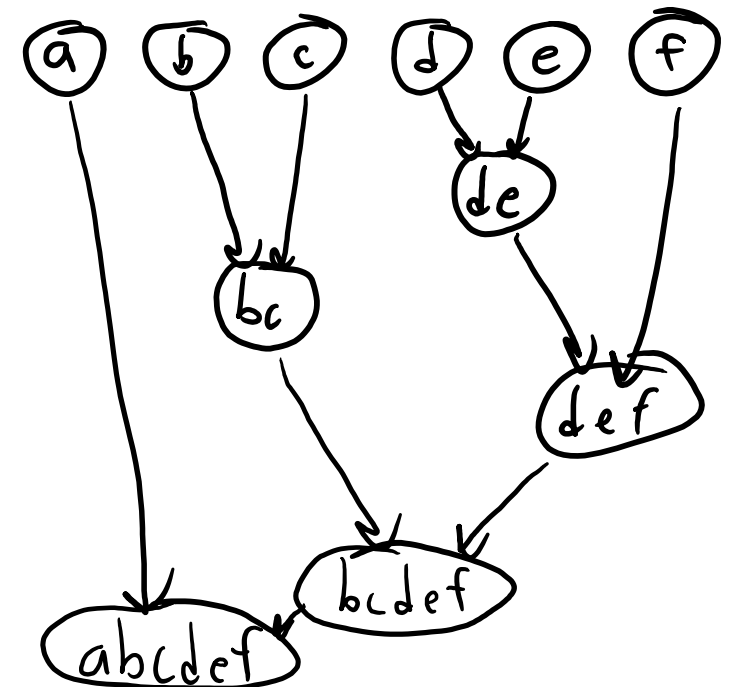
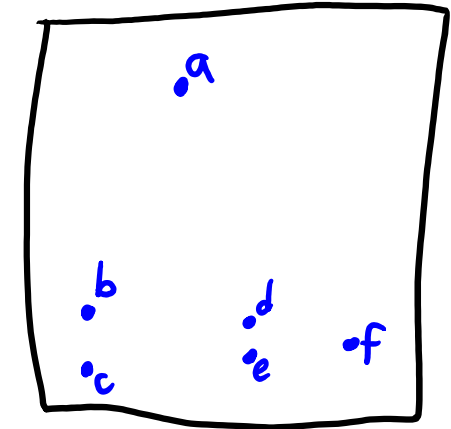


CPSC 340: Machine Learning and Data Mining

Outlier Detection

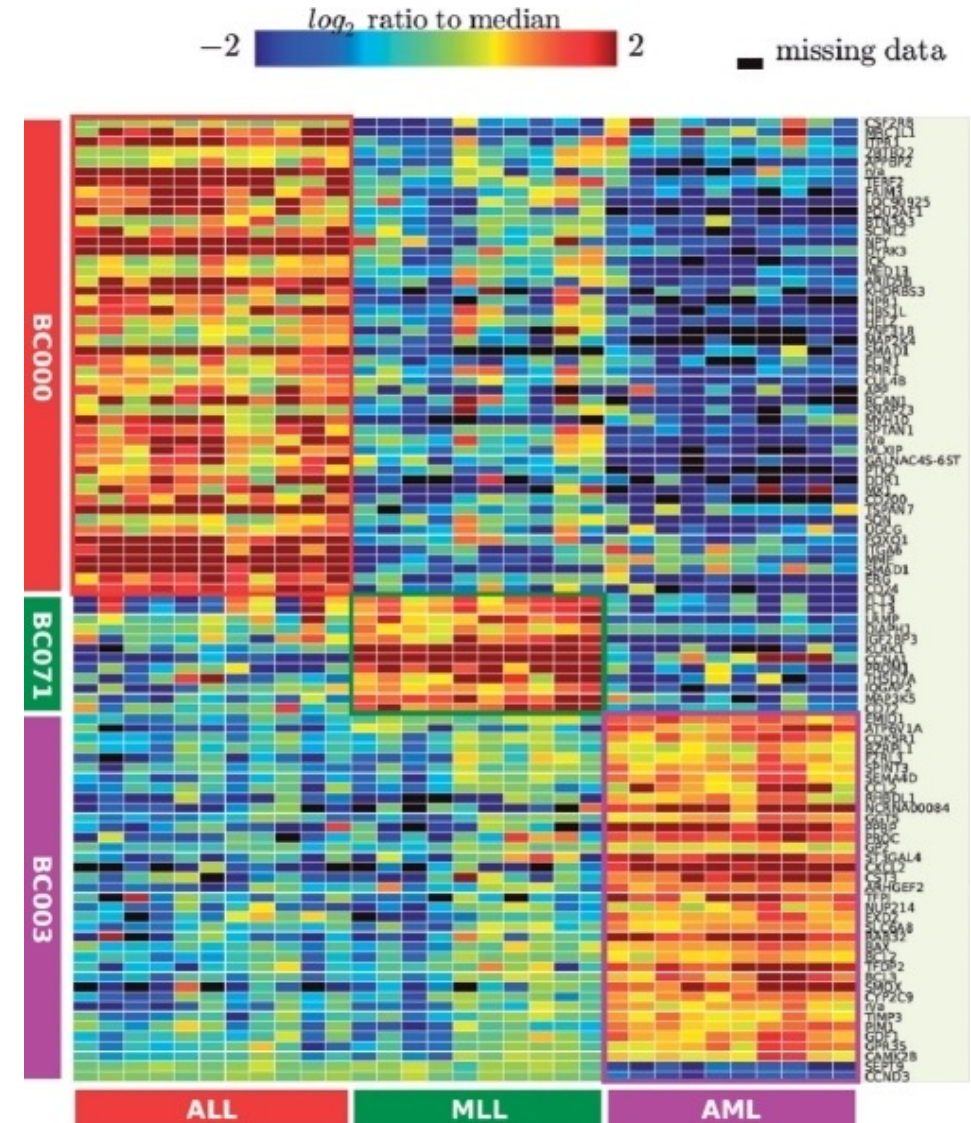
Last Time: Hierarchical Clustering

- We discussed **hierarchical clustering**:
 - Performs **clustering at multiple scales**.
 - Output is usually a **tree diagram** (“dendrogram”).
 - Reveals much more structure in data.
 - Usually non-parametric:
 - At finest scale, every point is its own clusters.
- There are various application areas:
 - Animals (phylogenetics).
 - Languages.
 - Stories.
 - Fashion.



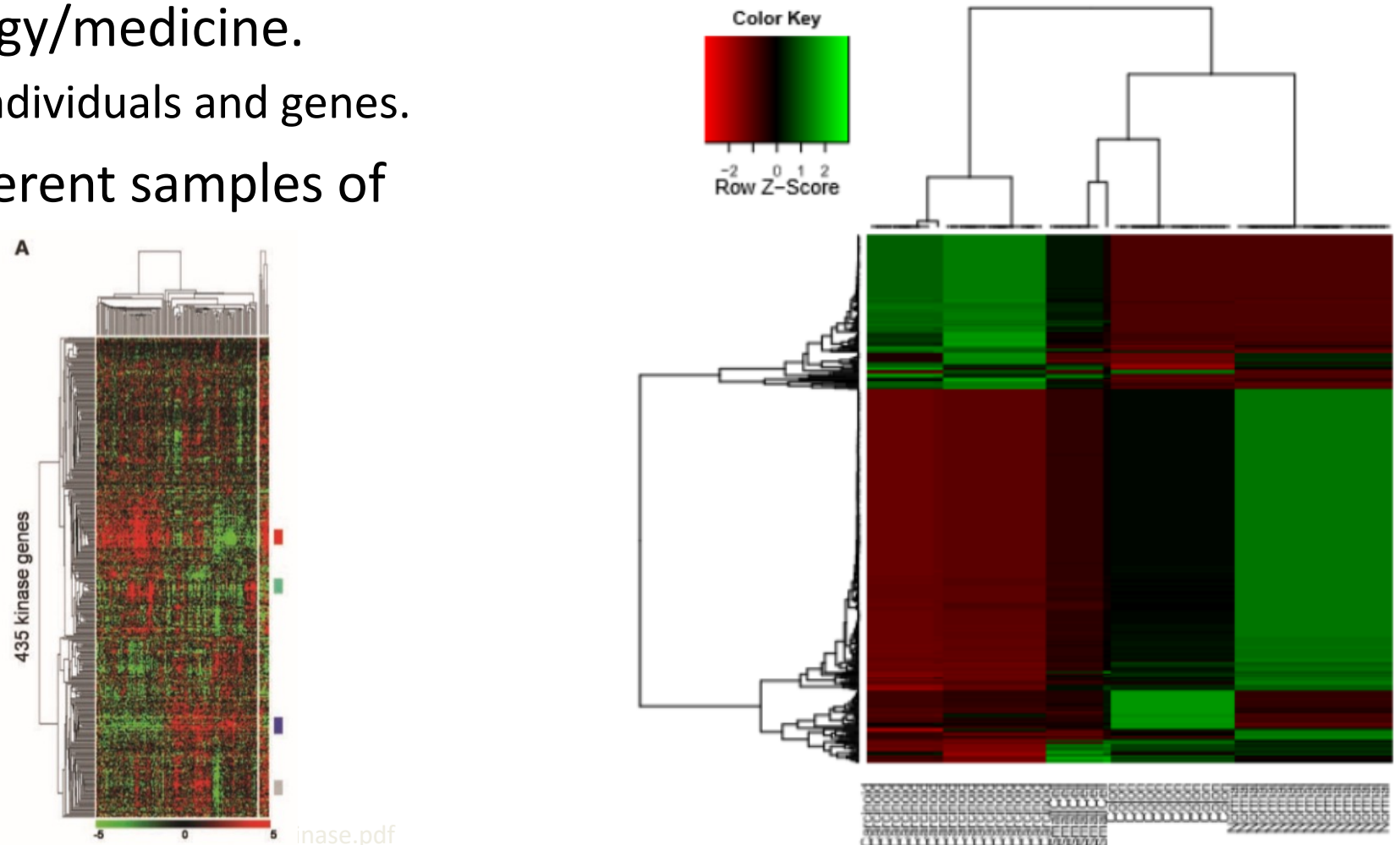
Biclustering

- **Biclustering:**
 - Cluster the training examples and features.
 - Also gives feature relationship information.
- Simplest and most popular method:
 - Run clustering method on 'X' (examples).
 - Run clustering method on 'X^T' (features).
- Often plotted with 'X' as a heatmap.
 - Where rows/columns arranged by clusters.
 - Helps you 'see' why things are clustered.



