CPSC 314

REVIEW 2

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LINES AND CURVES

- Explicit - one coordinate as function of the others

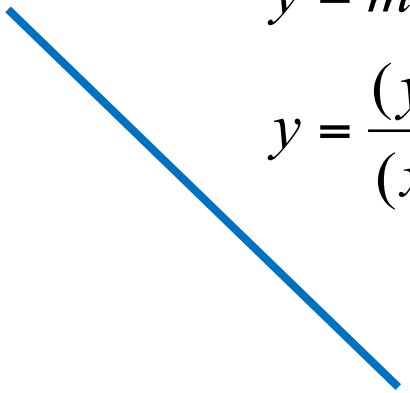
$$y = f(x)$$

$$z = f(x, y)$$

Line

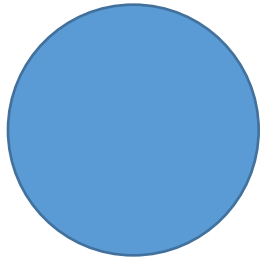
$$y = mx + b$$

$$y = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1) + y_1$$



Circle

$$y = \pm\sqrt{r^2 - x^2}$$



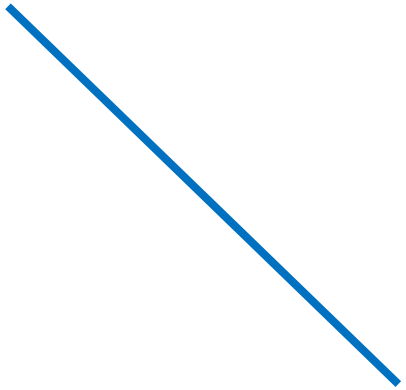
LINES AND CURVES

•Parametric – all coordinates as functions of common parameters

$$(x, y) = (f_1(t), f_2(t))$$

$$(x, y, z) = (f_1(u, v), f_2(u, v), f_3(u, v))$$

Line



$$x(t) = x_1 + t(x_2 - x_1)$$

$$y(t) = y_1 + t(y_2 - y_1)$$

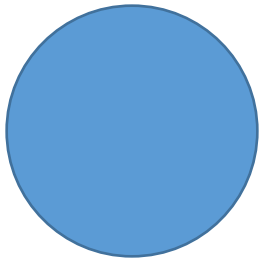
$$t \in [0,1]$$

Circle

$$x(\theta) = r\cos(\theta)$$

$$y(\theta) = r\sin(\theta)$$

$$\theta \in [0, 2\pi]$$



LINES AND CURVES

•Implicit - define as “zero set” of some function

$$\{(x, y) : F(x, y) = 0\}$$

$$\{(x, y, z) : F(x, y, z) = 0\}$$

- May define meaningful partition of space

$$\{(x, y) : F(x, y) > 0\}, \{(x, y) : F(x, y) = 0\}, \{(x, y) : F(x, y) < 0\}$$

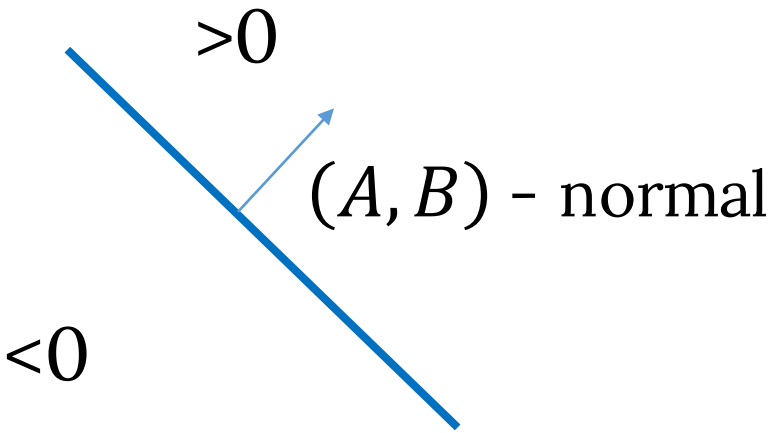
LINES AND CURVES - IMPLICITS

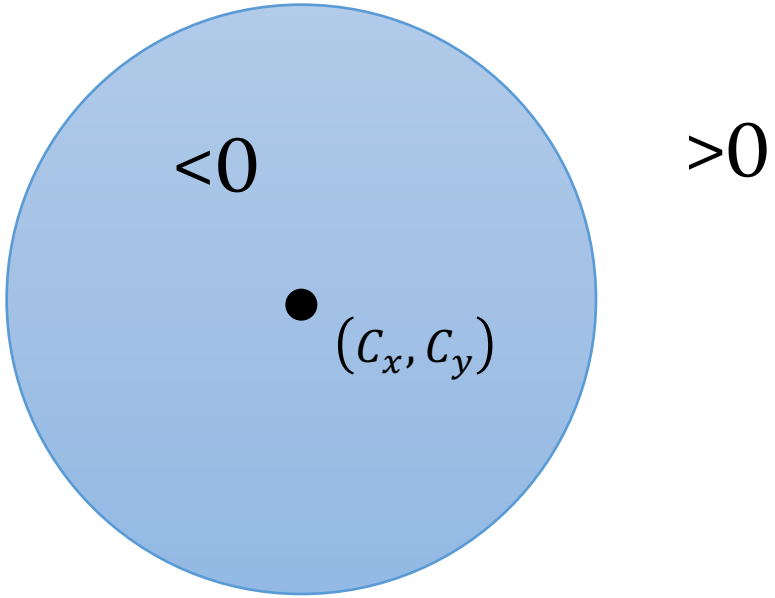
Line

$$Ax + By + C = 0$$

Circle

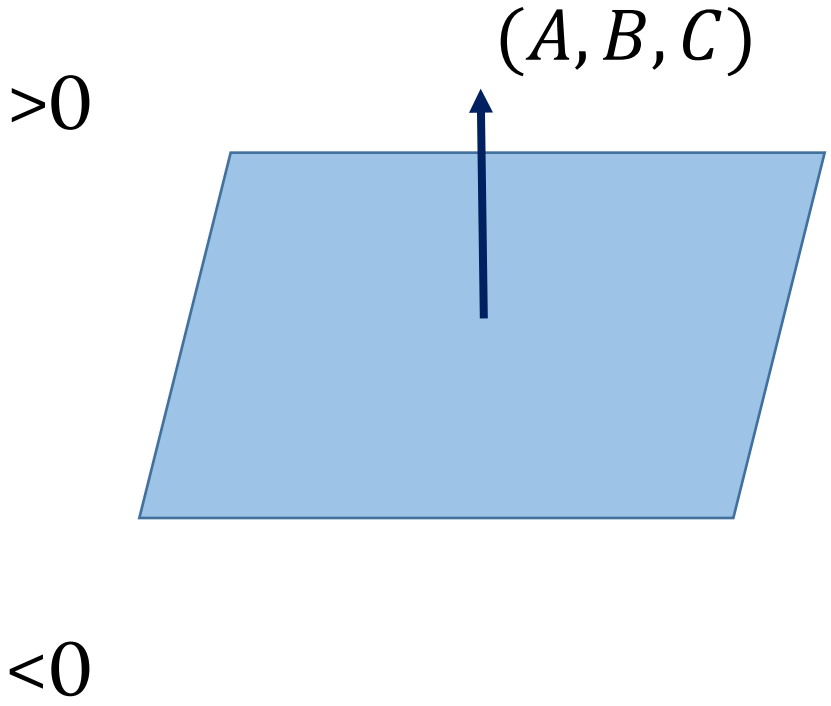
$$(x - C_x)^2 + \left(y - C_y\right)^2 - r^2 = 0$$





