmedicine.org

https://www.youtube.com/watch?v=9pNWVMxaFuM

More Mis-Leading Axes from "Calling Bullshit"

The Middle Class Tax Target

The amount of total taxable income (left scale) for all filers by adjusted gross income level for 2008

Y-axis values: $1.4 trillion, 1.2, 1.0, .8, .6, .4, .2, 0

X-axis labels: $0, 1-5k, 5-10k, 10-15k, 15-20k, 20-25k, 25-30k, 30-40k, 40-50k, 50-75k, 75-100k, 100-200k, 200-500k, 500k-1 mil., 1-1.5 mil., 1.5-2 mil., 2-5 mil., 5-10 mil., 10+ mil.
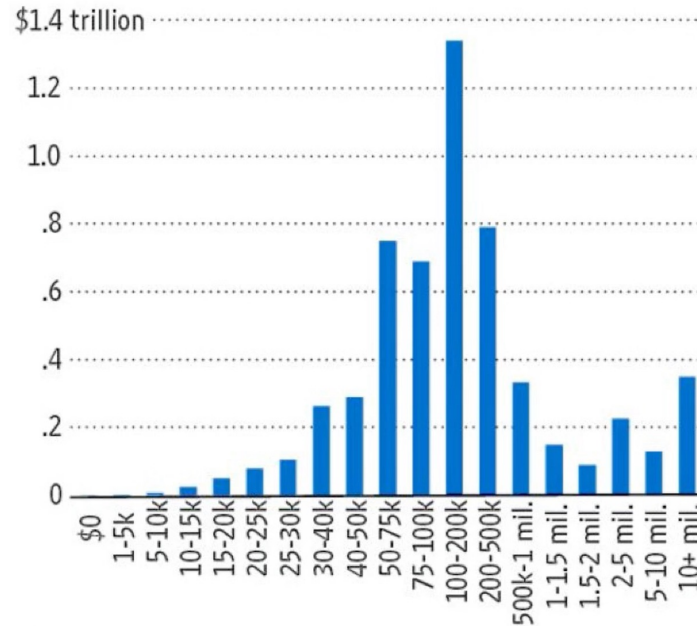
Source: IRS

"The rich, in short, aren't nearly rich enough to finance Mr. Obama's entitlement state ambitions—even before his health-care plan kicks in.

So who else is there to tax? Well, in 2008, there was about $5.65 trillion in total taxable income from all individual taxpayers, and most of that came from middle income earners. The nearby chart shows the distribution, and the big hump in the center is where Democrats are inevitably headed for the same reason that Willie Sutton robbed banks."

-The Wall Street Journal
April 17, 2011

- Look at the histogram bin widths.

https://www.youtube.com/watch?v=zAg1wsYfwsM

More Mis-Leading Axes from "Calling Bullshit"

- Axis is upside down.

- Looks like law makes murder go down, but number of murders go up!

