CPSC 320
2010/01/21

$$T(n) \le T(2^{\lceil \lg n \rceil}) = 2^{\lceil \lg n \rceil} + c2^{\lceil \lg n \rceil} \lg 2^{\lceil \lg n \rceil}$$

$$\le 2^{\lg n+1} + c2^{\lg n+1} \lg 2^{\lg n+1}$$

$$= 2n + c2n(\lg n + 1)$$

$$= 2n + c2n\lg n + c2n$$

$$= c2n\lg n + 2n(c+1)$$

$$\le c2n\lg n + 2n\lg n(c+1)$$ if $n \ge 2$ then $n\lg n \ge n$

$$= n\lg n(c2 + 2c + 2)$$

$d = 4c+2$

REDUCE

SORTING by COMP ($\in \Omega(n\lg n)$)

$\downarrow$ reduce in $o(n\lg n)$

SKYLINE

shows SKYLINE $\in \Omega(n\lg n)$

Let B INPUT: $A = [a_1, a_2, \ldots a_k]$

$\Downarrow$

$B = [(a_1, 1, a_1+1), (a_2, 1, a_2+1), \ldots (a_k, 1, a_k+1)]$

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OUTPUT: $(x_1, 1), (x_2, 0), \ldots (x_i, 1), (x_{i+1}, 0)$

$\Downarrow$

$x_1, \cancel{x_2}\, x_1+1, \ldots, x_2-1, \ldots, x_i, x_i+1, \ldots, x_{i+1}-$

SO: SKYLINE PROBLEM $\in \Omega(n \lg n)$ ~~and $O(n \lg n)$~~

SKYLINE_ALG $\in \Theta(n \lg n)$

DANGERS OF GUESS-AND-TEST

GUESS INCORRECTLY $T(n) \in O(n)$

$$T(1) = 1$$

$$T(n) = T(\lfloor \tfrac{n}{2} \rfloor) + T(\lceil \tfrac{n}{2} \rceil) + n$$

Guess: $T(n) = cn$

$\vdots$

Ind Hyp: ~~$T(k) = ck$~~ $\forall i < k,\ T(i) = ci$

Ind Step: Show $T(k+1) = c(k+1)$ $\Leftarrow$

$$T(k+1) = T(\lfloor \tfrac{k+1}{2} \rfloor) + T(\lceil \tfrac{k+1}{2} \rceil) + k+1$$

$$= c\lfloor \tfrac{k+1}{2} \rfloor + c\lceil \tfrac{k+1}{2} \rceil + k+1 \quad \text{by IH}$$

$$= c(k+1) + k+1$$

FAILED PROOF $\rightarrow$ 2 $= (c+1)(k+1)$ $\Leftarrow$ CANNOT RE-SELECT YOUR CONSTANT.

let $d = c+1$

$\boxed{\text{For ints } n: \lceil \tfrac{n}{2} \rceil + \lfloor \tfrac{n}{2} \rfloor = n}$

$$T(1) = 1$$

$$T(n) = T\left(\left\lceil \tfrac{n}{2} \right\rceil\right) + T\left(\left\lfloor \tfrac{n}{2} \right\rfloor\right) + 1$$

Guess-&-Test $T(n) \in O(n)$

WARNING #2: Not all $O(n)$ functions look like cn.

Guess: $T(n) = cn$

IH: $T(i) = ci$ for all $i < k$

IS: Show $T(k) = ck - 1$

$$T(k) = T\left(\left\lceil \tfrac{k}{2} \right\rceil\right) + T\left(\left\lfloor \tfrac{k}{2} \right\rfloor\right) + 1$$

$$= c\left\lceil \tfrac{k}{2} \right\rceil - 1 + c\left\lfloor \tfrac{k}{2} \right\rfloor - 1 + 1 \quad \text{by IH}$$