PLANE - IMPLICIT

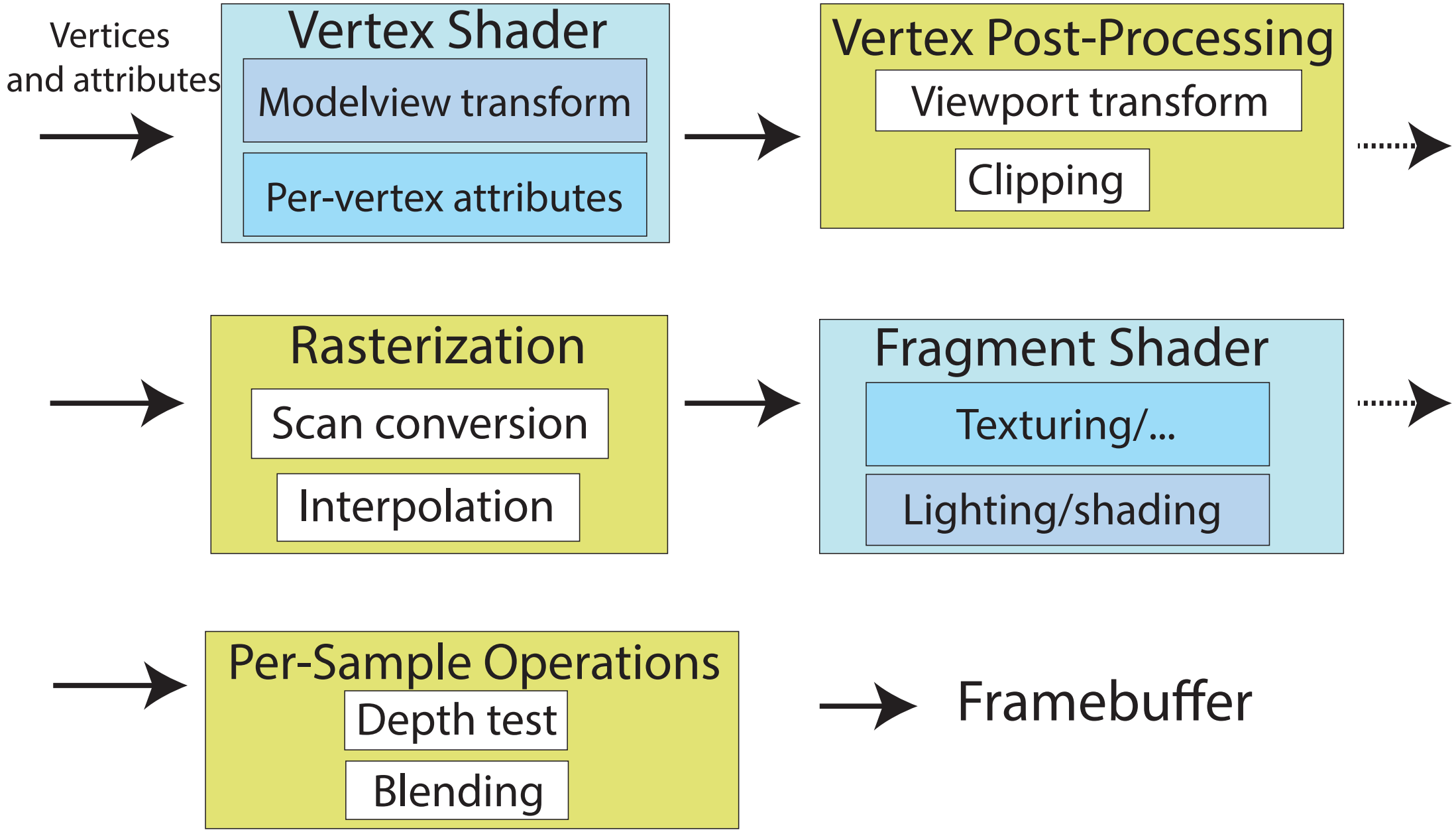
$$Ax + By + Cz + D = 0$$



ARBITRARY IMPLICIT FUNCTION

$$F(x, y, z) = 0$$

$$n(x, y, z) = \nabla F(x, y, z) = \begin{pmatrix} \partial F(x, y, z)/\partial x \\ \partial F(x, y, z)/\partial y \\ \partial F(x, y, z)/\partial z \end{pmatrix}$$


Vertices
and attributes
Vertex Shader
Modelview transform