Gun deaths in Florida

Number of murders committed using firearms

2005
Florida enacted
its 'Stand Your
Ground' law

873

721

1990s   2000s   2010s

Source: Florida Department of Law Enforcement

C. Chan 16/02/2014   REUTERS

Via http://blog.heapanalytics.com/how-to-lie-with-data-visualization/
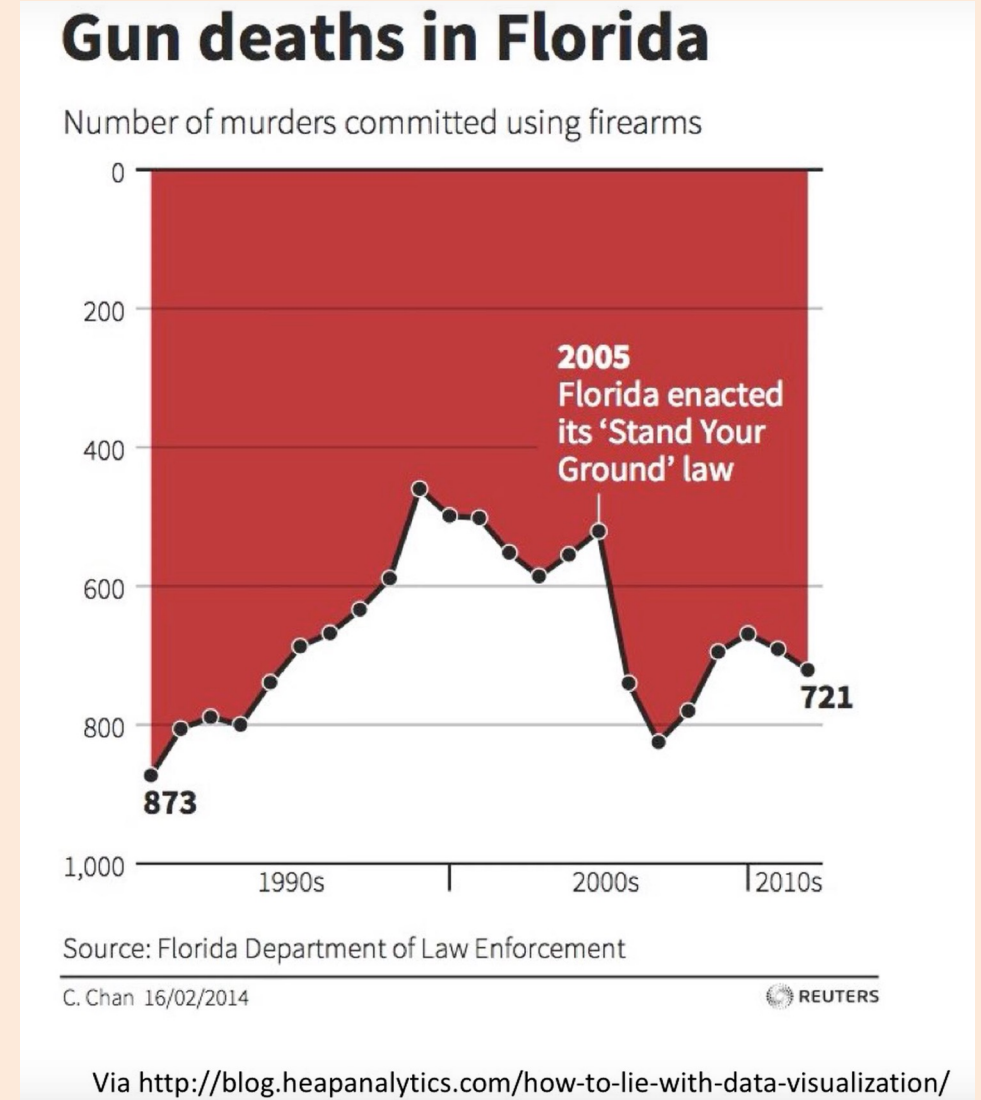
More Mis-Leading Axes from "Calling Bullshit"

- Calling BS gives this as another example:

- Actual numbers don't say much of anything:

Key findings of the survey include:

-- 39% of responding institutions reported a decline in international applications, 35% reported an increase, and 26% reported no change in applicant numbers.

- 39% vs. 35% (without sizes) doesn't mean "down nearly 40 percent".
  - Data can be used in mis-leading ways to "push agendas".
  - Even by reputed sources.
  - Even if you agree with the message.

More Mis-Leading Axes from Vancouver Housing

- A local example:
  - Are almost all Vancouver homes becoming empty?

  - Or did they use different y-axis scales for each line?