Biclustering

- Visualization: **hierarchical biclustering + heatmap + dendrograms.**
 - Popular in biology/medicine.
 - Might cluster individuals and genes.
 - Biclustering different samples of breast cancer:

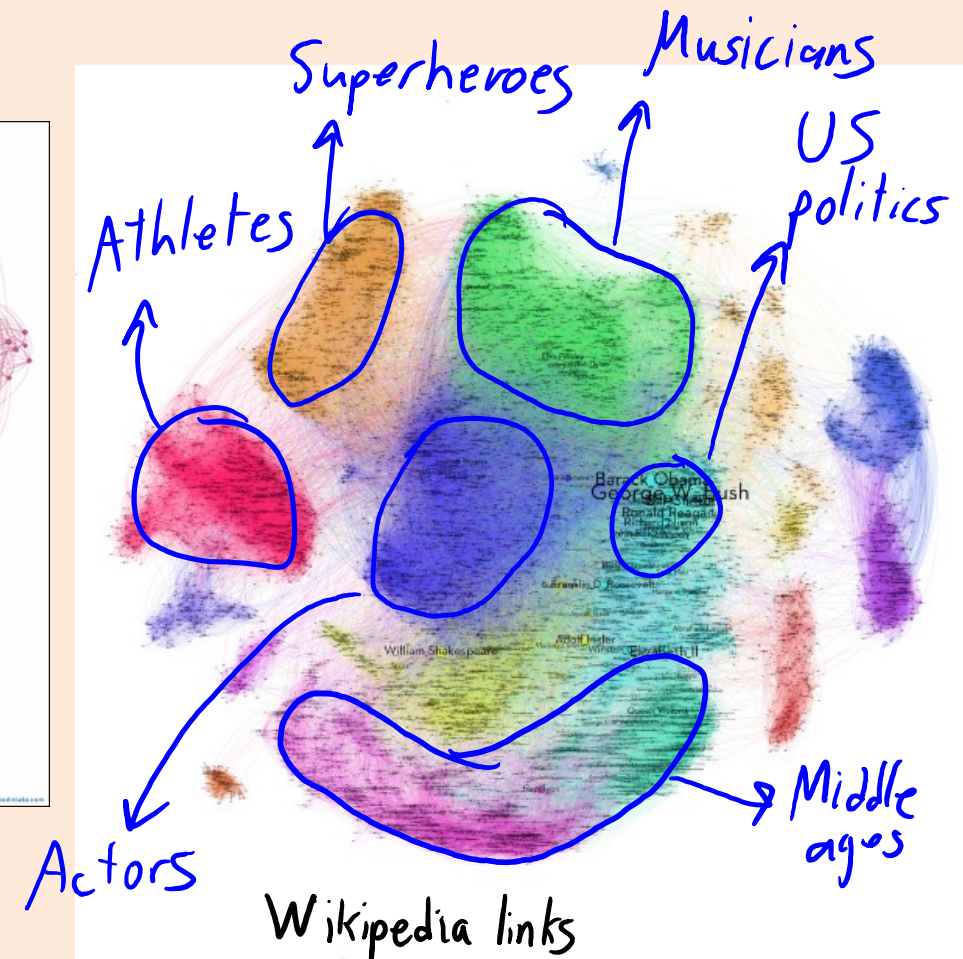
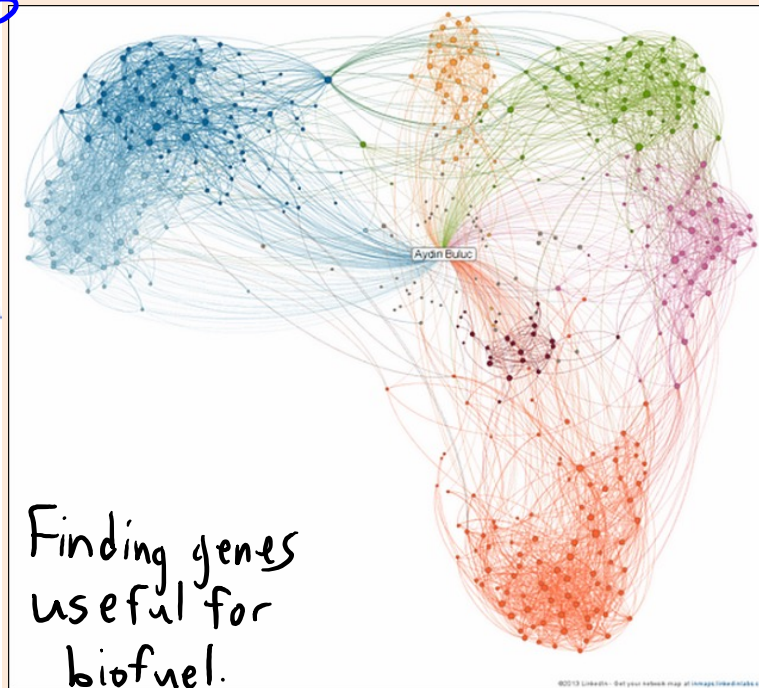
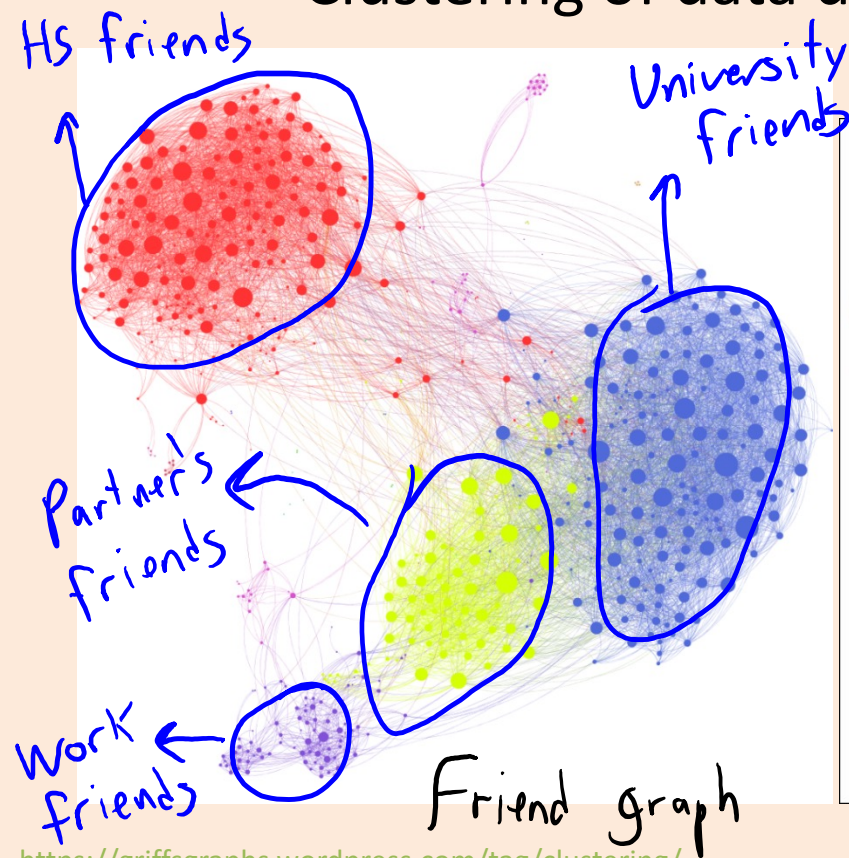


Other Clustering Methods

- **Mixture models:**
 - Probabilistic clustering.
- **Mean-shift clustering:**
 - Finds local “modes” in density of points.
 - Alternative approach to vector quantization.
- **Bayesian clustering:**
 - A variant on ensemble methods.
 - Averages over models/clustering, weighted by “prior” belief in the model/clustering.
- **Pairwise supervised clustering:**
 - Build a classifier that predicts whether 2 points are in the same cluster.
 - Based on training pairs from the same cluster, and pairs from different clusters.

Graph-Based Clustering

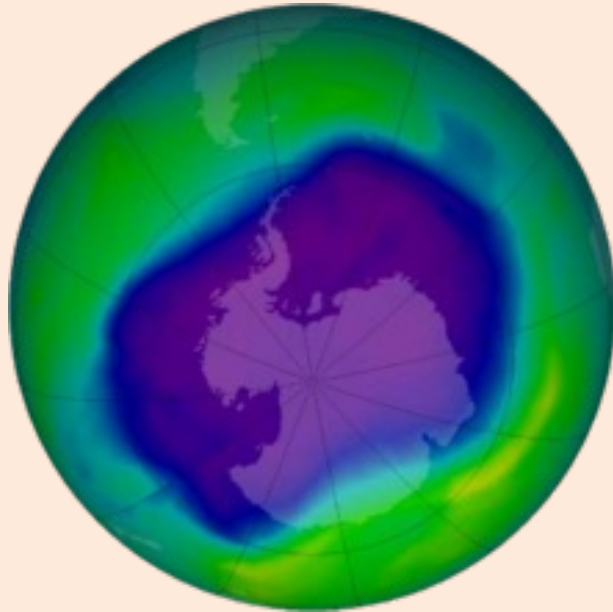
- Spectral clustering and graph-based clustering:
 - Clustering of data described by graphs.



Next Topic: Outlier Detection

Motivating Example: Finding Holes in Ozone Layer

- The huge Antarctic ozone hole was “discovered” in 1985.

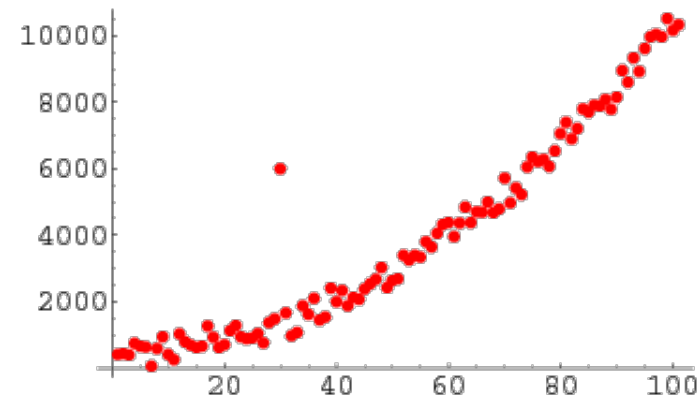
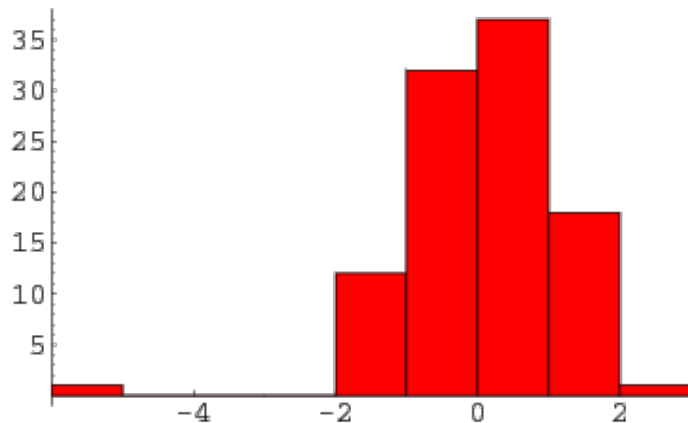


- It had been in satellite data since 1976:
 - But it was flagged and filtered out by a quality-control algorithm.

Outlier Detection

- **Outlier detection:**

- Find observations that are “unusually different” from the others.
- Also known as “anomaly detection”.
- May want to remove outliers, or be interested in the outliers themselves (security).




- **Some sources of outliers:**

- Measurement errors.
- Data entry errors.
- Contamination of data from different sources.
- Rare events.

Applications of Outlier Detection

- Data cleaning: removing outliers may lead to better models.
- Security and fault detection (network intrusion, DOS attacks).
- Fraud detection (credit cards, stocks, voting irregularities).

| Transaction Date | Posted Date | Transaction Details | Debit | Credit |
|------------------|---------------|---|---------|--------|
| Aug. 27, 2015 | Aug. 28, 2015 |  BEAN AROUND THE WORLD VANCOUVER, BC | \$10.95 | |

- Detecting natural disasters (underwater earthquakes).
- Astronomy (find new types of stars/planets).
- Genetics (identifying individuals with new/ancient genes).

5 Types of Methods for Outlier Detection

1. Model-based methods.
 2. Graphical approaches.
 3. Cluster-based methods.
 4. Distance-based methods.
 5. Supervised-learning methods.
- Warning: this is the topic with the most ambiguous “solutions”.
 - We will cover 5 types, but mainly argue that problem is hard.

But first...

- Usually it's good to do some **basic sanity checking**...

| Egg | Milk | Fish | Wheat | Shellfish | Peanuts | Peanuts | Sick? |
|------------|------|------|---------------|-----------|---------|----------------|----------|
| 0 | 0.7 | 0 | 0.3 | 0 | 0 | 0 | 1 |
| 0.3 | 0.7 | 0 | 0.6 | -1 | 3 | 3 | 1 |
| 0 | 0 | 0 | "sick" | 0 | 1 | 1 | 0 |
| 0.3 | 0.7 | 1.2 | 0 | 0.10 | 0 | 0 | 2 |
| 900 | 0 | 1.2 | 0.3 | 0.10 | 0 | 0 | 1 |

- Would any values in the column cause a Python/Julia **"type" error**?
- What is the **range of numerical features**?
- What are the **unique entries for a categorical feature**?
- Does it look like parts of the table are **duplicated**?
- These types of simple errors are **VERY common** in real data.
 - And have led to deaths (for example, over-dosing on medication).

