CPSC 320 Problem Set 2 25 September 2009
W. Evans Due: 2 October 2009 at the start of class.

1. We saw in class that $T(n) = T(\lceil n/2 \rceil) + T(\lfloor n/2 \rfloor) + cn$, $T(1) = 1$ is $O(n \log n)$. Show that it is $\Omega(n \log n)$ (and, thus, $\Theta(n \log n)$).

2. (Exercise 4.2-1 CLRS) Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = 3T(\lfloor n/2 \rfloor) + n$. Use the substitution method to verify your answer.

3. Use a recursion tree to guess a good (meaning small) upper bound on $T(n) = 4T(n-1) + 5T(n-2) + 5^n$, $T(0) = 0$, $T(1) = 1$. Use induction to prove your result.

4. (Problem 4-2 CLRS) **Finding the missing integer** An array $A[1 \ldots n]$ contains all the integers from 0 to n except one. It would be easy to determine the missing integer in $O(n)$ time by using an auxiliary array $B[0 \ldots n]$ to record which numbers appear in A. In this problem, however, we cannot access an entire integer in A with a single operation. The elements of A are represented in binary, and the only operation we can use to access them is "fetch the jth bit of $A[i]$," which takes constant time.

   Show that if we use only this operation, we can still determine the missing integer in $O(n)$ time.

5. (Exercise 4.3-2 CLRS) The recurrence $T(n) = 7T(n/2) + n^2$ describes the running time of an algorithm A. A competing algorithm A' has a running time of $T'(n) = aT'(n/4) + n^2$. What is the largest integer value for a such that A' is asymptotically faster than A?

   Hint: Use the master method.