Per-vertex attributes
Vertex Post-Processing
Viewport transform
Clipping
Rasterization
Scan conversion
Interpolation
Fragment Shader
Texturing/...
Lighting/shading
Per-Sample Operations
Depth test
Blending
Framebuffer

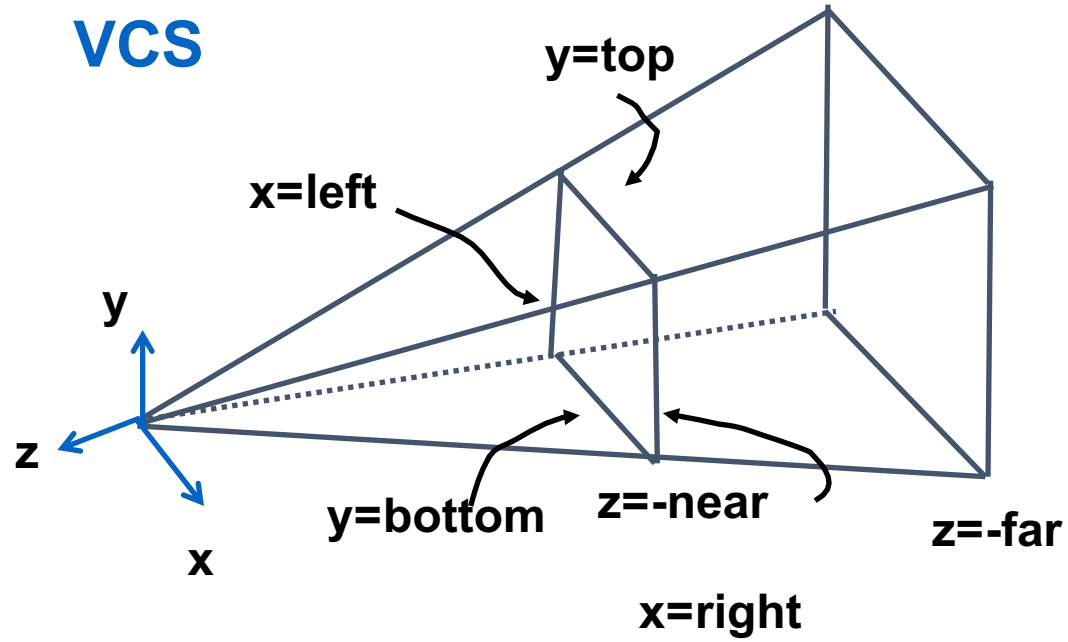
CLIPPING

What's the purpose of clipping?

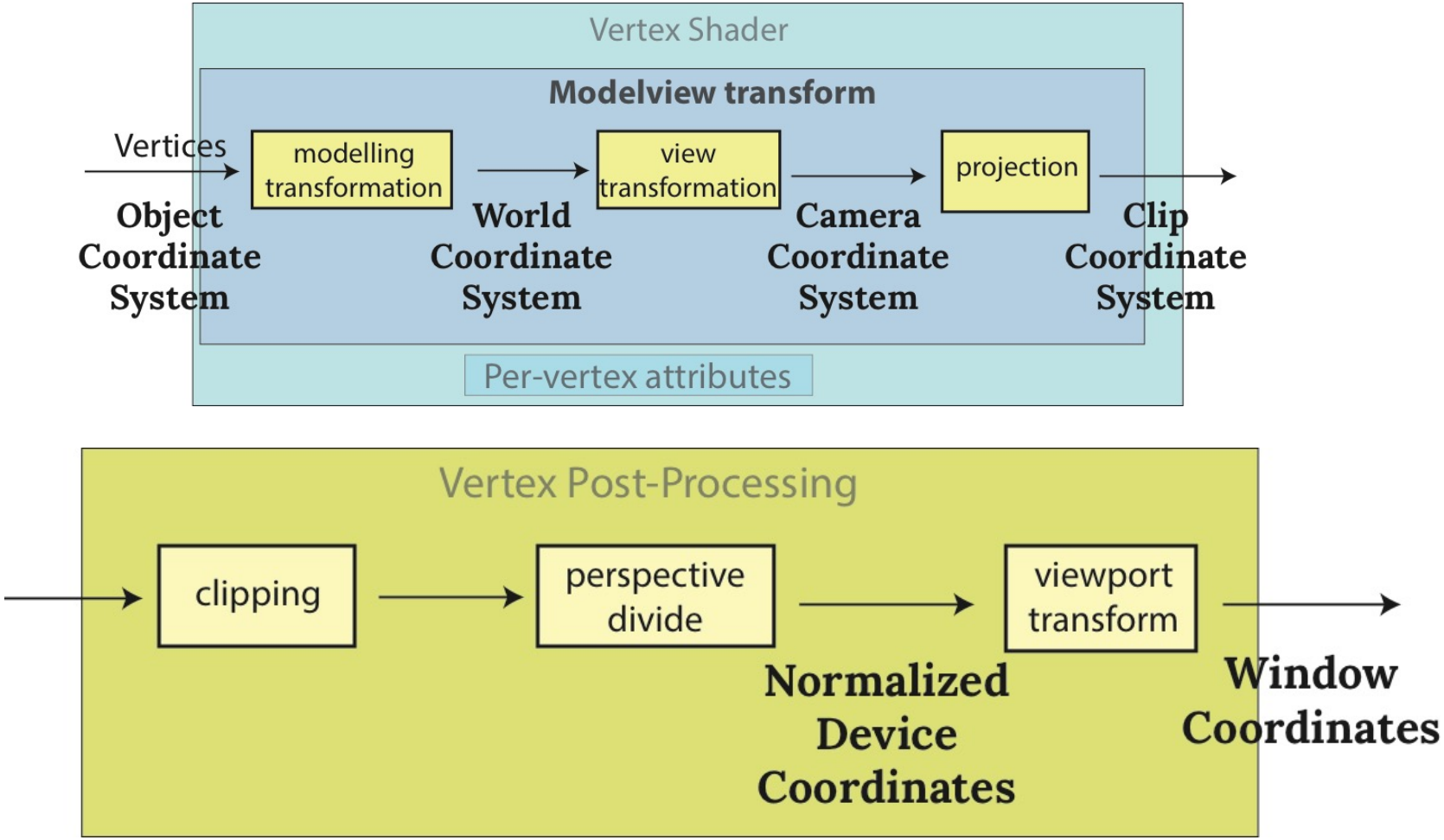
How is it done?