Outlier Detection Method 1: Model-Based

- Model-based outlier detection:
 1. Fit a probabilistic model.
 2. Outliers are examples with low density.

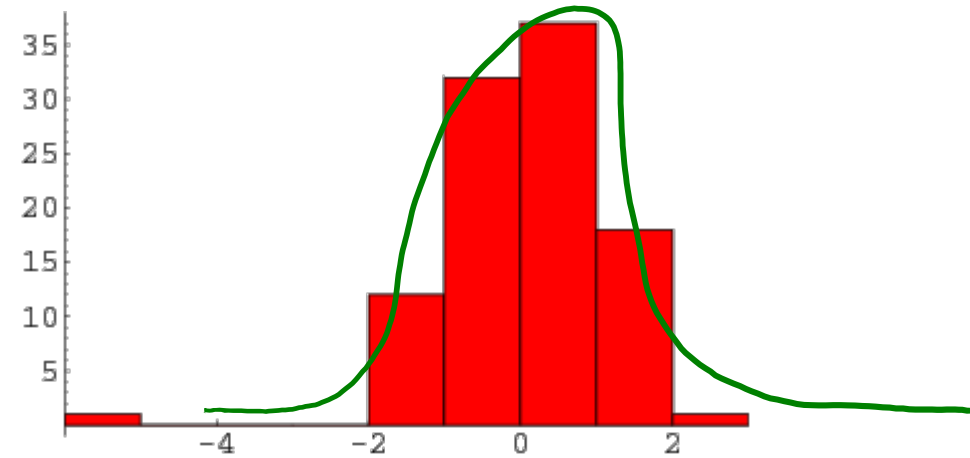
- Example:

- Assume data follows normal distribution.
- The z-score for 1D data is given by:

$$z_i = \frac{x_i - \mu}{\sigma}$$

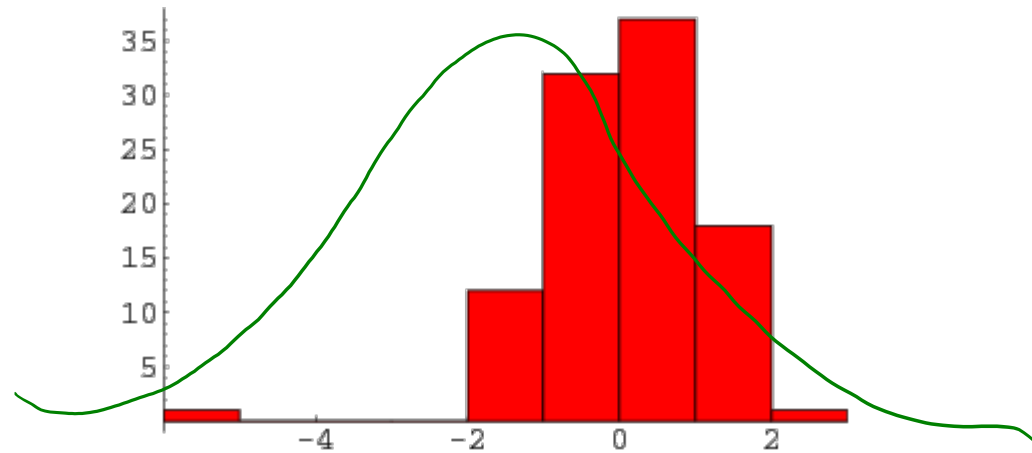
$$\text{where } \mu = \frac{1}{n} \sum_{i=1}^n x_i \quad \text{and} \quad \sigma = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2}$$

- “Number of standard deviations away from the mean”.
- Say “outlier” if $|z| > 4$, or some other threshold.

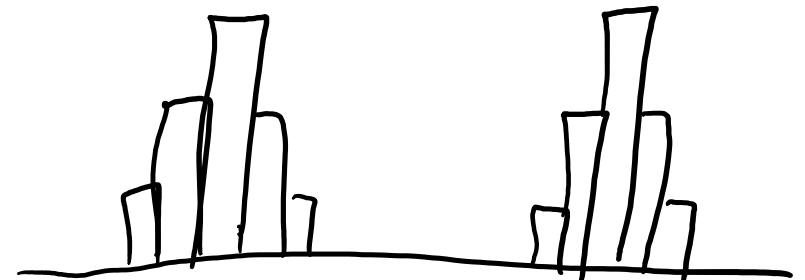


Problems with Z-Score

- Unfortunately, the **mean and variance are sensitive to outliers.**



- Possible fixes: **use quantiles, or sequentially remove worse outlier.**
- The z-score also assumes that data is “uni-modal”.
 - That data is concentrated around the mean.
 - See bonus slide for my e-mail regarding why the department should ***not*** use z-scores.



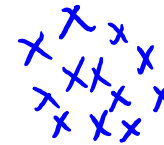
Global vs. Local Outliers

- Is the **red point** an outlier?



Global vs. Local Outliers

- Is the **red point** an outlier? What if we add the **blue points**?



Global vs. Local Outliers

- Is the **red point** an outlier? What if we add the **blue points**?



- Red point has the **lowest z-score**.
 - In the first case it was a “**global**” outlier (not within normal range of data).
 - In this second case it’s a “**local**” outlier:
 - Within normal data range, but **far from other points**.
- It’s hard to precisely define “outliers”.

Global vs. Local Outliers

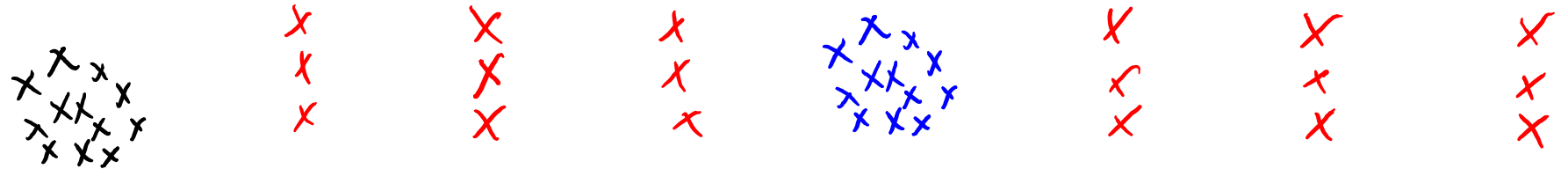
- Is the **red point** an outlier? What if we add the **blue points**?



- Red point has the **lowest z-score**.
 - In the first case it was a “**global**” outlier (not within normal range of data).
 - In this second case it’s a “**local**” outlier:
 - Within normal data range, but **far from other points**.
- It’s hard to precisely define “outliers”.
 - Can we have **outlier groups**?

Global vs. Local Outliers

- Is the **red point** an outlier? What if we add the **blue points**?

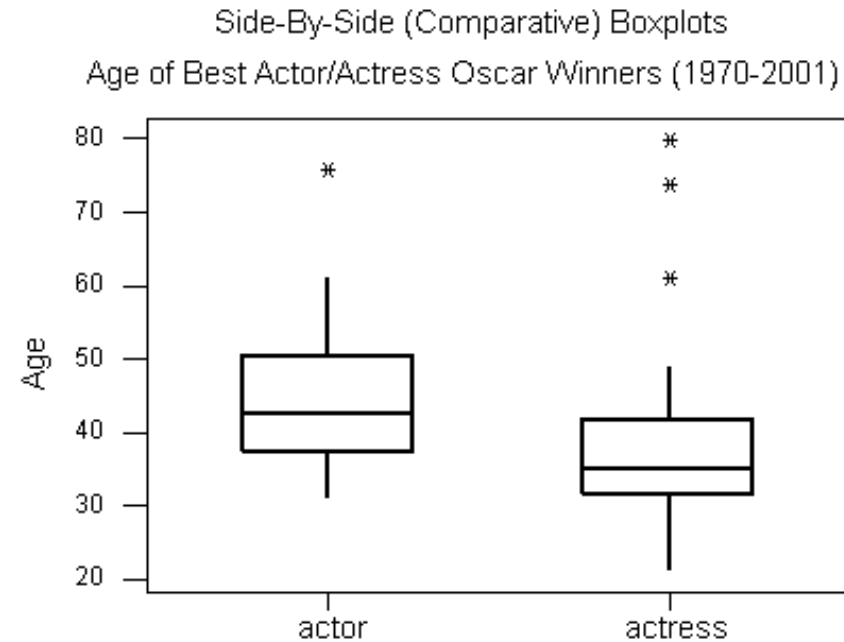


- Red point has the **lowest z-score**.
 - In the first case it was a “**global**” outlier (not within normal range of data).
 - In this second case it’s a “**local**” outlier:
 - Within normal data range, but **far from other points**.
- It’s hard to precisely define “outliers”.
 - Can we have **outlier groups**? What about repeating patterns?

Outlier Detection Method 2: Graphical

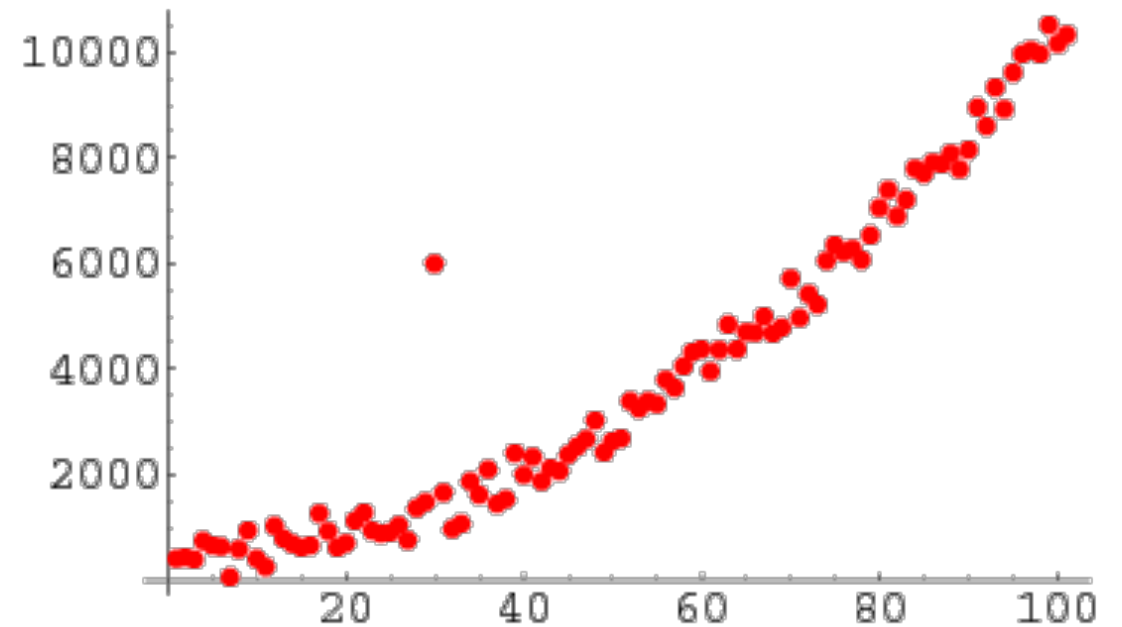
- Graphical approach to outlier detection:
 1. Look at a plot of the data.
 2. Human decides if data is an outlier.
- Examples:
 1. Box plot:
 - Visualization of quantiles/outliers.
 - Only 1 variable at a time.

Allegedly, when asked why outliers are those $> 1.5 \cdot \text{IQR}$, John Tukey said "Because 1 would have been too little, and 2 would have been too much."



Outlier Detection Method 2: Graphical

- Graphical approach to outlier detection:
 1. Look at a plot of the data.
 2. Human decides if data is an outlier.
- Examples:
 1. Box plot.
 2. Scatterplot:
 - Can detect complex patterns.

