CPSC 320
2010/03/02

ANNCTS

- EVALS DUE WED.
- MT#2 REF SHEET DUE END OF DAY TUE 9 MAR
- PRACTICE MT#2 COMING SOON

CONSIDER activities in OPT(S) that CONFLICT with a_i

$|ALT_i| =$

- $\emptyset$ not possible b/c then $OPT(S) \cup \{a_i\}$ would be better than $OPT(S)$ ✗
- 1 swap $\{a_i\}$ for ALT_i and get $OPT'(S)$ that looks more like GREEDY. RECURSE ✓
- ≥ 2 at least one elt of ALT_i finishes before a_i BUT a_i is the FIRST activity on which G & O differ. ✗

EVENTUALLY OPT & GREEDY DO NOT DIFFER

CONTRA of ASSUMPTION $|OPT| > |GREEDY|$

PRECISELY: $OPT'(S) = OPT(S) - ALT_i \cup \{a_i\}$

ALG#1: [PRIM'S]

CHOOSE a root
Add root to S
Make MST = ∅
Add all other ~~node~~ vert. v to a PQ, Q
with priority = $\begin{cases} \infty & \text{if no edge root} \to v \\ w(\text{root}, v) & \text{if edge root} \to v \text{ w/ weight } w(\text{root}, v) \end{cases}$
also set v.pred = root

Prim's Alg

While Q is not empty
 v = DELETE_MIN(Q) — bin heap: $\boxed{O(\lg |V|)}$
 Add v to S
 Add (v.pred, v) to MST
 Go through all vert. u adj to v:] $O(\deg(v))$
  if u ∉ S and u.priority > w(v,u): – O(1)
   u.priority = w(v,u) – O(1)
   u.pred = v – O(1)
   DECREASE_KEY(Q, u, u.priority)

O(|V|)

inner loop goes overall O(|E|) times

RETURN MST

??

bin heap $O(\lg |V|)$
Fib heap amort O(1)

$O(|E| + |V| \lg |V|)$

$O(\cancel{|V| + |E| \lg |V|})$
(w/ bin heap)

$O((|V| + |E|) \lg |V|) \overset{?}{=} O(|E| \lg |V|)$
(if connected)

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ALG #2 [KRUSKAL'S]

$O(1)$ $O(|V|)$ Initialize $MST = \{\}$

$O(|E| \lg |E|)$ Sort edges in increasing order by weight (e.g., using a stable MERGE-SORT) — $O(|E| \lg |E|)$ $\subseteq$ $\boxed{O(|E| \lg |V|)}$

$|E|$ ← For each edge (u,v) in order

(w/out path comp.) reps $O(\lg|V|)$ [if u & v are not already connected:

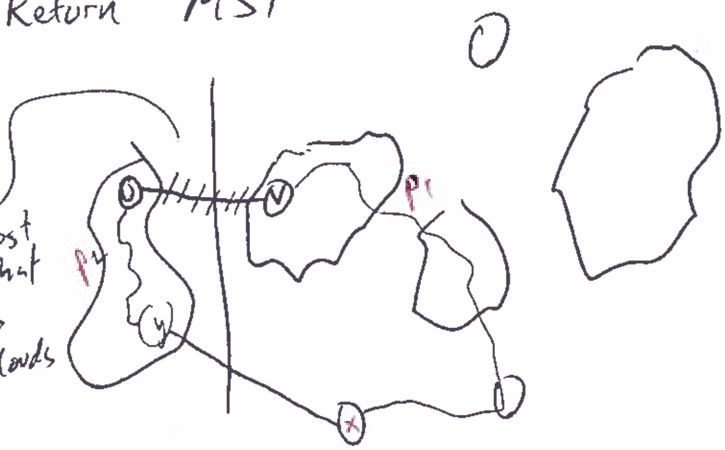
$MST = MST \cup \{(u,v)\}$

~~Connect u & v~~

$O(\lg|V|)$ — [Record that u & v are connected

Return MST

min cost edge that connects two clouds



MSF so far
(i.e. spanning forest)

same
$MSF \subseteq MST$

$(x,y) \notin MSF$

UP-TREES BY TARJAN

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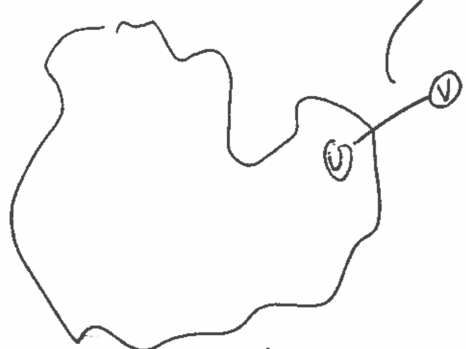
DOES PRIM'S WORK?

(u,v)
min. cost edge that "sticks out" of the cloud



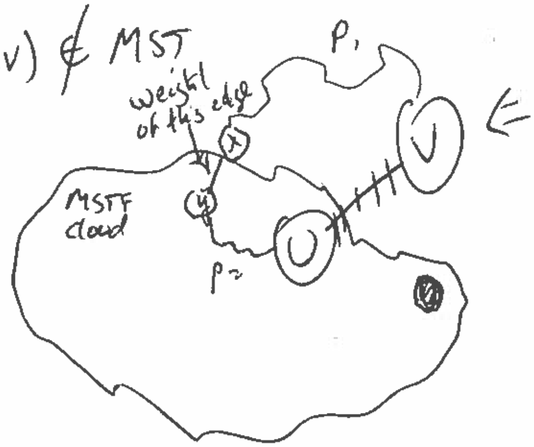
ALL NODES CONNECTED VIA MST FRAGMENT (in CLOUD)

every ~~edge in MSTF~~ is part of ~~some~~ MST

$MSTF \subseteq MST$ for some MST

is $MSTF \cup \{(u,v)\} \subseteq \overset{\text{some}}{MST}$

Consider MST s.t. $MSTF \subseteq MST$ but $(u,v) \notin MST$



$w(v,u) \le w(y,x)$

$MST' = MST - \{(y,x)\} \cup \{(u,v)\}$

sol'n: Yes. B/c del. (x,y) disconnects + adding (u,v) reconnects

opt: $w(MST') \le w(MST)$ b/c $\underset{(tot)}{w(MST')} = w(MST) - w(x,y) + w(u,v)$