

Approximation Algorithms

Final Exam
Dec 11 8:30a
LSK 200

A is a $\rho(n)$ -approximation algorithm if for every input I of size n with optimal solution value $OPT(I)$:

$$\max \left(\underbrace{\frac{A(I)}{OPT(I)}}_{\text{handles minimizing value}}, \underbrace{\frac{OPT(I)}{A(I)}}_{\text{handles maximizing value}} \right) \leq \rho(n)$$

Example (Minimum) Vertex Cover

Find smallest set of vertices $S \subseteq V$ such that all edges in G have at least one endpoint in S .

Approximation algorithm for Vertex Cover

Matching VC

Example

$$S = \emptyset$$

→ Pick edge $(u,v) \in G$ (any edge)

Remove u and v and all adjacent edges from G

Add u and v to S

Repeat until G contains no edges

Return S

Claim Matching VC is a 2-approximation alg. for min. VC

proof Let $VC(G)$ be the smallest vertex cover in G .

① $|VC(G)| \geq \# \text{ edges picked by MVC}$. Why? Each picked edge must be covered by $VC(G)$ and no two picked edges share the same endpoint.

② $|MVC(G)| = 2 \times \# \text{ edges picked by MVC}$

$$\Rightarrow \frac{|MVC(G)|}{|VC(G)|} \leq 2$$

Key idea: We don't know how big $VC(G)$ is but we can lower bound its size.

Approximation Algorithm for Δ TSP

The traveling-salesman problem is: Given a Δ complete undirected graph $G=(V,E)$ with a non-negative weight $w(u,v)$ on every edge. Find a simple cycle with $n=|V|$ edges (a tour) of minimum total weight.

$E=V \times V$

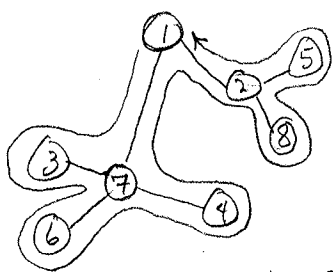
Δ TSP is the same problem but the edge weights obey the triangle inequality:

$$w(a,c) \leq w(a,b) + w(b,c) \quad \text{for all } a,b,c \in V$$

Δ TSP is NP-complete (so is general TSP)

Tree Tour (G)

- ① Find $T = \text{MST}(G)$ (minimum spanning tree)
- ② Let L be the list of vertices visited by a pre order traversal of T (using any vertex r as the root)
- ③ return $L \circ r$ as the tour



$L = 1, 7, 3, 6, 4, 2, 8, 5$

$L \circ r = 4, 7, 3, 6, 4, 2, 8, 5, 1$

Claim TreeTour(G) is a 2-approx. for Δ TSP.
proof Let $\text{OPT}(G)$ be the minimum weight tour of G

① $w(\text{OPT}(G)) \geq w(\text{MST}(G))$

If we delete any edge of $\text{OPT}(G)$ we get a spanning tree. The min weight spanning tree weighs even less (or the same).

- ② Let W be the list of vertices visited and returned to in a preorder traversal.
 $W = 1, 7, 3, 7, 6, 7, 4, 7, 1, 2, 8, 2, 5, 2, 1$

$w(W) = 2 w(\text{MST}(G))$

because each edge is traversed twice

$w(L \circ r) < w(W)$ by triangle inequality.

So $w(\text{TreeTour}(G)) \leq 2 w(\text{OPT}(G))$.