TEXTBOOK

CH 34

~~MIDTERM EXAM 3~~

CPSC 320

2010/04/01

LET'S LOOK @ PROBLEM DIFFICULTY

(UPPER-BOUND ON PERF OF BEST ALG TO SOLVE THE PROB)

EXAMPLES

P → POLYNOMIAL: BOUNDED BY $O(n^k)$ WHERE k is a constant & $k \in \mathbb{Z}^+$

$O(1)$	trivial	
$O(\lg n)$	e.g., binary search	FAST
$O(n)$	e.g., select, min, max	FAST
	online algorithms (if space use $\in O(1)$)	
$O(n^2)$	LCS	kinda fast
⋮		

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NP-COMPLETE
- TRAVELING SALESPERSON
- LONGEST (SIMPLE) PATH
- VERTEX COVER
- INDEPENDENT SET
- GRAPH COLOURING
- 0-1 KNAPSACK
- SATISFIABILITY
- MINESWEEPER
- AND SO ON...

PSPACE-COMPLETE
- QUANTIFIED SAT
- PLANNING
- CHESS/CHECKERS — SCALABLE
- ~~[illegible]~~ SOKOBAN AND SO ON...

UNDECIDABLE

HALTING: "GIVEN A PROGRAM P AND INPUT I, DETERMINE: DOES P $\boxed{\text{HALT}}$ WHEN RUN ON I."

ANY OTHER NON-TRIVIAL PROPERTY.

DEF'N OF NP:

A PROBLEM IS IN NP IFF:

GIVEN A PROBLEM INSTANCE AND A ~~CER~~ CERTIFICATE, WE CAN VERIFY OR REPUDIATE THE CERT. IN POLY TIME (IN # OF BITS IN PROB INSTANCE REPR.)

FOR EXAMPLE: GRAPH COLOURING

PROBLEM: CAN WE COLOUR w/ k COLOURS?

ORACLE WILL RETURN?

FOR EVERY VERTEX V,

GIVE A COLOUR FROM $\{1, \ldots, k\}$

VERIFY?

FOR EVERY $(U,V) \in$ EDGES

CHECK IF U's COLOUR == V's COLOUR:

IF SO, RETURN NO

RETURN YES

✓ PROVES $GC \in NP$.

DECISION PROBLEMS VS. OPTIMIZATION PROBLEMS

ONE IS IN P IF THE OTHER IS

$GC_{DEC} \equiv$ CAN YOU COLOUR W/ k COLOURS

$GC_{OPT} \equiv$ WHAT'S THE SMALLEST k^* S.T. YOU CAN COLOUR?

GC_{DEC} ALG: ASK FOR k^* FROM GC_{OPT}

RETURN $k^* \leq k$.

GC_{OPT} ALG: FOR $i = 1$ TO $|V|$:

ASK GC_{DEC} DOES i WORK?

IF YES: RETURN i

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SAT WAS PROVEN NP-COMPLETE IN ~1970
BY COOK, LEVIN
CANADIAN

$\underbrace{\text{IF } \underline{\text{SAT}} \in P, \quad P = NP.}$

NP-HARD + PROBLEM ∈ NP = NP-COMPLETE

TODO: PROVE THAT INDEPENDENT SET IS NP-COMPLETE.

IS: GIVEN GRAPH + k, IS THERE $S \subseteq V$ s.t. no two $u, v \in S$ ARE CONNECTED AND $|S| \geq k$

STRATEGY: ① SHOW IS ∈ NP

② REDUCE SOME NP-COMPLETE PROBLEM X TO IS.

THEN, IF IS ∈ P, X ∈ P
IF X ∈ P, P = NP (b/c X is NPC)

∴ IF IS ∈ P, P = NP.

