2010/01/28
CPSC 320

QUICK-SORT (A)
if $|A| \leq 1$
return A
else
pick an a_i in A (i is index of some elt in A)
Divide A into $A_L = [a_j \mid a_j < a_i \wedge a_j \in A]$
and $A_R = [a_j \mid a_j > a_i \wedge a_j \in A]$

Assume all $a \in A$ are distinct

QUICK-SORT(A_L)
QUICK-SORT(A_R)
return A_L ++ [a_i] ++ A_R

OK, so tag each a with its original position and break ties by tag.

WORST-CASE, a_i is an extreme element

$$T(n) = \underbrace{T(0)}_{\text{cut-out}} + \underbrace{T(n-1)}_{\text{all but pivot}} + \underbrace{cn}_{\text{partition}}$$

$$= T(n-1) + cn$$

$$= cn + c(n-1) + c(n-2) + \ldots + c1$$

$$= c\,\frac{n(n+1)}{2} \in \Theta(n^2)$$

BEST CASE: a_i is median

$$T(n) \leq 2T\left(\frac{n}{2}\right) + cn$$

MASTER THM

$a = 2$
$b = 2$
$f(n) = cn \in \Theta(n)$
$n^{\log_b a} = n^{\log_2 2} = n^1 = n$

$n^{\log_b a} \in \Theta(f(n))$

$T(n) \in O(n \lg n)$

and this is a comparison-based sort

so $T(n) \in \Omega(n \lg n)$

$T(n) \in \Theta(n \lg n)$

WAYS TO PICK PIVOT

1. FIRST ELEMENT — COMMON CASE IS WORST
2. RANDOM (unif @ random) — WORST ONLY AT RANDOM
3. CLOSE TO MEAN
4. MEDIAN — CAN THIS BE DONE IN $O(n)$
5. MEDIAN-OF-THREE
6. TAKE A SAMPLE MEDIAN OF $k \ll n$ elts

REMEMBER SUPER-LUCKY-SORT: $O(n)$ b/c it discarded $\frac{n}{c}$ elts at each round

