

THM: Perfect matching produced by our alg is stable.

Assume for contradiction that the matching has an instability.

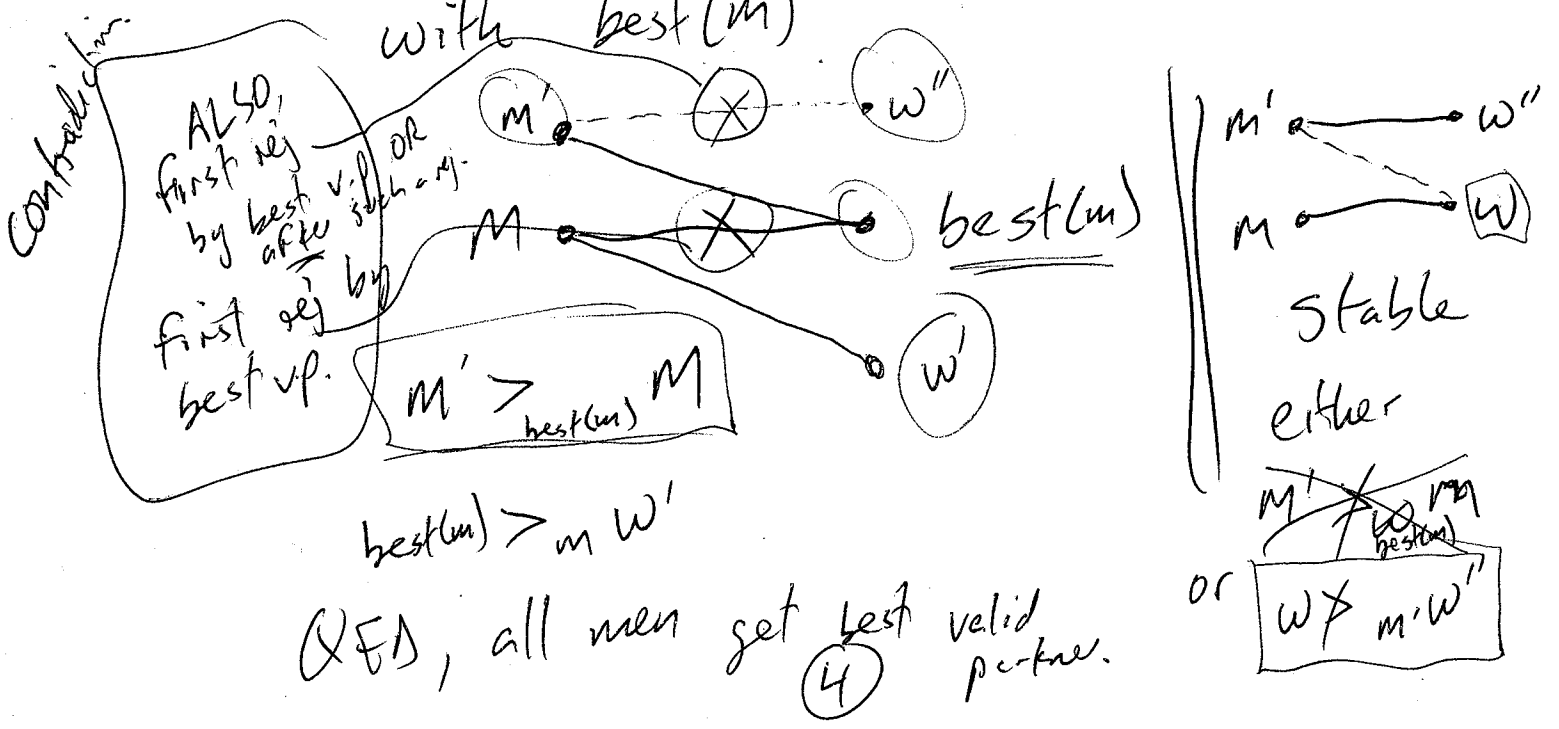
From proof on board:  
contradiction.

SEE NEXT PAGE

QED: our matching is stable.

$best(m) = w$ , where  $\exists$  stable matching with  $(m, w)$  and  $\nexists$  stable matching  $w' / (m, w')$  where  $w' \succ_m w$ .

All men  $m_i$  in G-S's output are paired with  $best(m)$



(EXPANDED FROM ABOVE)  
~~PROOF~~ PROOF BEFORE LAST

ASSUME AN INSTABILITY.

THEN, we have  $m, w, m', w'$  s.t:

$$m \longrightarrow w$$

$$m' \longrightarrow w'$$

where  $w' \succ_m w$

AND  $m \succ_{w'} m'$

But from  $m$  must have proposed to  $w'$  before  $w$  (since men propose in preference order) and been rejected in favor of some man  $m''$  (either rebuffed then or dropped later for  $m''$ ).

So,  $m'' \succ_{w'} m$ .

And, either  $m' = m''$  or  $w'$  "gets to"  $m'$  later in her sequence than  $m''$ .

So,  $m' \succeq_{w'} m''$ .

$m' \succeq_{w'} m''$  and  $m'' \succ_{w'} m \rightarrow m' \succ_{w'} m$

This contradicts  $m \succ_{w'} m'$  above.

QED: Our matching has no instabilities.