CPSC 320
2014/01/26

Handy identities

$a^{\log_b c} = c^{\log_b a}$

(Since:

$\log_b a^{\log_b c} = \log_b c \cdot \log_b a$

$= \log_b a \cdot \log_b c$

$= \log_b c^{\log_b a}$

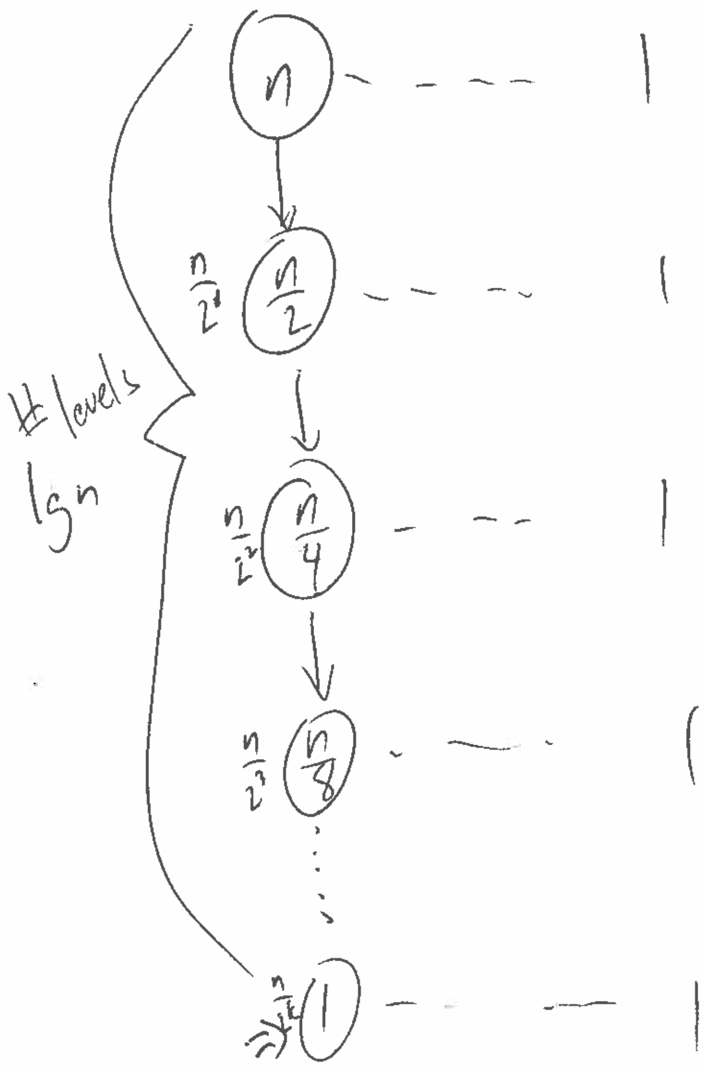
for $0 < a < 1$:

