The Master Theorem

Let $a \geq 1$, $b > 1$ be real constants, let $f(n) : \mathbf{N} \rightarrow \mathbf{R}^+$, and let $T(n)$ be defined by

$$T(n) = \begin{cases} aT(n/b) + f(n) & \text{if } n \geq n_0 \\ \Theta(1) & \text{if } n < n_0 \end{cases}$$

where n/b might be either $\lfloor n/b \rfloor$ or $\lceil n/b \rceil$. Then

1. If $f(n) \in O(n^{(\log_b a) - \varepsilon})$ for some $\varepsilon > 0$ then $T(n) \in \Theta(n^{\log_b a})$.

2. If $f(n) \in \Theta(n^{\log_b a} \log^k n)$ for some constant $k \geq 0$, then $T(n) \in \Theta(n^{\log_b a} \log^{k+1} n)$.

3. If $f(n) \in \Omega(n^{(\log_b a) + \varepsilon})$ for some $\varepsilon > 0$, and $af(n/b) < \delta f(n)$ for some $0 < \delta < 1$ and all n large enough, then $T(n) \in \Theta(f(n))$.