1. Proof by induction k. For $k = 0$, a set of no shims allows us to create thicknesses from 0 to $2^1 - 1 = 0$. Assume that the claim is true for k shims. A set of $k+1$ shims contains shims of thickness $2^0, 2^1, \ldots, 2^{k-1}$ — and so, by induction, can create all thicknesses from 0 to $2^k - 1$ using only these shims — plus it contains a shim of thickness 2^k. If we add this shim to the each of the subsets that created thicknesses 0 to $2^k - 1$, we obtain thicknesses 2^k to $2^{k+1} - 1$. Thus we obtain all thicknesses from 0 to $2^{k+1} - 1$.

2. The idea is to swap the red elements from A into the first positions of A, then swap the white elements into the first positions of A following the red elements. To do this we first maintain the loop invariant: At step i (for i from 1 to $n+1$), all of the red elements in $A[1 \ldots i-1]$ are in $A[1 \ldots r-1]$ (and the non-red elements are in $A[r \ldots i-1]$). Initially, $r = i = 1$ so the loop invariant holds. To maintain the loop invariant from step i to $i+1$, we execute:

   If $A[i]$ is RED then
       swap $A[i]$ with $A[r]$
       $r = r + 1$
   $i = i + 1$

   When i reaches $n+1$, we are done with the first phase. The second phase, moving the white elements into the positions following the red elements, uses a similar loop invariant: At step i (for i from r to $n+1$), all of the white elements in $A[r \ldots i-1]$ are in $A[r \ldots w-1]$ (and the blue elements are in $A[w \ldots i-1]$). Initially, $w = i = r$ so the loop invariant holds. To maintain the loop invariant from step i to $i+1$, we execute:

   If $A[i]$ is WHITE then
       swap $A[i]$ with $A[w]$
       $w = w + 1$
   $i = i + 1$

   When i reaches $n+1$, we are done.

3. The function is the sum of:

$$2^{\lg(n)/2} n \lg(n/4) = n^{1/2} n (\lg n - \lg 4) = n^{1.5} \lg n - 2n^{1.5}$$

   and

$$4^{\lg(n)} \log_5(n^{1.3}) = (2^{\lg n})^2 (1.3) \log_5 n = (1.3 \log_5 2) n^2 \lg n.$$

   The second term is asymptotically greater than the first, so $f(n) \in O(n^2 \log n)$.

4. The idea is to maintain the height of the merged skyline in a variable y as we increase x from $x = -\infty$ to $x = +\infty$. Whenever y changes, we record the pair (x, y). The loop invariant we maintain is that at step k (in the pseudocode k is $i + j - 2$), we have produced the merged skyline for x ranging from $-\infty$ to the kth smallest x-coordinate of U and V, and that y is the height of the merged skyline at that x-coordinate.

```
Merge(U, V)
    y = uy = vy = 0
    i = j = 1
    while( i + j < |U| + |V| )
        ux = U[i].x or ∞ if i > |U|
        vx = V[j].x or ∞ if j > |V|
        if ux ≤ vx then
            x = ux
            uy = U[i].y
            i++
        if ux ≥ vx then
            x = vx
            vy = V[j].y
            j++
        maxy = max(uy, vy)
        if maxy ≠ y then
            y = maxy
            add (x, y) to the end of S
    return S
```

5.

$$(\log n)^5, \quad \sqrt{n}, \quad n^2(\log n)^4, \quad \sum_{i=1}^{n} i^2, \quad n^3 \log n, \quad n^{\log_2 n}, \quad 2^n, \quad n!, \quad 2^{(2^n)}$$

6. (a) $T(n) = 3\lfloor n/2 \rfloor$.
   (b) $T(n) = \lg n$.
   (c) $T(n) = n^2 - n$.