$$\sum_{i=0}^{n} a^i = \frac{a^{n+1} - 1}{a - 1}$$

$$\sum_{i=0}^{\infty} a^i = \frac{1}{1-a}$$

$T(n) = T(n/2) + 1$ $\quad T(n) \in \Theta(\lg n)$

$T(1) = 1$



$\frac{n}{2^k} = 1$

$n = 2^k$

$\lg n = k$

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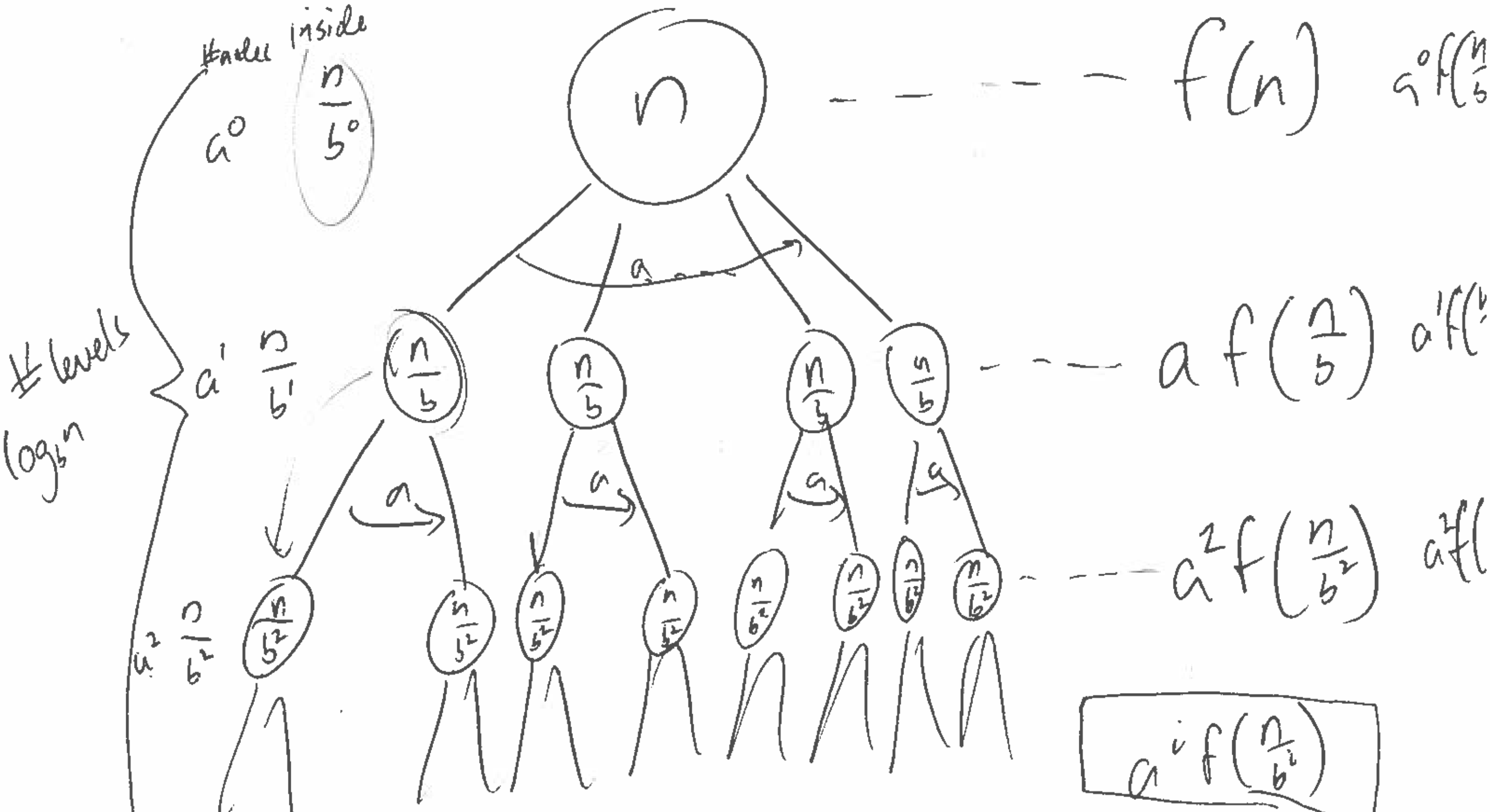
$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$



$$\boxed{a^i f\left(\frac{n}{b^i}\right)}$$

if $f(n) \in \Theta(n^{\log_b a})$, then

$$a^i f\left(\frac{n}{b^i}\right) \in \Theta\left(a^i \left(\frac{n}{b^i}\right)^{\log_b a}\right)$$

$$a^i \left(\frac{n}{b^i}\right)^{\log_b a} = a^i \frac{n^{\log_b a}}{b^{i \log_b a}} = a^i \frac{n^{\log_b a}}{b^{\log_b a^i}} = \cancel{a^i} \frac{n^{\log_b a}}{\cancel{a^i}} = n^{\log_b a}$$

$a^k \mid = \frac{n}{b^k}$

leaves

$n = b^k$

$\log_b n = k$

a^k leaf nodes each contributing $\Theta(1)$

$$a^{\log_b n} \cdot c = n^{\log_b a} \cdot c$$

internal contribution: $\displaystyle\sum_{i=0}^{\log_b n - 1} a^i f\left(\frac{n}{b^i}\right)$