$$= ck \cancel{+1} - 1$$

TRY $T(n) = cn - 1$

BC: $T(1) = c \cdot 1 - 1 = 2 \cdot 1 - 1 = 1$ ✓

NEED $T(1) = 1$

Let $c = 2$

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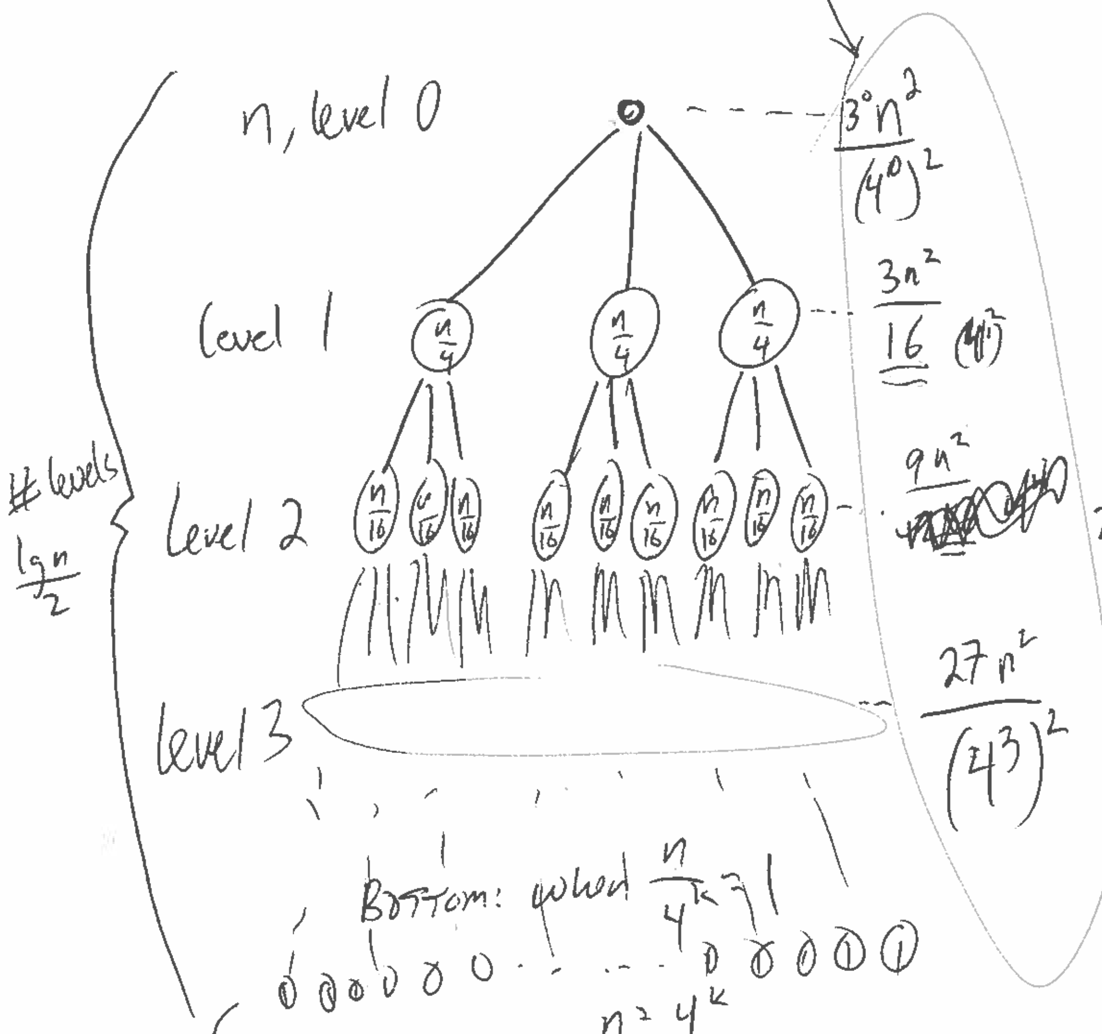
$T(1) = 1$ $\quad \in O(n^2)$ by below

$T(n) = \underline{3}T(\frac{n}{4}) + \underline{n^2}$ $\quad \in \Omega(n^2)$ clearly

Assume n is a power of 4



level k

$$\frac{3^k n^2}{(4^k)^2} = \frac{3^k n^2}{(4^2)^k} = \left(\frac{3}{16}\right)^k n^2$$

$0 < a < 1$

$$\sum_{i=0}^{\infty} a^k = \frac{1}{1-a}$$

$n = 4^k$

$\lg n = \lg 4^k = k \lg 4 = 2k$

$k = \frac{\lg n}{2}$

$3^{\frac{\lg n}{2}}$ nodes

$$T(n) = \left(\sum_{i=0}^{k-1} \left(\frac{3}{16}\right)^k n^2\right) + 3^{\frac{\lg n}{2}}$$

$$= n^2 \left(\sum_{i=0}^{k-1} \left(\frac{3}{16}\right)^k\right) + 3^{\frac{\lg n}{2}}$$

$$\leq n^2 \left(\sum_{i=0}^{\infty} \left(\frac{3}{16}\right)^k\right) + 3^{\lg n/2} \; (4) = n^2\left(\frac{1}{1-\frac{3}{16}}\right) + 3^{\frac{\lg n}{2}} \leq n^2 c + 4^{\lg n/2}$$

$$= n^2 c + 2^{\lg n} = n^2 c + n$$

$3^{\lg n/2} = (3^{0.5})^{\lg n}$

$x = \left(\frac{3^{0.5}}{4}\right)^{\lg n}$