CPSC 320
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$\boxed{\text{Textbook}}$
$\boxed{\text{Tutorials}}$
Syllabus

Text: CLRS or The Big White Book

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I. PROBLEM

INPUT: ~~GIVEN~~: A set of hospitals and their preferences
A set of residents and their preferences

OUTPUT: A "good" matching of residents & hospitals

WANT: residents "like" their hospitals
hospitals "like" their residents

Consider this ~~problem~~ situation:

Hospital h has resident s but prefers s'
Hospital $h' \neq h$ has s'
s' prefers h to h'

No resident should be matched to more than one hospital.
No hospital slot should be " " " " " resident

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A hospital ~~w~~ has a fixed # of slots ≥ 1

A ~~preference~~ list for a hospital is a permutation of the contents of the set of residents. ~~$s >_h s'$~~

$s >_h s'$ means h prefers s to s'

$>_h$ will be transitive:

if $a >_h b$ and $b >_h c$ then $a >_h c$

anti-symmetric:

either $a >_h b$ or $b >_h a$ but not both $(a \neq b)$

anti-reflexive:

$a \not>_h a$

II. Simplify/Abstract

Hospitals $\Rightarrow$ women

Residents $\Rightarrow$ men

Residency $\Rightarrow$ holy matrimony

$|W| = |M|$

III Examples.

IV. ALGORITHM

VI. ANALYSIS

INVARIANT: Once women become engaged, they remain engaged; sequence of fiancés improves

A man's seq of fiancées deteriorates.

INVARIANT: The set of engagements is a matching.

Assume the loop terminates with a free man. Since $|M| = |W|$ & the set of engagements is a matching, there must be a free women.

She must never have been engaged or proposed to (see above).

Yet, since the loop terminated, the man must have proposed to everyone on his list (including the free woman).

This is a contradiction.

QED: when the loop terminates, there are no free men; since, $|M| = |W|$ and the set of engagements is a matching, there are also no free women. ∴ The set of engagements is a perfect matching.