When is it done in pipeline?

Why do we need near/far planes?



PIPELINE EXPANDED



VIEWPORT TRANSFORM

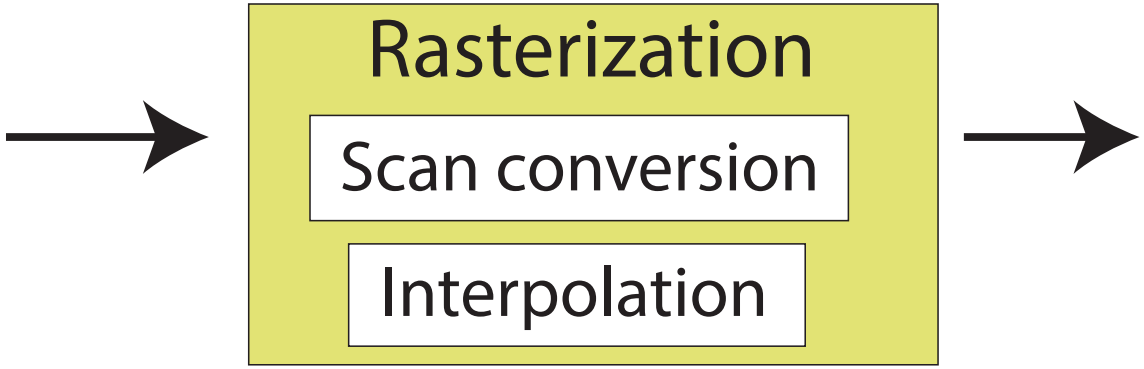
- What does it do?

$$\begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix} = \begin{bmatrix} W/2 & 0 & 0 & (W-1)/2 \\ 0 & H/2 & 0 & (H-1)/2 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_n \\ y_n \\ z_n \\ 1 \end{bmatrix}$$

RASTERIZATION

- This is part of the fixed function pipeline

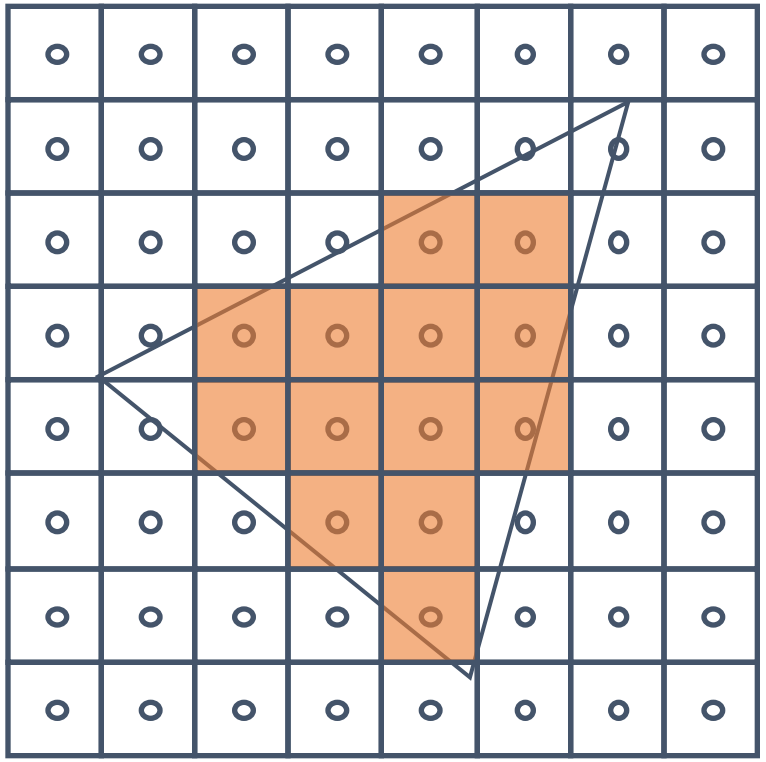
- Input: clipped polygons
- Output: fragments (with **varying variables** interpolated)