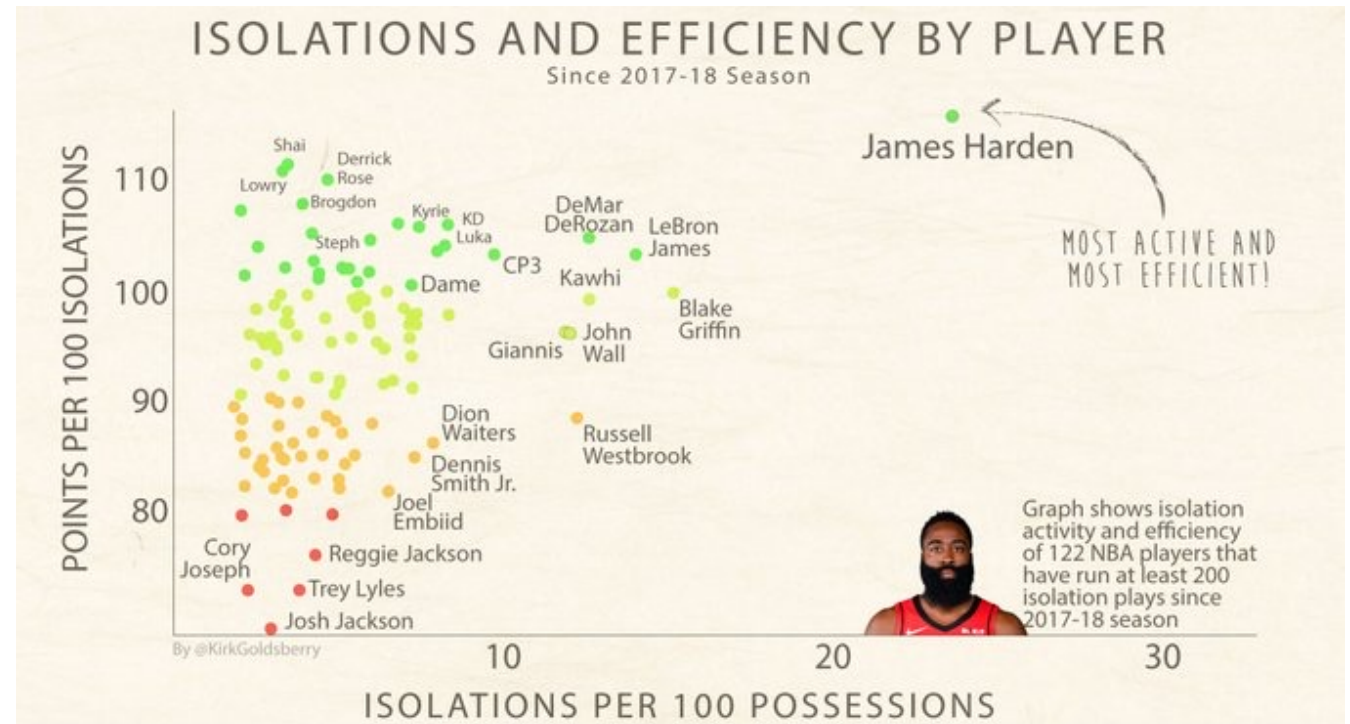


Outlier Detection Method 2: Graphical

- Graphical approach to outlier detection:
 1. Look at a plot of the data.
 2. Human decides if data is an outlier.

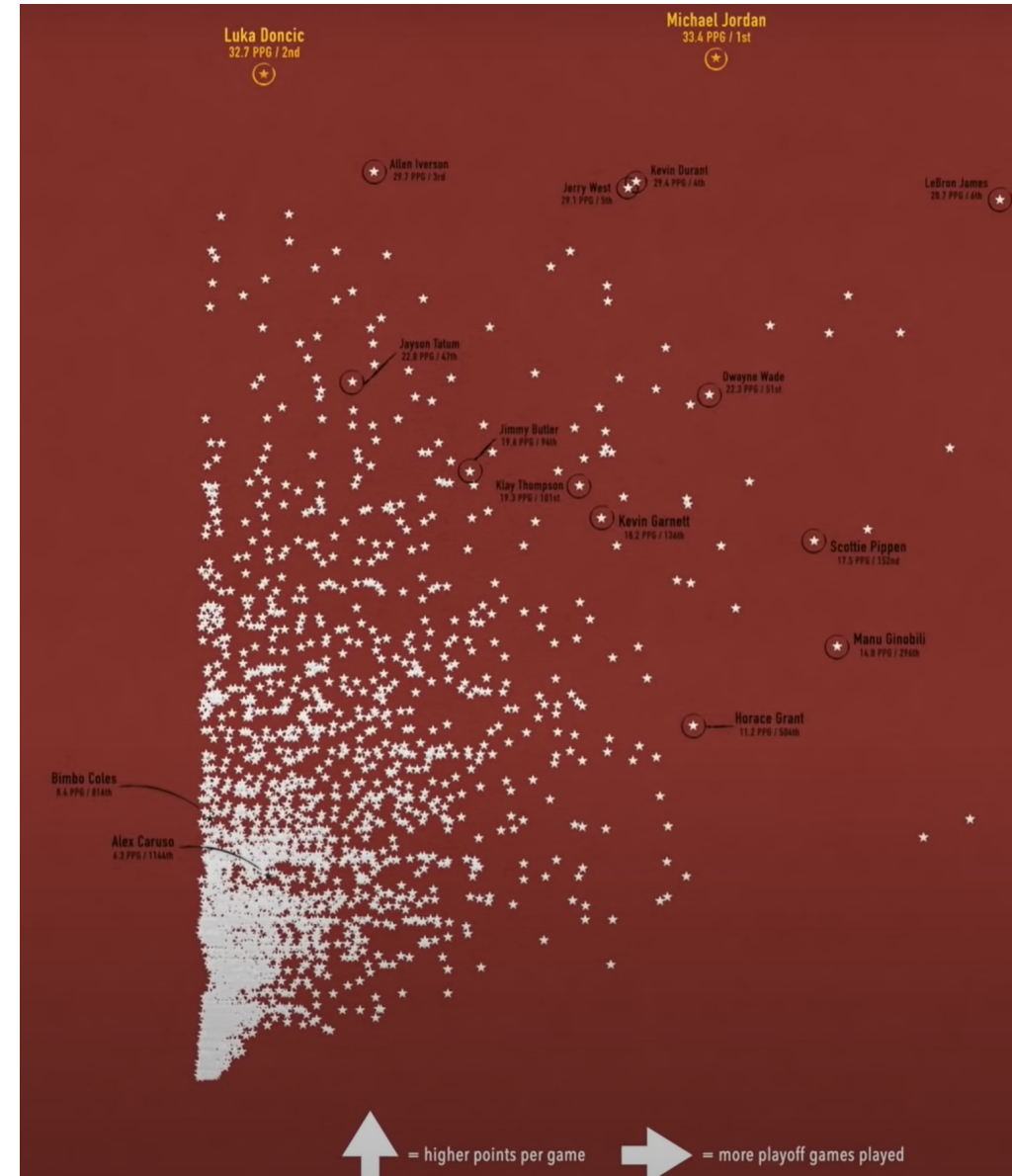
- Examples:

1. Box plot.
2. Scatterplot:
 - Can detect complex patterns.



Outlier Detection Method 2: Graphical

- Graphical approach to outlier detection:
 1. Look at a plot of the data.
 2. Human decides if data is an outlier.
- Examples:
 1. Box plot.
 2. Scatterplot:
 - Can detect complex patterns.
 - Gives an idea of “how different” outliers are.
 - But **only 2 variables at a time**.



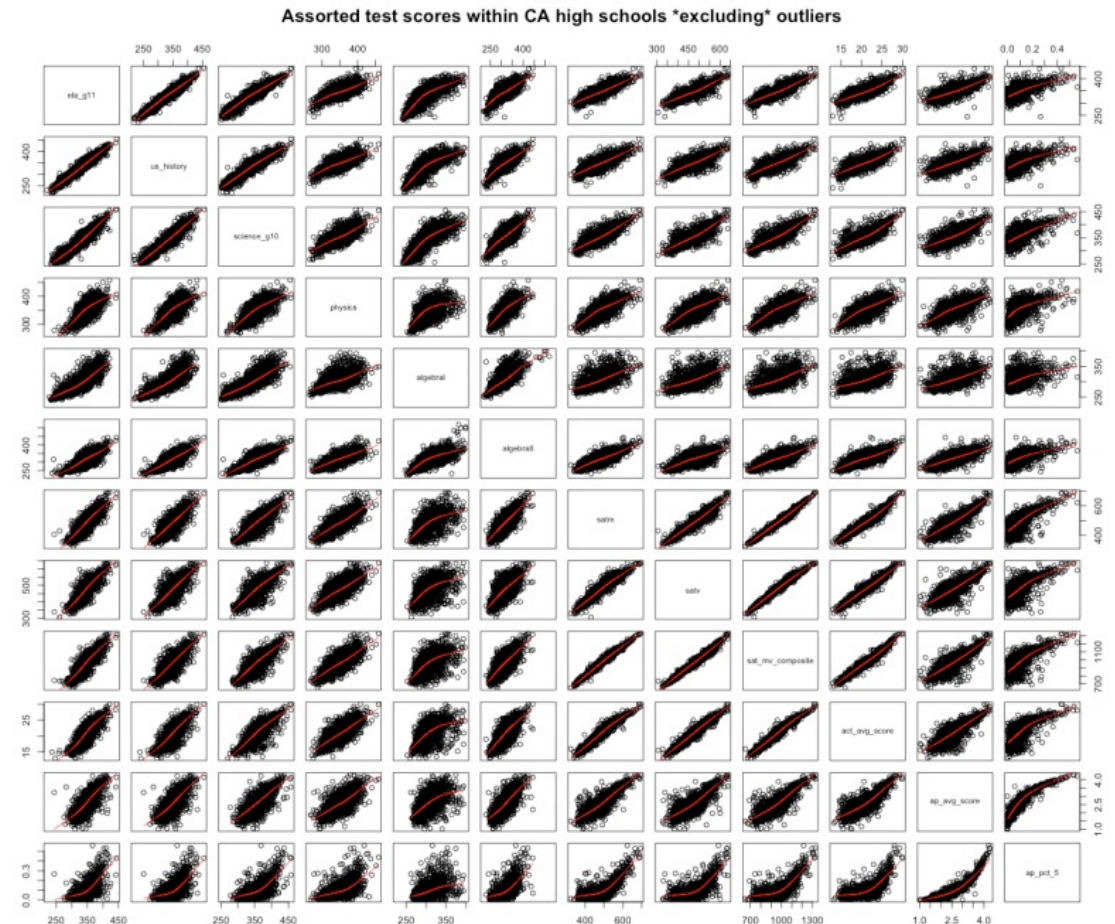
Outlier Detection Method 2: Graphical

- Graphical approach to outlier detection:

1. Look at a plot of the data.
2. Human decides if data is an outlier.

- Examples:

1. Box plot.
2. Scatterplot.
3. Scatterplot array:
 - Look at all combinations of variables.
 - But laborious in high-dimensions.
 - And still **only 2 variables at a time**.