Total Private Dwellings and Unoccupied or Occupied by Temporary Resident

— Total Private Dwellings   — Unoccupied or Occupied by Temporary Resident
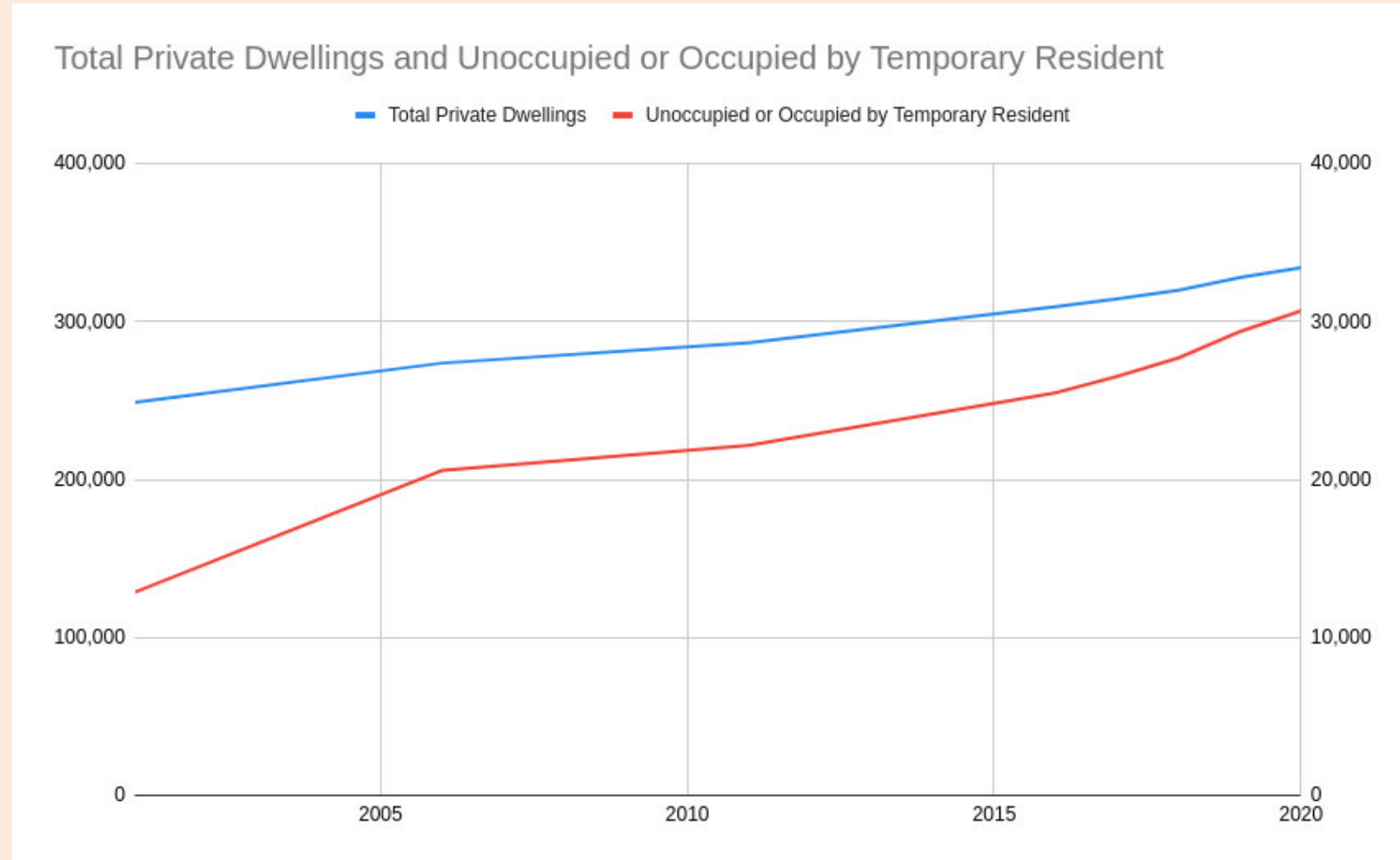
Hamming Distance vs. Jaccard Coefficient

A	B
1	0
1	0
1	0
0	1
0	1
1	0
0	0
0	0
0	1

- These vectors agree in 2 positions.
  - Normalizing Hamming distance by vector length, similarity is 2/9.

- If we're really interested in predicting 1s, we could find set of 1s in both and compute Jaccard:
  - A -> {1,2,3,6}, B -> {4,5,9}
  - No intersection so Jaccard similarity is actually 0.

Hamming Distance vs. Jaccard Coefficient
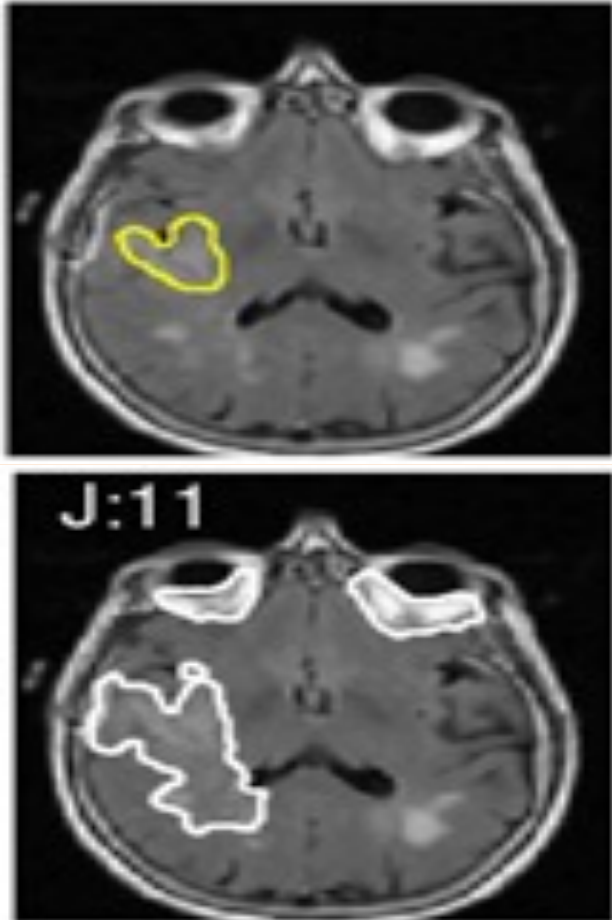
- Let's say we want to find the tumour in an MR image.
- We have an expert label (top) and a prediction from our ML system (bottom).