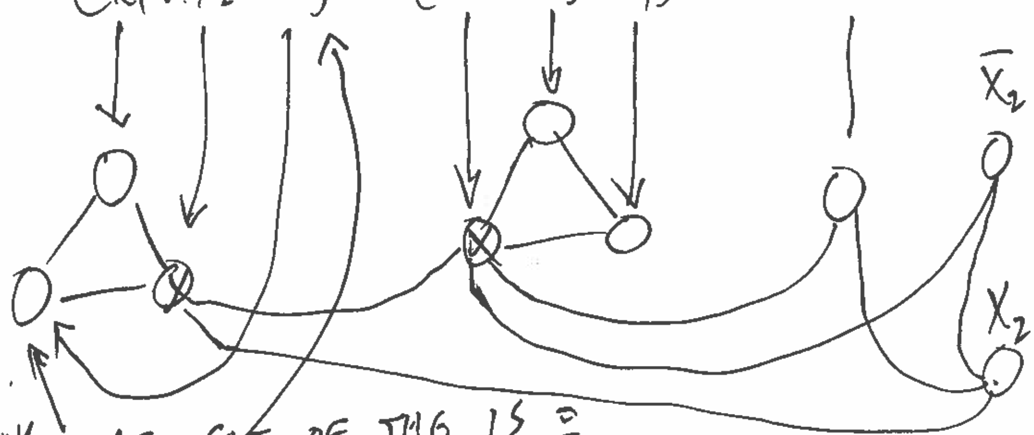
① CERT: The set S.
VERIFIER: CHECK THAT $|S| \geq k$.
FOR EACH PAIR $u, v \in S$:
CHECK THAT $(u,v) \notin E$ ✓

(2) REDUCE 3SAT TO IS

3SAT is about satisfiability of

THINGS LIKE

$$(x_1 \vee \bar{x}_2 \vee x_3) \wedge (x_2 \vee x_3 \vee \bar{x}_4) \wedge \dots (\bar{x}_2 \vee \bar{x}_1 \vee x_4 \dots)$$



CHOOSING AS ELT OF THE IS ≡

SETTING TO BE TRUE.

THE IS HAS ≤ 1 OF EACH △'s NODES.

IF THERE ARE C CLAUSES,

WE MAKE C △s.

ASK FOR AN IS w/ ≥ C NODES,

FOR EVERY PAIR OF NODES ~~RE~~ REPR A VAR + IT'S NEGATION, CONNECT W/AN EDGE.

TO PROVE: ANY SOL'N TO 3SAT ⟷ SOL'N TO IS.

1. REDUCTION TAKES POLY TIME ✓
2. ANY SOL'N TO 3SAT BECOMES A SOL'N IN IS ✓
3. ANY SOL'N TO IS REPR A SOL'N TO 3SAT ✓

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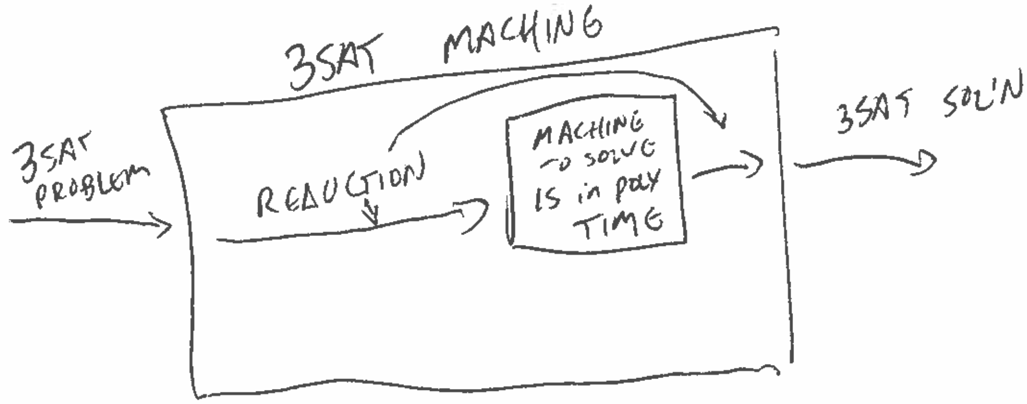

WE KNOW SAT $\in$ NPC

HOW DO WE PROVE 3SAT $\in$ NPC?

1. PROVE 3SAT $\in$ NP
2. REDUCE IN POLY TIME SAT $\rightarrow$ 3SAT
   a. SHOW REDUCTION TAKES POLY TIME
   b. SHOW SAT SOL'N $\rightarrow$ 3SAT SOL'N
   c. SHOW 3SAT SOL'N $\rightarrow$ SAT SOL'N

NOTE: ANY 3SAT PROBLEM IS A SAT PROBLEM.
SO, 3SAT $\in$ NP

CONSIDER A CLAUSE:

$$(x_1 \vee x_2 \vee x_3 \vee \dots \vee x_n)$$

$$\underbrace{\overline{(x_1 \vee x_2)} \rightarrow x'}, \quad \underbrace{x' \rightarrow (x_3 \vee \dots \vee x_n)}$$

$$\overline{\overline{(x_1 \vee x_2)}} \vee x' = (x_1 \vee x_2 \vee x') \qquad (\bar{x}' \vee x_3 \vee \dots \vee x_n)$$