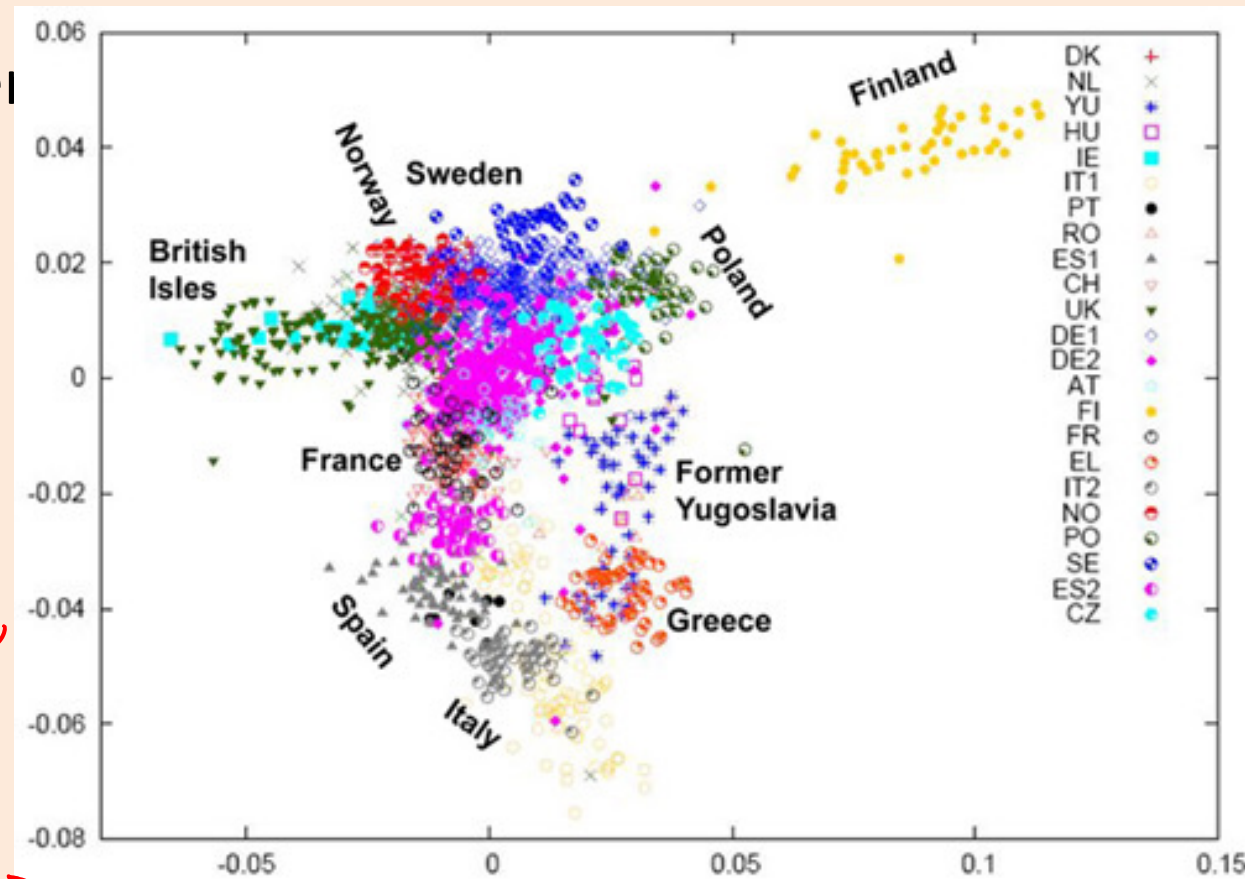
Outlier Detection Method 2: Graphical

- Graphical approach to outlier detection:

1. Look at a plot of the data.
2. Human decides if data is an outlier

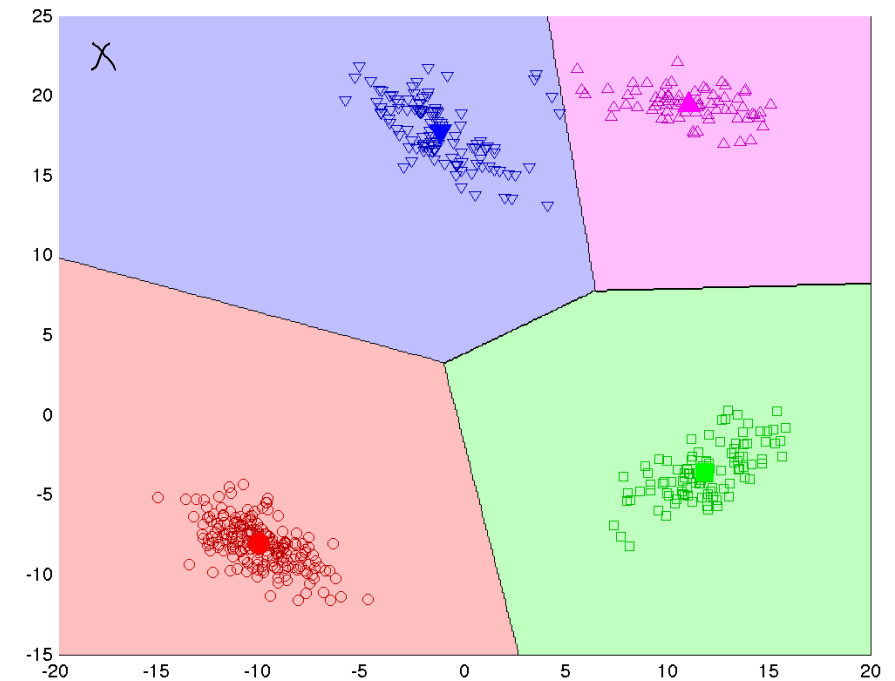
- Examples:

1. Box plot.
2. Scatterplot.
3. Scatterplot array.
4. Scatterplot of 2-dimensional PCA:
 - 'See' high-dimensional structure.
 - But loses information and sensitive to outliers.



Outlier Detection Method 3: Cluster-Based

- Detect outliers based on **clustering**:
 1. Cluster the data.
 2. Find **points that do not belong to clusters**.
- Examples:
 1. K-means:
 - Find points that are far away from any mean.
 - Find clusters with a small number of points.

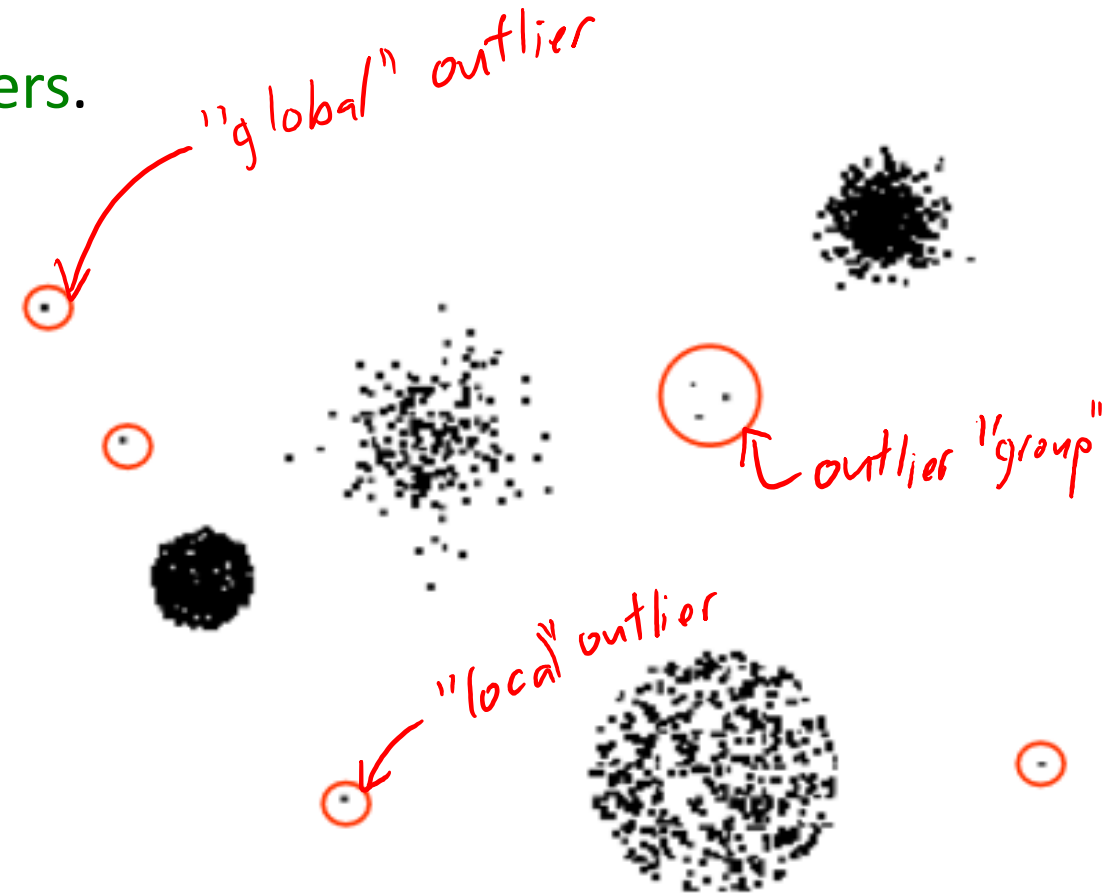


Outlier Detection Method 3: Cluster-Based

- Detect outliers based on clustering:
 1. Cluster the data.
 2. Find points that do not belong to clusters.

- Examples:

1. K-means.
2. Density-based clustering:
 - Outliers are points not assigned to cluster.



Outlier Detection Method 3: Cluster-Based

- Detect outliers based on clustering:

1. Cluster the data.

2. Find points that do not belong to clusters.

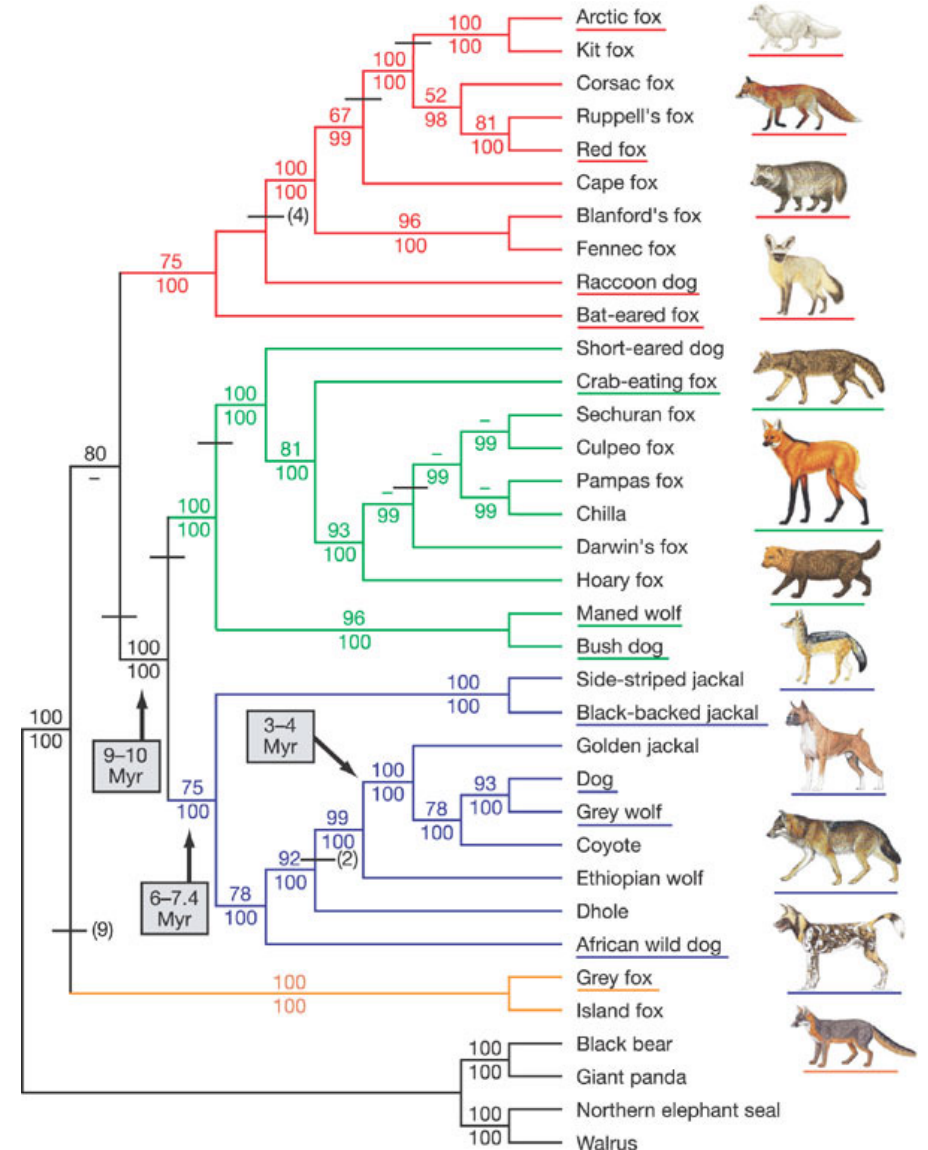
- Examples:

1. K-means.

2. Density-based clustering.

3. Hierarchical clustering:

- Outliers take longer to join other groups.
- Also good for outlier groups.