SCAN CONVERSION: IDEA

A point is inside ⇔

$$A_i x + B_i y + C > 0, i = 1, \ldots, 3$$

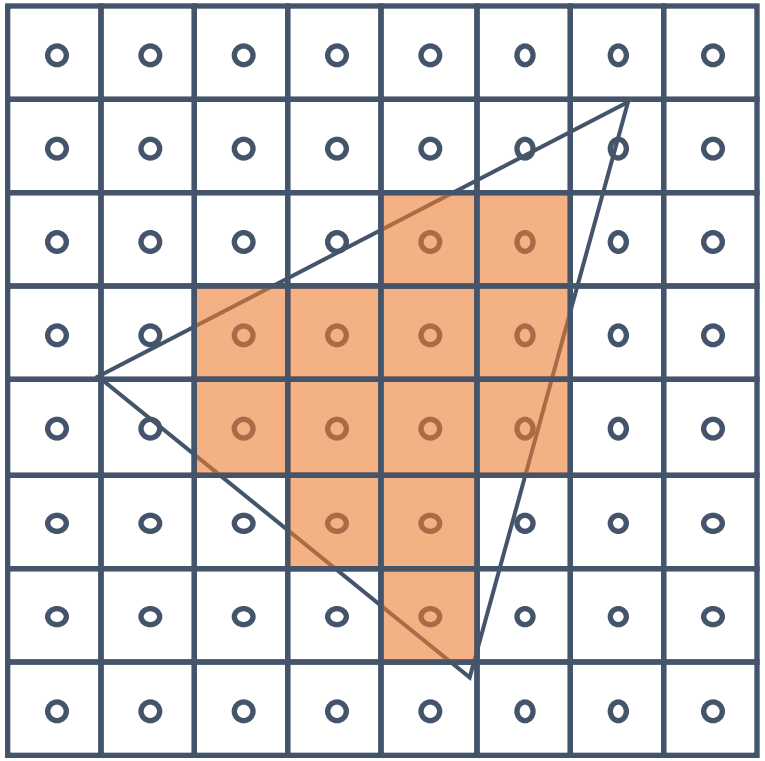


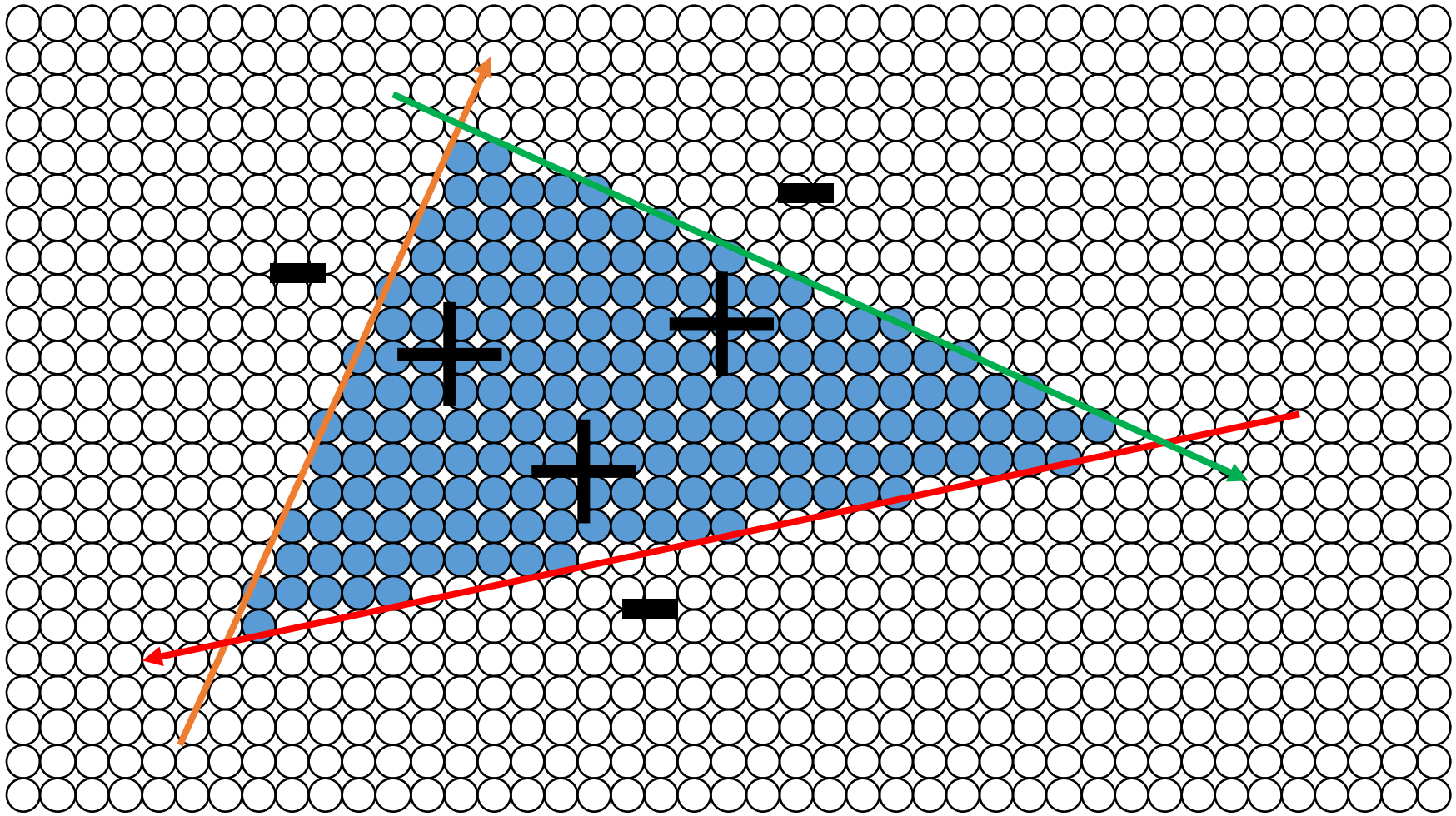
HOW TO TREAT BOUNDARY?

