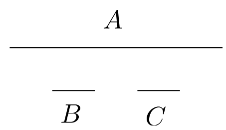
CPSC 320 Problem Set 7 solutions 24 November 2009

1. Consider the three jobs $A = (0, 5)$, $B = (1, 2)$, and $C = (3, 4)$.



The optimal solution is $\{A\}$ with length 5.

(a) This version considers the jobs in the order A, C, B. It calculates last$[1, 2, 3] = [0, 0, 2]$ and $L[0, 1, 2, 3] = [0, 5, 5, 6]$, returning 6.

(b) This version considers the jobs in the order A, B, C. It calculates last$[1, 2, 3] = [0, 0, 2]$ and $L[0, 1, 2, 3] = [0, 5, 5, 6]$, returning 6.

(c) works. See problem set 6 explanation.

2. The cheapest way to get from city 1 to city n must stop at some last city k and then go directly from city k to city n. Let $F[i]$ be the cheapest rental cost to canoe from city 1 to city i.

$$F[i] = \min_{1 \le k < i} \{F[k] + p_{ki}\} \qquad \text{for } i = 2, 3, \ldots, n$$
$$F[1] = 0$$

The actual algorithm is a simple for-loop:

CheapestCost(p_{ij} for $1 \le i < j \le n$)
    $F[1] = 0$
    For $i = 2$ to n
        $F[i] = \min_{1 \le k < i}\{F[k] + p_{ki}\}$
    Return $F[n]$

3. The maximum sum contiguous subsequence that ends at position j is either the empty subsequence or the maximum sum contiguous subsequence that ends at position $j - 1$ plus a_j. Let $S[j]$ be the sum of the maximum sum contiguous subsequence that ends at position j.

$$S[j] = \max\{0, S[j-1] + a_j\} \qquad \text{for } j = 1, 2, \ldots, n$$
$$S[0] = 0$$

MaxSumSubsequence($a_1, a_2, \ldots, a_n$)
    $S[0] = 0$, $m = 0$
    For $j = 1$ to n
        $S[j] = \max\{0, S[j-1] + a_j\}$
        If $S[j] > S[m]$ then $m = j$
    $j = m$
    While $j > 0$ and $S[j] = S[j-1] + a_j$
        $j = j - 1$
    Return $a_{j+1}, a_{j+2}, \ldots a_m$