Outlier Detection Method 4: Distance-Based

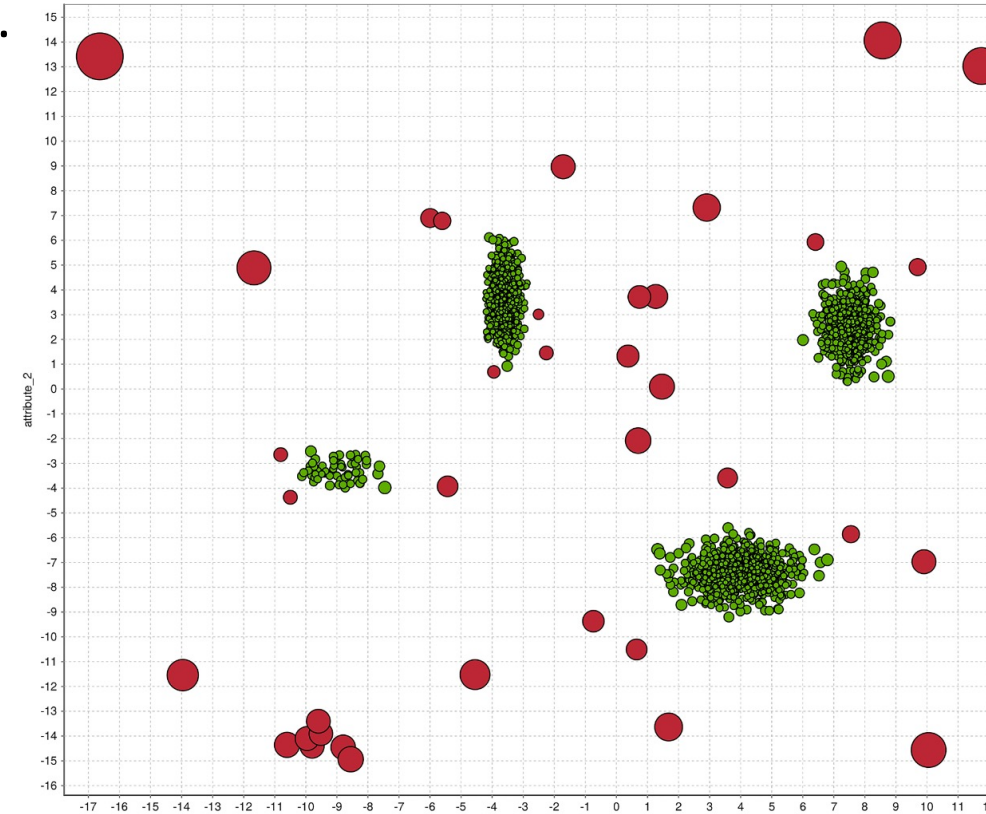
- Most outlier detection approaches are **based on distances**.
- Can we skip the model/plot/clustering and **just measure distances**?
 - How many points lie in a radius 'epsilon'?
 - What is distance to k^{th} nearest neighbour?
- UBC connection (first paper on this topic):

Algorithms for Mining Distance-Based Outliers in Large Datasets

Edwin M. Knorr and Raymond T. Ng
Department of Computer Science
University of British Columbia

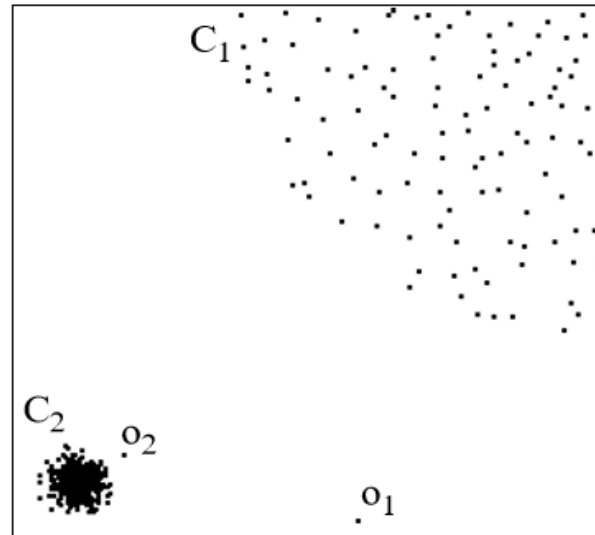
Global Distance-Based Outlier Detection: KNN

- KNN outlier detection:
 - For each point, compute the **average distance to its KNN**.
 - Choose points with biggest values (or values above a threshold) as outliers.
 - “Outliers” are points that are far from their KNNs.
- Goldstein and Uchida [2016]:
 - Compared 19 methods on 10 datasets.
 - **KNN best for finding “global” outliers.**
 - “Local” outliers best found with **local distance-based** methods...



Local Distance-Based Outlier Detection

- As with density-based clustering, **problem with differing densities:**



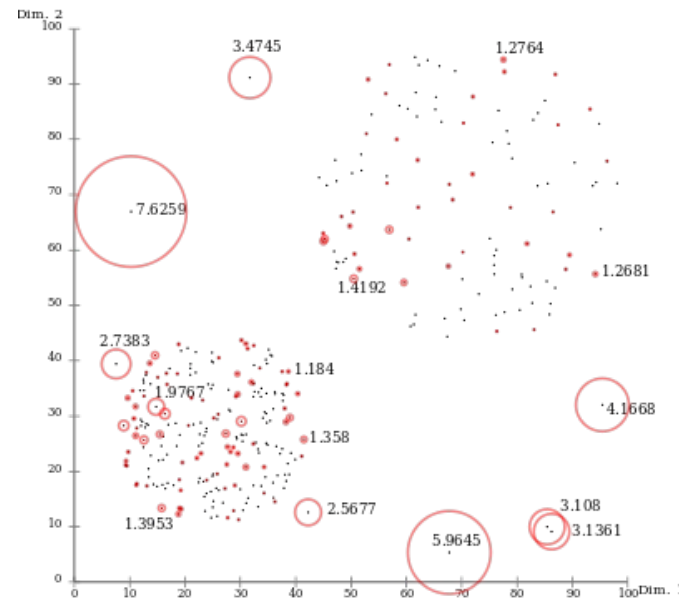
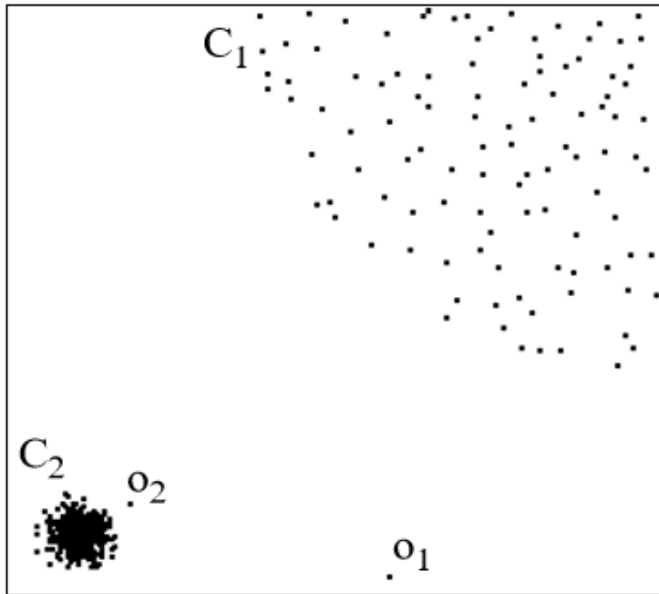
- Outlier o_2 has similar density as elements of cluster C_1 .
- Basic idea behind **local distance-based** methods:
 - Outlier o_2 is “**relatively**” **far** compared to its neighbours.

Local Distance-Based Outlier Detection

- “Outlierness” ratio of example ‘i’:

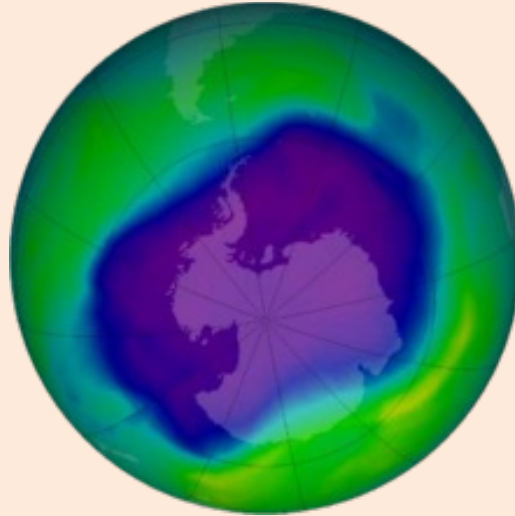
average distance of ‘i’ to its KNNs
average distance of neighbours of ‘i’ to their KNNs

- If outlierness > 1 , x_i is further away from neighbours than expected.



Problem with Unsupervised Outlier Detection

- Why wasn't the hole in the ozone layer discovered for 9 years?



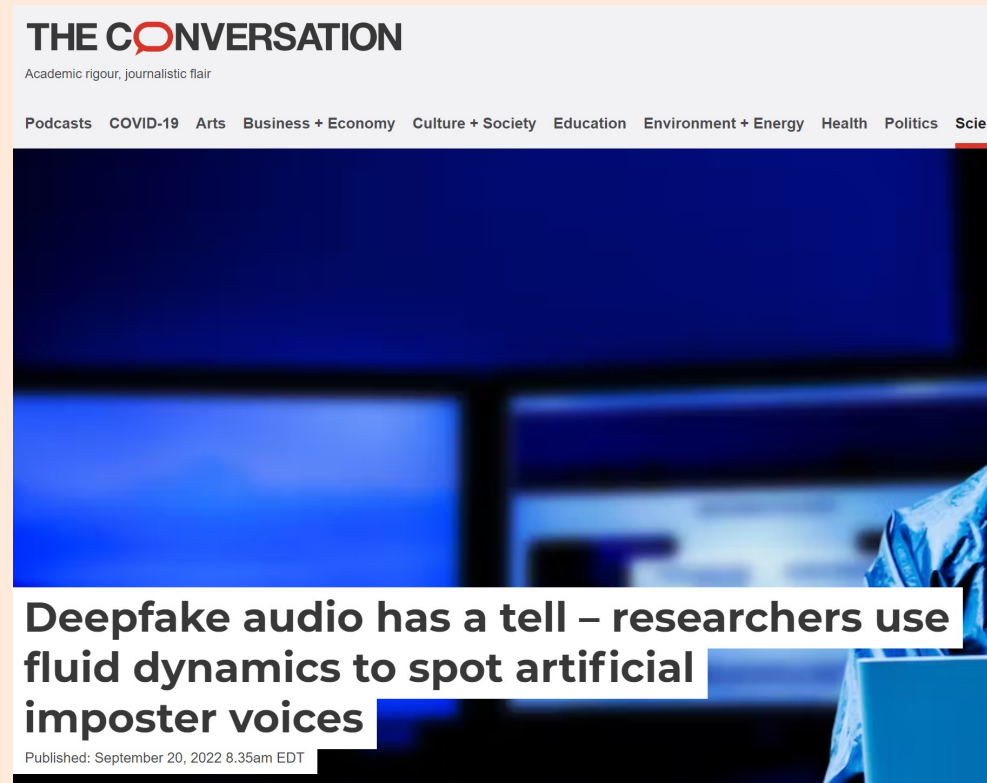
- Can be **hard to decide when to report** an outlier:
 - If **you report too many non-outliers, users will turn you off.**
 - Most antivirus programs do not use ML methods (see ["base-rate fallacy"](#))

Outlier Detection Method 5: Supervised

- Final approach to outlier detection is to use supervised learning:
 - $y_i = 1$ if x_i is an outlier.
 - $y_i = 0$ if x_i is a regular point.
- We can use our methods for supervised learning:
 - We can find very complicated outlier patterns.
 - Classic credit card fraud detection methods used decision trees.
- But it needs supervision:
 - We need to know what outliers look like.
 - We may not detect new “types” of outliers.