- If two triangles share an edge, scan conversion should be consistent
  - No pixel drawn twice
  - No gaps

- E.g. draw left edge,

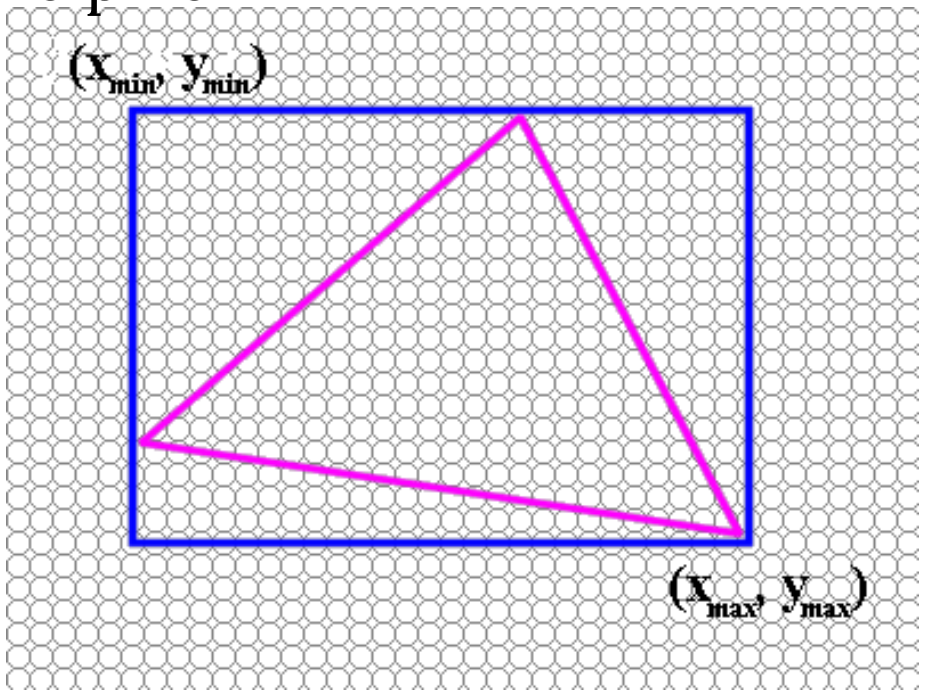
don't draw right one





SCANLINE IDEA (SIMPLIFIED)

- Basic structure of code:
  - Setup: compute edge equations, bounding box
  - (Outer loop) For each scanline in bounding box…
  - (Inner loop) …check each pixel on scanline, evaluating edge equations and drawing the pixel if all three are positive



SCANLINE: CODE

```
findBoundingBox(xmin, xmax, ymin, ymax);
setupEdges (a0,b0,c0,a1,b1,c1,a2,b2,c2);

for (int y = yMin; y <= yMax; y++) {
  for (int x = xMin; x <= xMax; x++) {
    float e0 = a0*x + b0*y + c0;
    float e1 = a1*x + b1*y + c1;
    float e2 = a2*x + b2*y + c2;
    if (e0 > 0 && e1 > 0 && e2 > 0)
        Image[x][y] = TriangleColor;
  }
}
```

SCANLINE: OPTIMIZED CODE

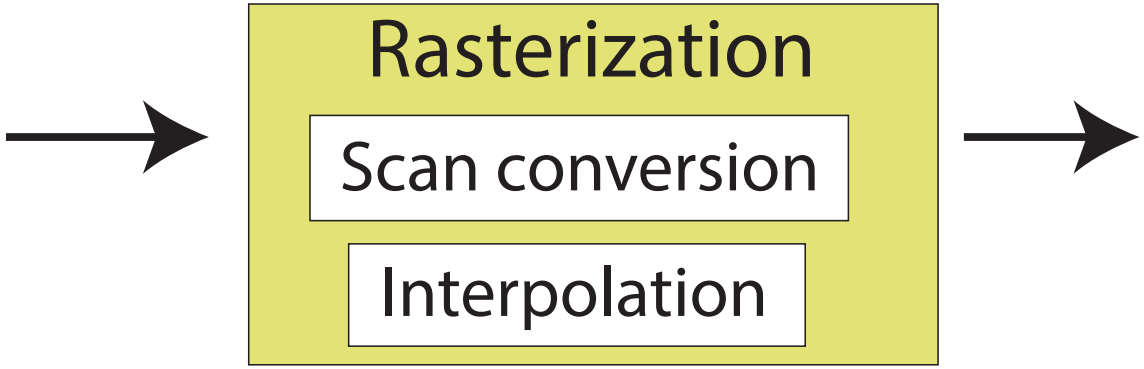
```
// more efficient inner loop
for (int y = yMin; y <= yMax; y++) {
 float e0 = a0*xMin + b0*y + c0;
 float e1 = a1*xMin + b1*y + c1;
 float e2 = a2*xMin + b2*y + c2;
 for (int x = xMin; x <= xMax; x++) {
  if (e0 > 0 && e1 > 0 && e2 > 0)
    Image[x][y] = TriangleColor;

  e0 += a0;   e1+= a1;    e2 += a2;
 }
}
```

SCAN CONVERSION

- What are problems of scan conversion?
- How to find a bounding box?
- How to scan-convert an arbitrary polygon?

