1. Let $F[i, X]$ be the value of the most valuable combination of items of weight at most X from items $1, 2, \ldots, i$. The optimal solution to obtain value $F[i, X]$ either (1) includes item i (if it fits) and an optimal solution using items $1, 2, \ldots, i-1$ with knapsack capacity $X - w_i$, or (2) excludes item i and is an optimal solution using items $1, 2, \ldots, i-1$ with knapsack capacity X.

$$F[i, X] = \begin{cases} \max\{F[i-1, X-w_i] + v_i, F[i-1, X]\} & \text{if } w_i \leq X \\ F[i-1, X] & \text{otherwise} \end{cases}$$

$F[0, X] = 0$ for integers X between 0 and W. $F[i, 0] = 0$ for integers i between 0 and n.

```
Knapsack(v_1, ..., v_n, w_1, ..., w_n, W)
    for X := 0 to W F[0, X] := 0
    for i := 0 to n F[i, 0] := 0
    for i := 1 to n
        for X := 0 to W
            F[i, X] := F[i-1, X]
            if w_i ≤ X and F[i, X] < F[i-1, X-w_i] + v_i then
                F[i, X] := F[i-1, X-w_i] + v_i
    X := W
    for i := n to 1
        if F[i, X] > F[i-1, X] then
            print i
            X := X - w_i
```

2. For $i \leq j$, let $A[i, j]$ be the length of a longest palindromic subsequence of $X[i \ldots j]$. If $X[i] = X[j]$ then such a palindromic subsequence is $X[i]$ followed by a longest palindromic subsequence of $X[i+1 \ldots j-1]$ followed by $X[j]$. If $X[i] \neq X[j]$ then such a palindromic subsequence is either a longest palindromic subsequence of $X[i+1 \ldots j]$ or of $X[i \ldots j-1]$ since both $X[i]$ and $X[j]$ cannot contribute. For $i < j$,

$$A[i, j] = \begin{cases} A[i+1, j-1] + 2 & \text{if } X[i] = X[j] \\ \max\{A[i+1, j], A[i, j-1]\} & \text{otherwise} \end{cases}$$

$A[i, i] = 1$ for all i between 1 and n. $A[i, i-1] = 0$ for all i between 2 and n.

```
PalindromicSubseq(X[1...n])
    for i := 1 to n A[i, i] := 1
    for i := 2 to n A[i, i-1] := 0
    for d := 1 to n-1
        for i := 1 to n-d
            j := i + d
            if X[i] = X[j] then A[i, j] := A[i+1, j-1] + 2
            else A[i, j] := max{A[i+1, j], A[i, j-1]}
    return A[1, n]
```