Limitations of Supervised Outlier Detection

- News article from last week:
 - Detects fake voices at outliers.
 - Using a fluid dynamic model.



- A model-based outlier detection method.
- In the evaluation, it works on **specific ways** to generate fake voices.
- I am 99% sure you **could design other ways that would fool it.**
 - There is no guarantee this would detect new “types” of outliers.

End of Part 2: Key Concepts

- We focused on 2 unsupervised learning tasks:
 - Clustering.
 - Partitioning (k-means) vs. density-based.
 - “Flat” vs. hierarchical (agglomerative).
 - Vector quantization.
 - Label switching.
 - Outlier Detection.
 - Difficulty in even defining the task.
 - 5 common approaches (model, graphs, clustering, distances, supervised).
 - Difficulty in deciding when to report.

Skipped Content: Finding Similar Items

- Due to lack of time, we removed the Part 2 topic “finding similar items”.
- Topics that are normally covered:
 - Original Amazon product recommendation algorithm.
 - It uses an unsupervised version of k-nearest neighbours.
 - How to solve huge k-nearest problems and other “closest point” problems.
 - Inverted indices, grid-based pruning, and very-fast approximate nearest neighbour methods.
 - Shingling, where we divide object into parts and match parts.
 - Detecting plagiarism, biological sequence alignment, anti-virus software, fingerprinting.
 - Frequent itemsets, where we find items that are often bought together.
 - The “a priori” algorithm an often-effective pruning strategy for doing this.
- If you are interested in these topics, we put our slides here:
 - <https://www.cs.ubc.ca/~schmidtm/Courses/340-F22/L10.5.pdf>

Summary

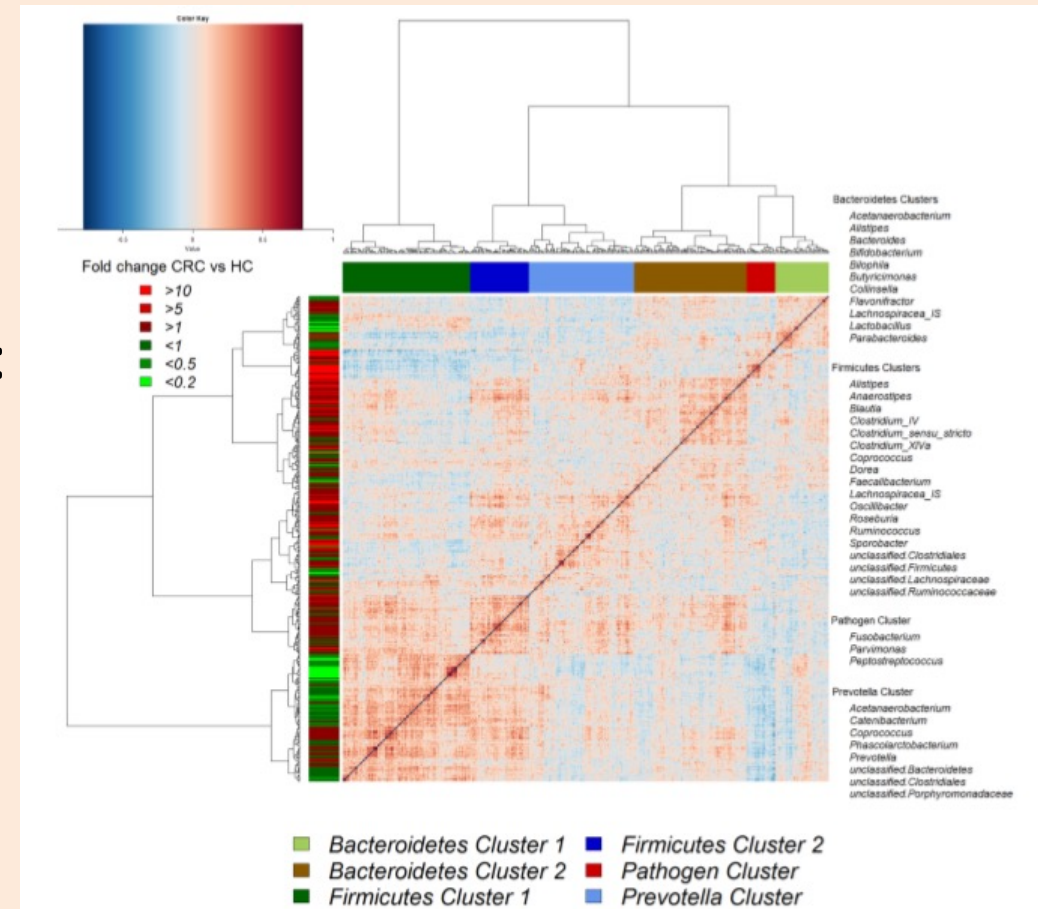
- **Biclustering**: clustering of the examples *and* the features.
- **Outlier detection** is task of finding unusually different example.
 - A concept that is very difficult to define.
- **5 approaches** for outlier detection:
 - **Model-based** find unlikely examples given a model of the data.
 - **Graphical** methods plot data and use human to find outliers.
 - **Cluster-based** methods check whether examples belong to clusters.
 - **Distance-based outlier detection**: measure (relative) distance to neighbours.
 - **Supervised-learning for outlier detection**: turns task into supervised learning.
- Next time: how do we do supervised learning with a *continuous* y_i ?

Application: Medical data

- Hierarchical clustering is very common in **medical data analysis**.
 - Clustering different samples of colorectal cancer:

– This plot is different, it's not a biclustering:

- The matrix is 'n' by 'n'.
- Each matrix element gives correlation.
- Clusters should look like “blocks” on diagonal.
- **Order of examples is reversed in columns.**
 - This is why diagonal goes from bottom-to-top.
 - Please don't do this reversal, it's confusing to me.



Issues with using z-scores for grades

I definitely sympathize with issues regarding baseline grades in different classes. The ideal solution is to encourage grades to have a standardized meaning across courses, and for courses to have a standardized difficulty, but obviously this is incredibly hard (and probably impossible).

The use of z-scores seems to be a nice solution, but I wanted to point out some potential issues:

1. Z-scores are quite sensitive to outliers. Basically, the mean will be pulled in the direction of outliers, and the variance will be made much larger by outliers. See Slide 8 here:

<https://www.cs.ubc.ca/~schmidtm/Courses/540-W20/L6.pdf>

The major way this manifests is if you have a relatively-small class, and one person just catastrophically fails the course. This has weird effects on the z-score compared to if that person was not in the class: since the average moves lower, people who are slightly below average will actually appear slightly above average. This isn't a big deal, but the more serious issue is that since the variance is made larger the people who are a bit below average will appear very-far below average. (And students well above average get pushed way above average.)

The effect is much smaller in big classes, unless you have a cluster of catastrophic fails and in that case the effect is the same.

There are easy solution to this issue by using statistics based on more-robust measures that allow outliers (for examples, see Slide 9 in that lecture).

2. Z-scores assume the distribution is unimodal. See Slide 10 here:

<https://www.cs.ubc.ca/~schmidtm/Courses/540-W20/L6.pdf>

If you have a group of "good" students and a group of "bad" students, it may reward the good group and punish the bad group more than their grade difference would justify. I think this is a less serious issue, and it's also harder to fix (you would probably need to use historic grade distribution data). In 340, I would expect the grade distribution to roughly look like this.

3. It doesn't address "skew" in the distribution. This could be the case if you have a lot of people at the very top and then the grades drop off slowly from there (another effect I've noticed in 340 grades). Similar to 2, I view this as a less-serious issue than 1 since the shifts probably aren't huge.

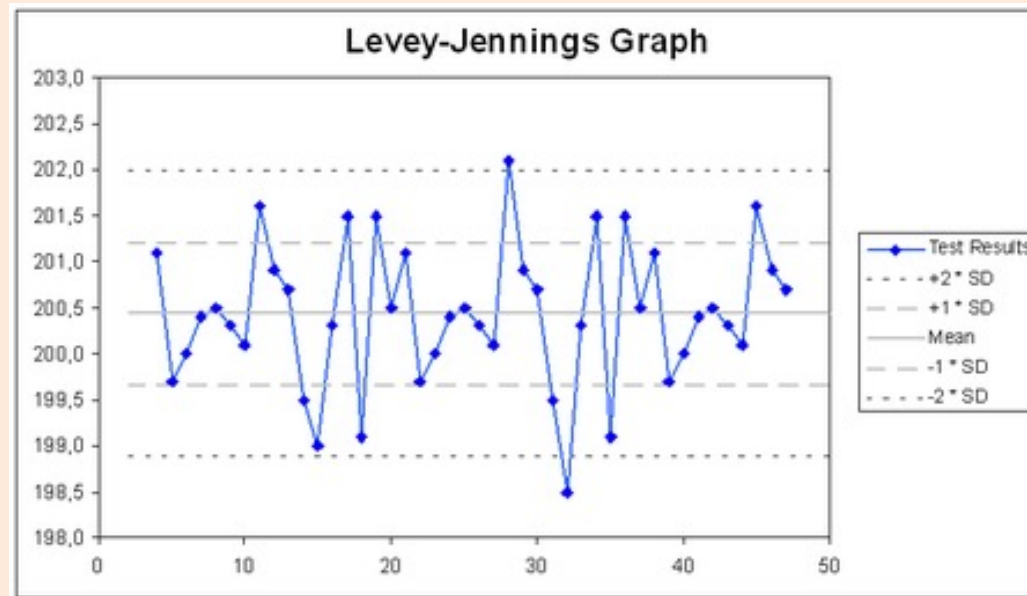
4. If you compare z-scores *across* classes, there is a confounding factor that the students may not come from the same distribution. E.g., one class may attract more strong students and one class may attract more weak students. In a simple setting where only top students take one class and only weak students take another class, the weaker "top" students will be hurt and the stronger "weak" students will be helped.

A simple approach that would address 1-3 is using quantiles. For example, just saying "student A ranked in the top 38% of grades" is simple and avoids some of the issues above. It's not perfect since it doesn't give the real spread (problematic if many students are really close, since it will push them apart). It also doesn't address issue 4, but I would be more comfortable making decisions with this than z-scores. Indeed, my criterion for whether I will write reference letters for students in class is based on ranking rather than absolute score. It's even-more informative to give the class size, like "student A ranked 14 out of 76", but that might be more-difficult to use in automated ways.

For addressing issue 4, you would really need data across classes and I would have to think about whether there is a simple/fair solution.

“Quality Control”: Outlier Detection in Time-Series

- A field primarily focusing on outlier detection is **quality control**.
- One of the main tools is plotting z-score thresholds over time:



- Usually don't do tests like " $|z_i| > 3$ ", since this happens normally.
- Instead, identify problems with tests like " $|z_i| > 2$ twice in a row".