Not QUITE ANALOGOUS

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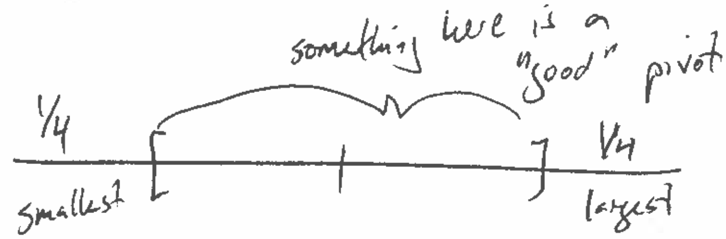
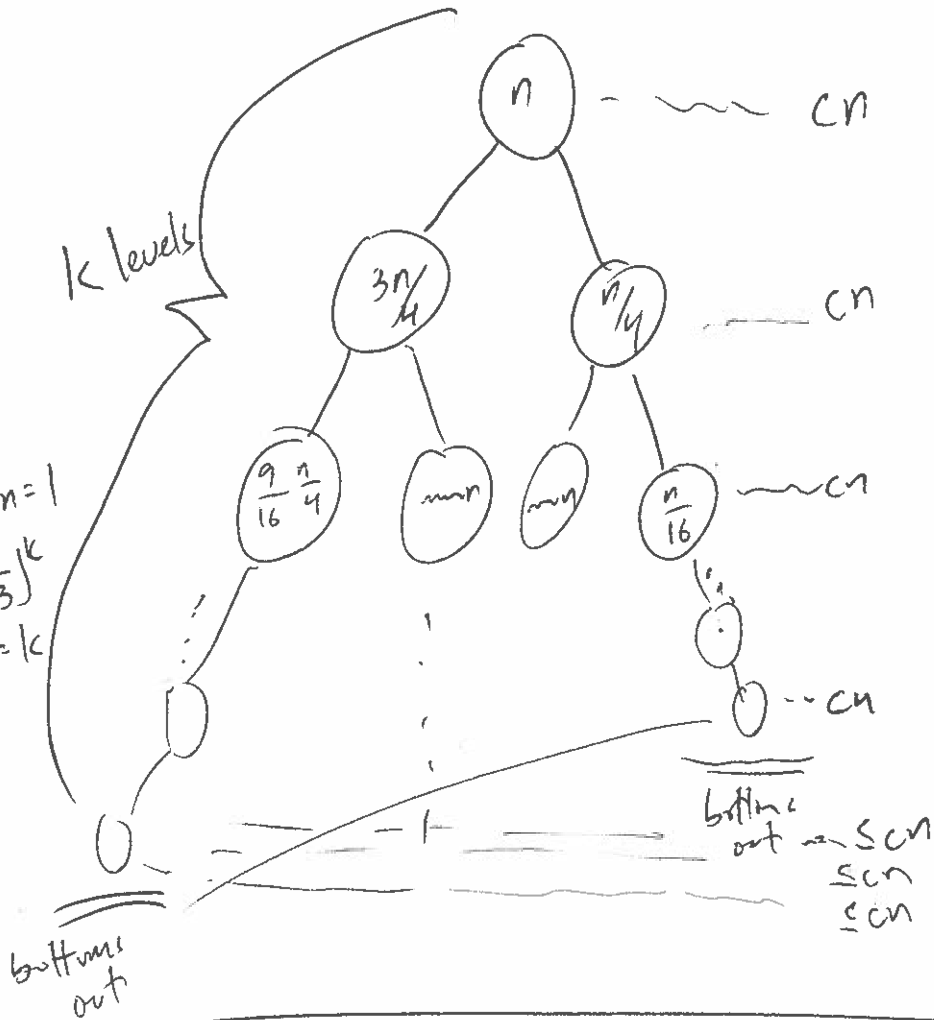


Odds of good pivot: 50%

$$T(n) \le T\left(\frac{n}{4}\right) + T\left(\frac{3n}{4}\right) + cn$$



$\left(\frac{3}{4}\right)^k n = 1$

$n = \left(\frac{4}{3}\right)^k$

$\log_{4/3} n = k$

$$\le cn \log_{4/3} n \in \Theta(n \lg n)$$

How many coin flips before heads? X = # of flips before heads

$$E[X] = \underbrace{\sum_{i=0}^{\infty} x \Pr[X=x]}_{\text{def'n}} = \sum_{i=1}^{\infty} i \Pr[X=i]$$

$$= 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} + 3 \cdot \frac{1}{8} + \dots$$

1/4 2/4 3/8

3

$$2E[X] = 1 + \frac{2}{2} + \frac{3}{4} + \frac{4}{8} + \dots$$

$$2E[X] - E[X] = E[X] = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = \frac{1}{1-\frac{1}{2}}$$

Let X_i = time spent on recursion tree levels when largest A fragments are of size $n\left(\frac{3}{4}\right)^i \ldots n\left(\frac{3}{4}\right)^{i+1}$

$$E[\text{Sum over all } X_i] = E[X_0] + E[X_1] + \ldots + E[X_k]$$
$$\leq E[X_0] + E[X_1] + \ldots + E[X_{\log_{4/3} n}]$$

$E[X_i] = ?$

$n(3/4)^i$ Assume bad pivots do not help
pay cn for each bad pivot

$\vdots$

$n(3/4)^{i+1}$ One good pivot completes this range and we pay cn.

$$E[X_i] \leq 2cn$$

$$E[\text{Sum of } X_i] \leq \sum_{i=0}^{\log_{4/3} n} 2cn = (\log_{4/3} n + 1) 2cn$$

EXPECTED PERFORMANCE $\in O(n \lg n)$
OF QUICK-SORT w/ RANDOM PIVOT is $\Theta(n \lg n)$

QUICK-MEDIAN: