CPSC 320: Tutorial 1 solutions

1. Let $S_n = \sum_{i=0}^{n} a^i$

   Now, consider $aS_n - S_n = a\sum_{i=0}^{n} a^i - \sum_{i=0}^{n} a^i$

$$
\begin{aligned}
aS_n - S_n &= a\sum_{i=0}^{n} a^i - \sum_{i=0}^{n} a^i \\
(a-1)S_n &= (a^{n+1} + a^n + \cdots + a^2 + a^1) - (a^n + a^n - 1 \cdots + a^1 + a^0) \\
(a-1)S_n &= a^{n+1} - 1 \\
S_n &= \frac{a^{n+1}-1}{a-1} = \sum_{i=0}^{n} a^i
\end{aligned}
$$

2. Let's try $n_0 = a$.

$$
\begin{aligned}
(n+a)^b \leq (n+n)^b &\leq cn^b \\
(2n)^b &\leq cn^b \\
2^b n^b &\leq cn^b \\
2^b &\leq c
\end{aligned}
$$

   so take $c = 2^b$ for $(n+a)^b \epsilon O(n^b)$

   Now for $(n+a)^b \epsilon \Omega(n^b)$. The case of $a \geq 0$ is trivial, since, by inspection, $(n+a)^b \geq cn^b$ for $c = 1$.

   Consider $a < 0$. Let's try $n_0 = 2|a|$.

$$
\begin{aligned}
(n+a)^b &\geq (n - \frac{n}{2})^b \\
&\geq (\frac{n}{2}^b) \\
&\geq 2^{-b} n^b
\end{aligned}
$$

so take $c = 2^b$

3. We will disprove this by counterexample.

   Take $f(n) = 2n$, and $g(n) = n$. Clearly, $f(n) \epsilon O(g(n))$.

   However, $2^{f(n)} = 2^{2n} = 4^n$, while $2^{g(n)} = 2^n$

4. We know that the algorithm will terminate exactly when the last woman is proposed to. (Women are always paired from their first proposal, and so if all women have been proposed to, then they are all paired, giving us n pairs, so no men are left unpaired.) Now, if a man is paired with the last woman on his list, this means that he has proposed to all the other women, and so they must all be paired. Consequently, the moment he proposes to the last woman on his list, the algorithm will terminate. The algorithm only terminates once, so only one man can terminate it by proposing the the last women on his list.