Outlierness (Symbol Definition)

- Let $N_k(x_i)$ be the **k-nearest neighbours** of x_i .
- Let $D_k(x_i)$ be the **average distance** to k-nearest neighbours:

$$D_k(x_i) = \frac{1}{k} \sum_{j \in N_k(x_i)} \|x_i - x_j\|$$

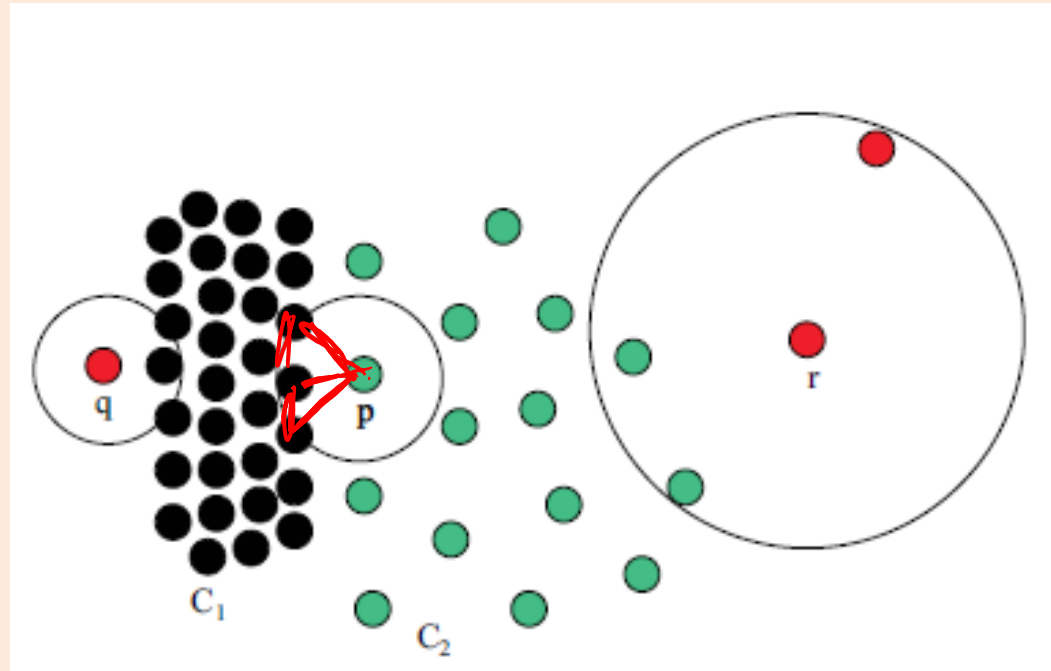
- **Outlierness** is ratio of $D_k(x_i)$ to average $D_k(x_j)$ for its neighbours 'j':

$$O_k(x_i) = \frac{D_k(x_i)}{\frac{1}{k} \sum_{j \in N_k(x_i)} D_k(x_j)}$$

- If outlierness > 1 , x_i is **further away from neighbours** than expected.

Outlierness with Close Clusters

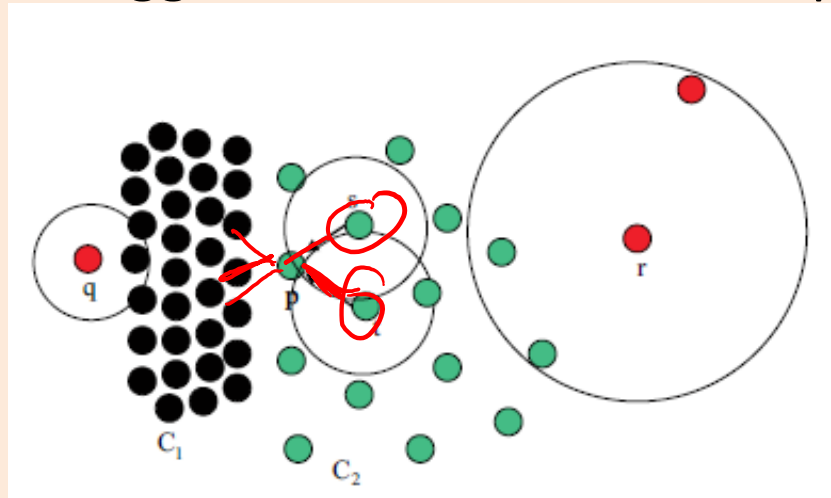
- If clusters are close, outlierness gives unintuitive results:



- In this example, 'p' has higher outlierness than 'q' and 'r':
 - The green points are not part of the KNN list of 'p' for small 'k'.

Outlierness with Close Clusters

- ‘Influenced outlierness’ (INFLO) ratio:
 - Include in denominator the ‘reverse’ k-nearest neighbours:
 - Points that have ‘p’ in KNN list.
 - Adds ‘s’ and ‘t’ from bigger cluster that includes ‘p’:



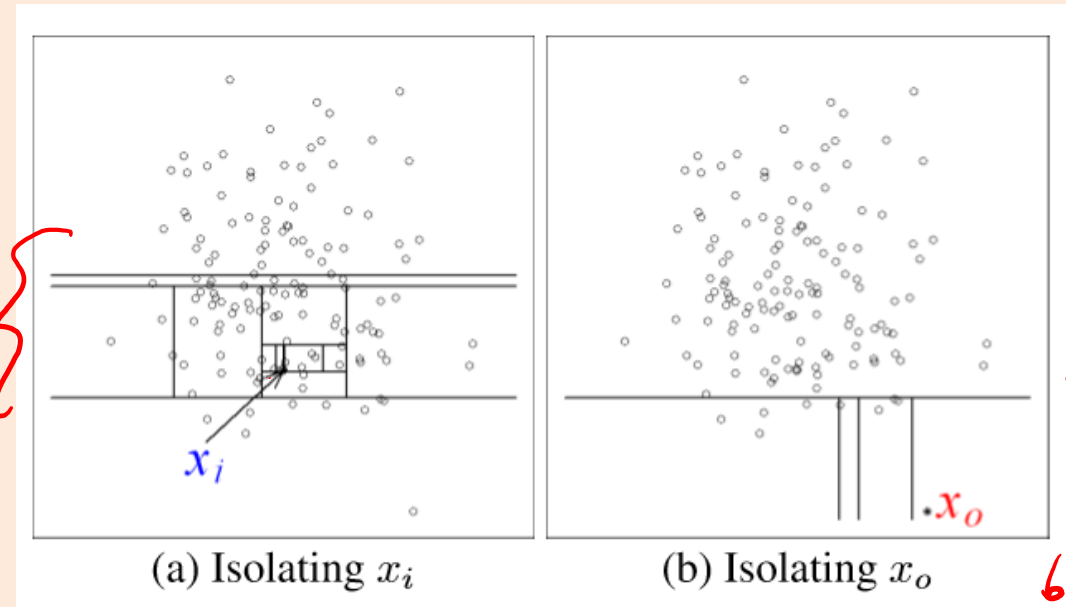
- But still has problems:
 - Dealing with hierarchical clusters.
 - Yields many false positives if you have “global” outliers.
 - Goldstein and Uchida [2016] recommend just using KNN.

Isolation Forests

- Recent method based on random trees is **isolation forests**.
 - Grow a tree where **each stump uses a random feature and random split**.
 - Stop when each example is “isolated” (each leaf has one example).
 - The “**isolation score**” is the depth before example gets isolated.
 - Outliers should be isolated quickly, inliers should need lots of rules to isolate.

Depth 12:
- needed 12
rules to isolate
so may be inlier.

- Repeat for different random trees, take average score.



depth 4
so more likely to be outlier

Training/Validation/Testing (Supervised)

- A typical supervised learning setup:
 - Train parameters on dataset D_1 .
 - Validate hyper-parameters on dataset D_2 .
 - Test error evaluated on dataset D_3 .
- What should we choose for D_1 , D_2 , and D_3 ?
- Usual answer: should all be IID samples from data distribution D_s .

Training/Validation/Testing (Outlier Detection)

- A typical outlier detection setup:
 - Train parameters on dataset D_1 (there may be no “training” to do).
 - For example, find z-scores.
 - Validate hyper-parameters on dataset D_2 (for outlier detection).
 - For example, see which z-score threshold separates D_1 and D_2 .
 - Test error evaluated on dataset D_3 (for outlier detection).
 - For example, check whether z-score recognizes D_3 as outliers.
- D_1 will still be samples from D_s (data distribution).
- D_2 could use IID samples from another distribution D_m .
 - D_m represents the “none” or “outlier” class.
 - Tune parameters so that D_m samples are outliers and D_s samples aren't.
 - Could just fit a binary classifier here.

Training/Validation/Testing (Outlier Detection)

- A typical outlier detection setup:
 - Train parameters on dataset D_1 (there may be no “training” to do).
 - For example, find z-scores.
 - Validate hyper-parameters on dataset D_2 (for outlier detection).
 - For example, see which z-score threshold separates D_1 and D_2 .
 - Test error evaluated on dataset D_3 (for outlier detection).
 - For example, check whether z-score recognizes D_3 as outliers.
- D_1 will still be samples from D_s (data distribution).
- D_2 could use IID samples from another distribution D_m .
- D_3 could use IID samples from D_m .
 - How well do you do at recognizing “data” samples from “none” samples?

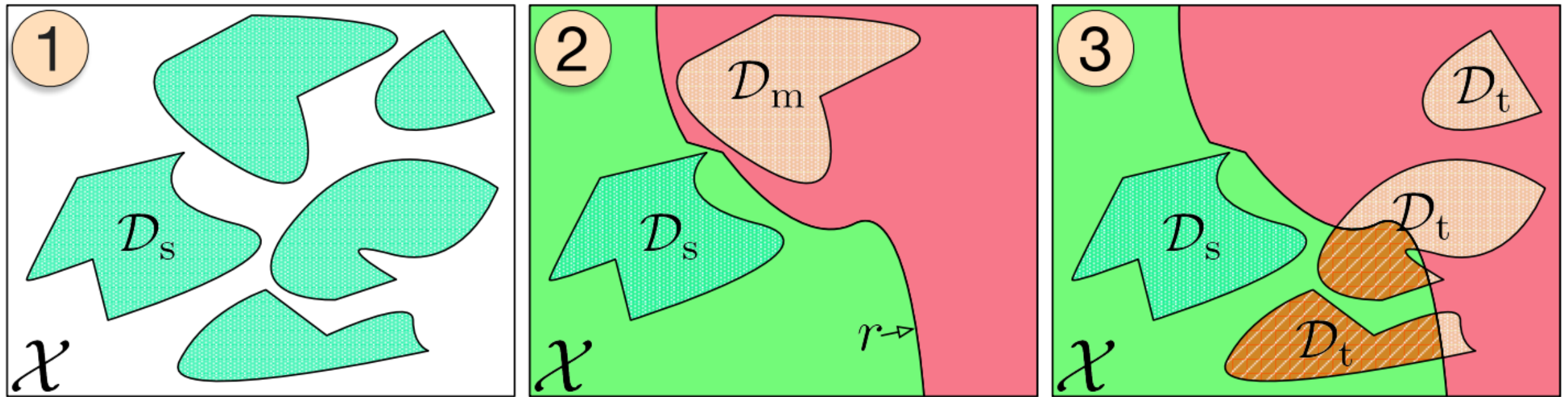
Training/Validation/Testing (Outlier Detection)

- Seems like a reasonable setup:
 - D_1 will still be **samples from D_s** (data distribution).
 - D_2 could use **IID samples from another distribution D_m** .
 - D_3 could use **IID samples from D_m** .
- What can go wrong?
- You **needed to pick a distribution D_m** to represent “none”.
 - But in the wild, your **outliers might follow another “none” distribution**.
 - This procedure can overfit to your D_m .
 - You can **overestimate your ability to detect outliers**.

OD-Test: a better way to evaluate outlier detections

- A reasonable setup:
 - D_1 will still be **samples from D_s** (data distribution).
 - D_2 could use **IID samples from another distribution D_m** .
 - ~~D_3 could use **IID samples from D_m** .~~
 - D_3 could use **IID samples from yet-another distribution D_t** .
- “How do you perform at detecting different types of outliers?”
 - Seems like a harder problem, but arguably closer to reality.

OD-Test: a better way to evaluate outlier detections



- “How do you perform at detecting different types of outliers?”