- The normalized Hamming distance between the predictions at each pixel is 0.91. This sounds good, but since there are so many non-tumour pixels this is misleading.
- The ML system predicts a much bigger tumour so hasn't done well. The Jaccard coefficient between the two sets of tumour pixels is only 0.11 so reflects this.

Coupon Collecting

- Consider trying to collect 50 uniformly-distributed states, drawing at random.

- The probability of getting a new state if there 'x' states left: p=x/50.

- So expected number of samples before next "success" (getting a new state) is 50/x.
    (mean of geometric random variable with p=x/50)

- So the expected number of draws is the sum of 50/x for x=1:50.

- For 'n' states instead of 50, summing until you have all 'n' gives:

$$\sum_{i=1}^{n} \frac{n}{i} = n \sum_{i=1}^{n} \frac{1}{i} \leq n(1 + \log(n)) = O(n \log n)$$

Huge Datasets and Parallel/Distributed Computation

- Most sufficient statistics can be computed in linear time.
- For example, the mean of 'n' numbers is computed as:

$$mean(x_1, x_2, x_3, \ldots, x_n) = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$$

- This costs O(n), which is great.

- But if 'n' is really big, we can go even faster with parallel computing…

Huge Datasets and Parallel/Distributed Computation

- Computing the mean with multiple cores:
  - Each of the 'c' cores computes the sum of $O(n/c)$ of the data:

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n/c} \end{bmatrix} \longrightarrow \text{Sum1}$$

$$\begin{bmatrix} x_{n/c+1} \\ x_{n/c+2} \\ \vdots \\ x_{2n/c} \end{bmatrix} \longrightarrow \text{Sum2}$$

$$\begin{bmatrix} x_{(c-1)n/c+1} \\ x_{(c-1)n/c+2} \\ \vdots \\ x_n \end{bmatrix} \longrightarrow \text{SumC}$$

Huge Datasets and Parallel/Distributed Computation

- Computing the mean with multiple cores:
  - Each of the 'c' cores computes the sum of O(n/c) of the data:
  - Add up the 'c' results from each core to get the mean.

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \\ x_n \end{bmatrix} \longrightarrow \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n/c} \end{bmatrix} \longrightarrow sum1$$

$$\begin{bmatrix} x_{n/c+1} \\ x_{n/c+2} \\ \vdots \\ x_{2n/c} \end{bmatrix} \longrightarrow sum2$$