$\underline{\text{Not}}$ case 1: $f(n) \in O\left(\frac{n^{\log_b a}}{\lg n}\right)$

$T(n) = \cancel{3}\,3T(\lfloor \frac{n}{4} \rfloor) + n^2$

$T(1) = 1$

$n^{\log_b a} = n^{\log_4 3} \approx n^{0.79}$

$\frac{a = 3}{b = 4}$

$f(n) = n^2$

Is $f(n) \in \Omega(n^{\log_b a + \epsilon})$ for $\epsilon > 0$

Let $\epsilon \approx 0.01$

Is $n^2 \in \Omega(n^{0.8})$? YES

Is it true for some $c < 1$ and $n \geq n_0$

that $3f(\lfloor \frac{n}{4} \rfloor) \leq c f(n)$

Is $3 \lfloor \frac{n}{4} \rfloor^2 \leq c n^2$

Let $c = 1/2$

$3 \lfloor \frac{n}{4} \rfloor^2 \leq 3 \frac{n}{4}^2 = \frac{3n^2}{16} \leq \frac{1}{2} n^2 = c$

✓

$\therefore T(n) \in \Theta(f(n)) = \Theta(n^2)$

$$T(n) = \cancel{X}\, T\left(\frac{n}{4}\right) + n^2$$

For balanced case: $4^{\log_4 x} = 4^2$ in other words $n^{\log_4 x} = n^2$

$a = x$

$b = 4$

$n^{\log_b a} = n^{\log_4 x}$

$x = 4^2 = 16$

INPUT: X and Y Size of input: # of digits in X and Y

OUTPUT: X * Y

$aT\left(\frac{n}{b}\right) + f(n)$

$$\begin{array}{r} \overbrace{8412}^{n} \\ \times\ 320 \\ \hline 00000 \\ 16824 \\ +\ 25236 \\ \hline \underbrace{2691840}_{\sim 2n} \end{array}$$

$\}n \Leftarrow O(n^2)$

$\Leftarrow O(n^2)$

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$$X * Y$$

$$\begin{array}{ccccccc} & & & 10^3 & 10^2 & 10^1 & 10^0 \\ x_1 & x_2 & x_3 \dots & x_{n-3} & x_{n-2} & x_{n-1} & x_n \\ y_1 & y_2 & y_3 \dots & y_{n-3} & y_{n-2} & y_{n-1} & y_n \end{array}$$

$$\left\lfloor \frac{n}{2} \right\rfloor = n - \left\lceil \frac{n}{2} \right\rceil$$

$$\overbrace{x_1 x_2 x_3 \dots x_{\lfloor \frac{n}{2} \rfloor}}^{X_L} * 10^{\lceil \frac{n}{2} \rceil} + \overbrace{x_{\lfloor \frac{n}{2} \rfloor + 1} \dots x_n}^{X_R}$$

$$y_L * 10^{\lceil \frac{n}{2} \rceil} + Y_R$$

$$(X_L 10^{\lceil \frac{n}{2} \rceil} + X_R)(Y_L 10^{\lceil \frac{n}{2} \rceil} + Y_R) =$$

CALC

$$* \; X_L Y_L 10^{\lceil \frac{n}{2} \rceil \cdot 2} + X_L Y_R 10^{\lceil \frac{n}{2} \rceil} + X_R Y_L 10^{\lceil \frac{n}{2} \rceil} + X_R Y_R \; *$$

CALC

NOT GOOD ENOUGH

$$T(n) = 4 T\left(\frac{n}{2}\right) + cn \qquad T(n) \in \Theta(n^2)$$

$4 \to 3$

$a = 4$

$b = 2$

$f(n) \in \Theta(n)$

$$n^{\log_b a} = n^{\lg 4} = n^2$$

CALC

$$(X_L + X_R)(Y_L + Y_R) = X_L Y_L + \boxed{X_L Y_R + X_R Y_L} + X_R Y_R$$