$$mean(x) = \frac{sum1 + sum2 + \cdots sumc}{n}$$

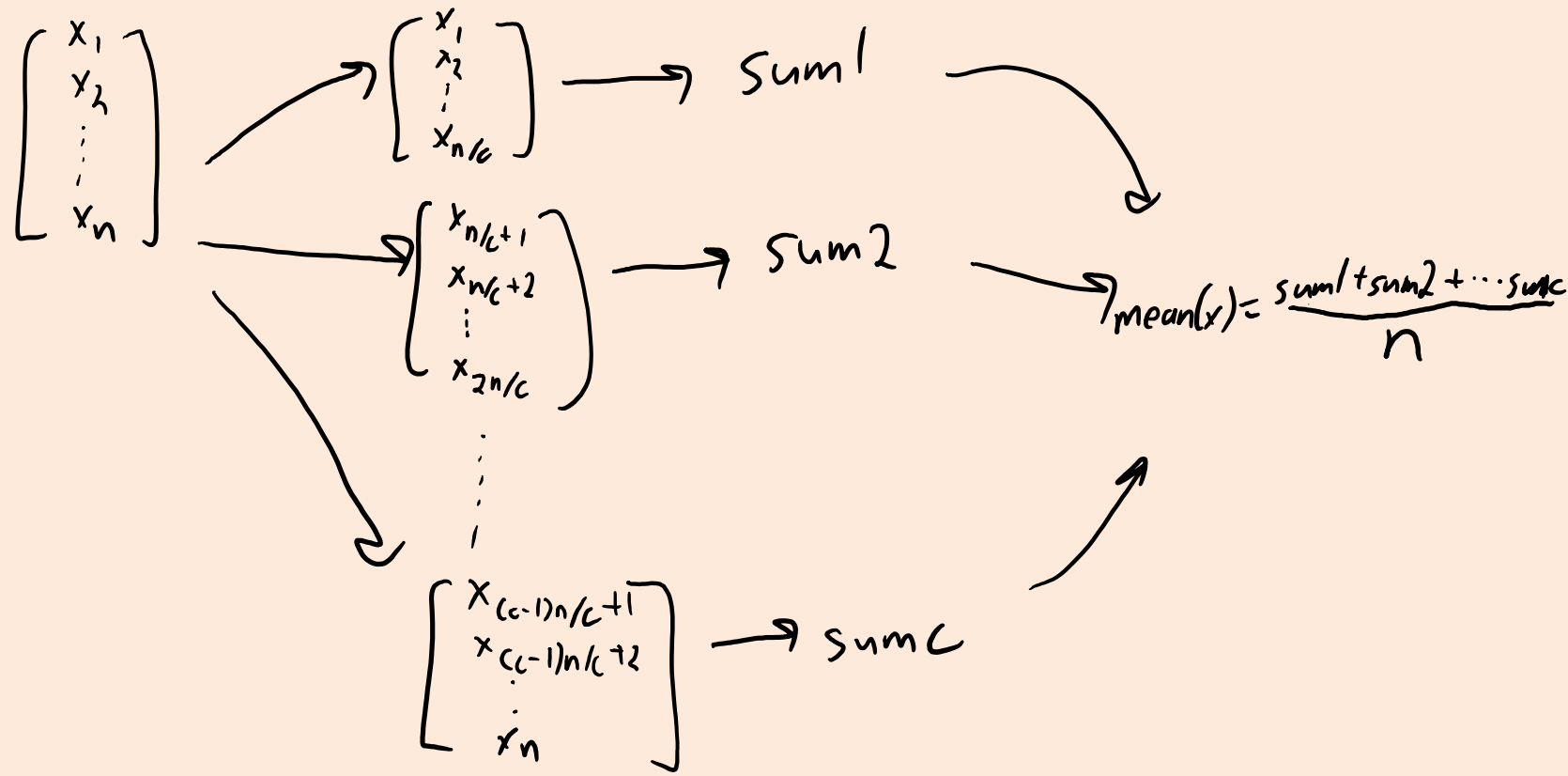$$\begin{bmatrix} x_{(c-1)n/c+1} \\ x_{(c-1)n/c+2} \\ \vdots \\ x_n \end{bmatrix} \longrightarrow sumc$$

Huge Datasets and Parallel/Distributed Computation

- Computing the mean with multiple cores:
    - Each of the 'c' cores computes the sum of $O(n/c)$ of the data.
    - Add up the 'c' results from each core to get the mean.
    - Cost is only $O(n/c + c)$, which can be much faster for large 'n'.

- This assumes cores can access data in parallel (not always true).
- Can reduce cost to $O(n/c)$ by having cores write to same register.
    - But need to "lock" the register and might effectively cost $O(n)$.

Huge Datasets and Parallel/Distributed Computation

- Sometimes 'n' is so big that data can't fit on one computer.

- In this case the data might be distributed across 'c' machines:
  - Hopefully, each machine has $O(n/c)$ of the data.

- We can solve the problem similar to the multi-core case:
  - "Map" step: each machine computes the sum of its data.
  - "Reduce" step: each machine communicates sum to a "master" computer, which adds them together and divides by 'n'.

Huge Datasets and Parallel/Distributed Computation

- Many problems in DM and ML have this flavour:
  - "Map" computes an operation on the data on each machine (in parallel).
  - "Reduce" combines the results across machines.
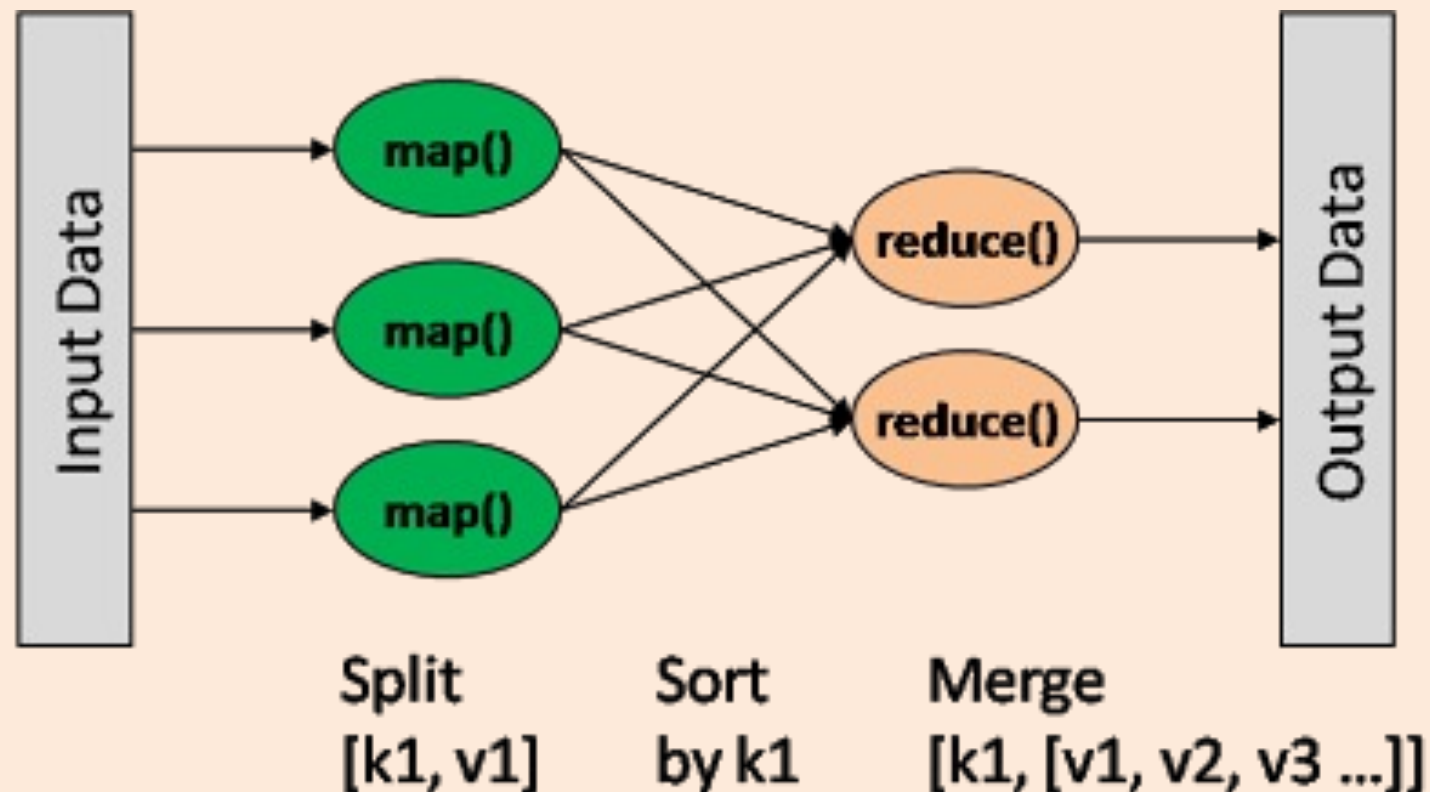
Huge Datasets and Parallel/Distributed Computation

- Many problems in DM and ML have this flavour:
  - "Map" computes an operation on the data on each machine (in parallel).
  - "Reduce" combines the results across machines.
  - These are standard operations in parallel libraries like MPI.

- Can solve many problems almost 'c' times faster with 'c' computers.

- To make it up for the high cost communicating across machines:
  - Assumes that most of the computation is in the "map" step.
  - Often need to assume data is already on the computers at the start.

Huge Datasets and Parallel/Distributed Computation

- Another challenge with "Google-sized" datasets:
  - You may need so many computers to store the data,
    that it's inevitable that some computers are going to fail.
- Solution to this is a distributed file system.

- Two popular examples are Google's MapReduce and Hadoop DFS:
  - Store data with redundancy (same data is stored in many places).
    - And assume data isn't changing too quickly.
  - Have a strategy for restarting "map" operations on computers that fail.
  - Allows fast calculation of more-fancy things than sufficient statistics:
    - Database queries and matrix multiplications.