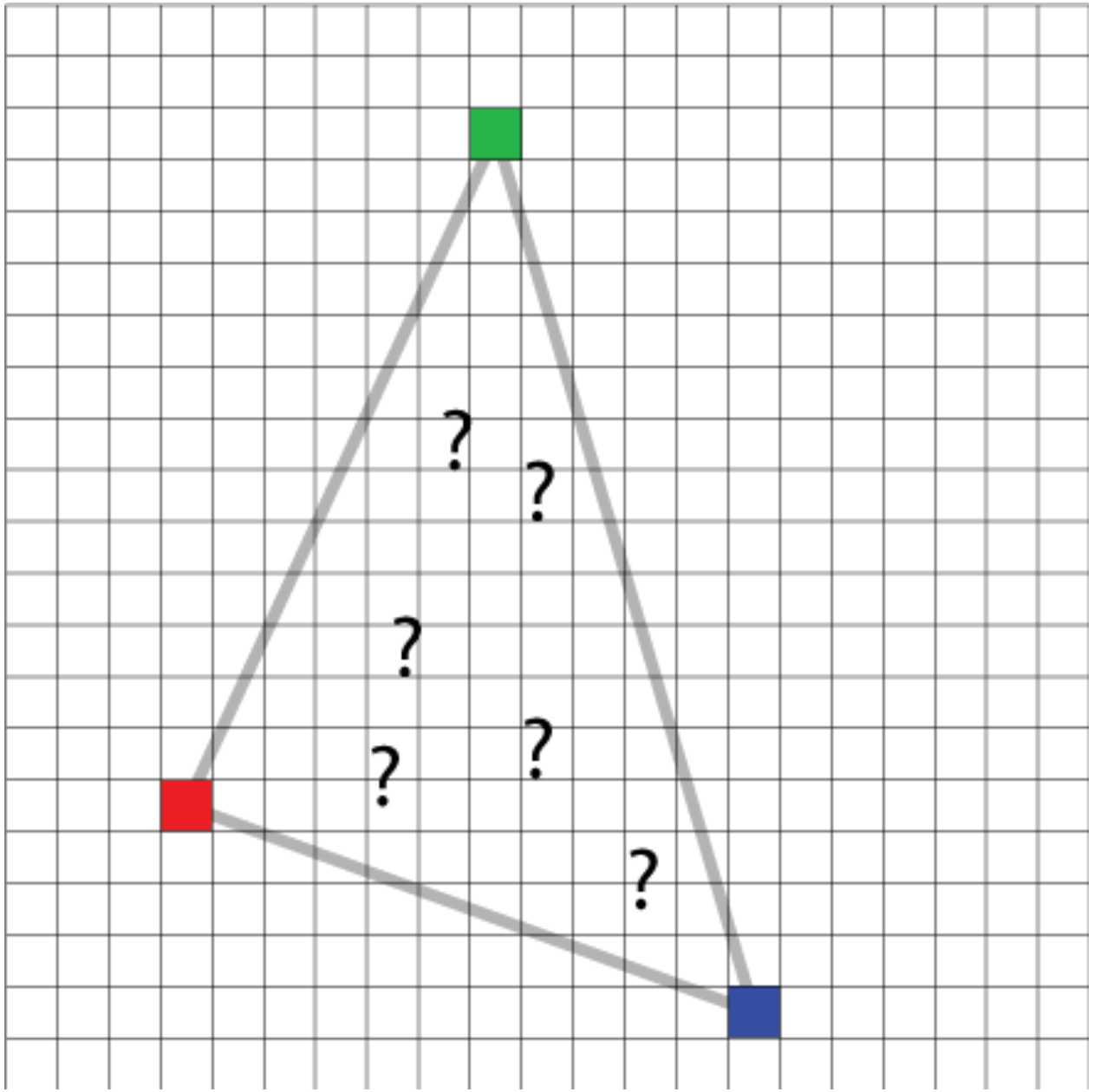
INTERPOLATION

- What does it do?



INTERPOLATION – ACCESS TRIANGLE INTERIOR

- Interpolate between vertices:
  - z
  - r,g,b - colour components
  - u,v - texture coordinates
  - N_x, N_y, N_z - surface normals
- Equivalent
  - Barycentric coordinates
  - Bilinear interpolation
  - Plane Interpolation

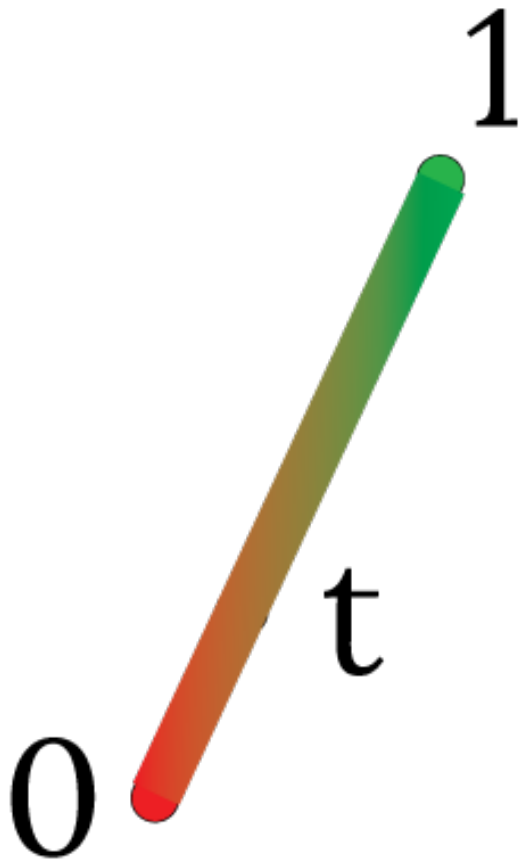


SIMPLER:

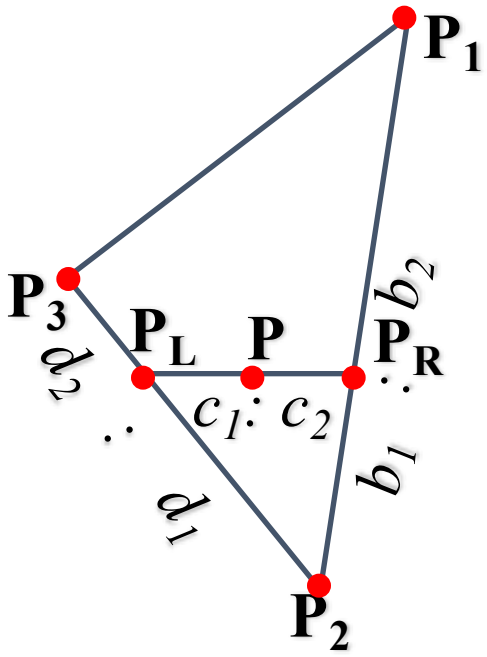
How to interpolate color between two points?

$$c(t) \approx c(0) \cdot (1 - t) + c(1) \cdot t$$

Linear interpolation



BI-LINEAR INTERPOLATION



$$P = \frac{c_2}{c_1 + c_2} \cdot P_L + \frac{c_1}{c_1 + c_2} \cdot P_R$$

$$P_L = \frac{d_2}{d_1 + d_2} P_2 + \frac{d_1}{d_1 + d_2} P_3$$

$$P_R = \frac{b_2}{b_1 + b_2} P_2 + \frac{b_1}{b_1 + b_2} P_1$$

$$P = \frac{c_2}{c_1 + c_2}\left(\frac{d_2}{d_1 + d_2} P_2 + \frac{d_1}{d_1 + d_2} P_3\right) + \frac{c_1}{c_1 + c_2}\left(\frac{b_2}{b_1 + b_2} P_2 + \frac{b_1}{b_1 + b_2} P_1\right)$$

BARYCENTRIC COORDINATES

- Area

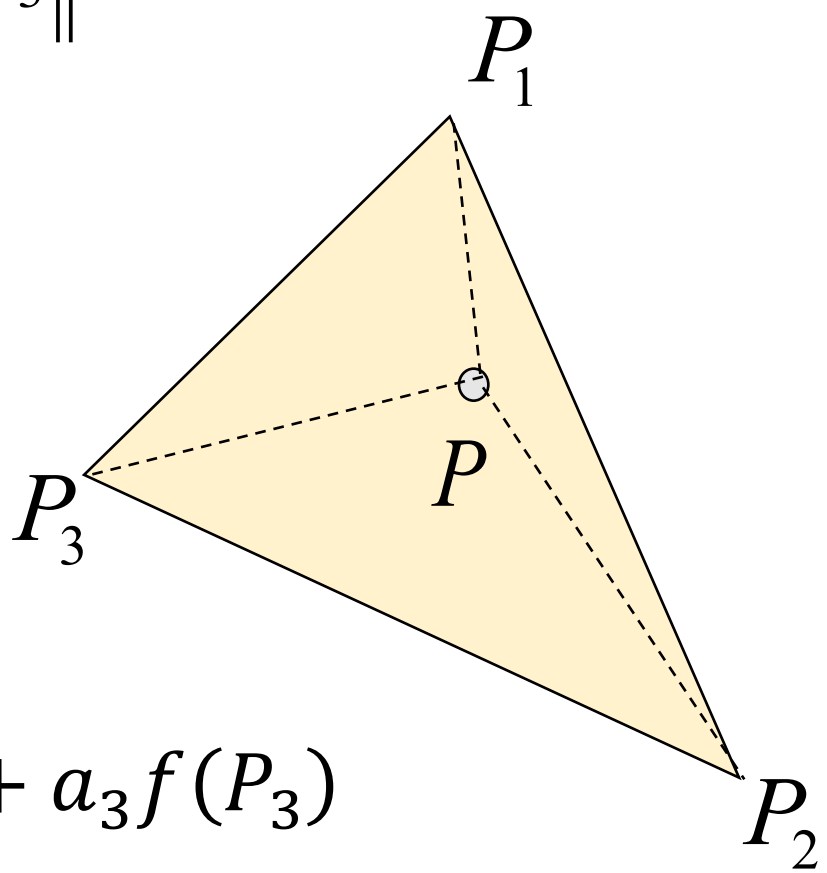
$$A = \frac{1}{2}\left\|\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}\right\|$$

- Barycentric coordinates

$$a_1 = A_{P_2P_3P} / A, a_2 = A_{P_3P_1P} / A,$$

$$a_3 = A_{P_1P_2P} / A,$$

$$P = a_1P_1 + a_2P_2 + a_3P_3$$



$$f(P) \approx a_1 f(P_1) + a_2 f(P_2) + a_3 f(P_3)$$

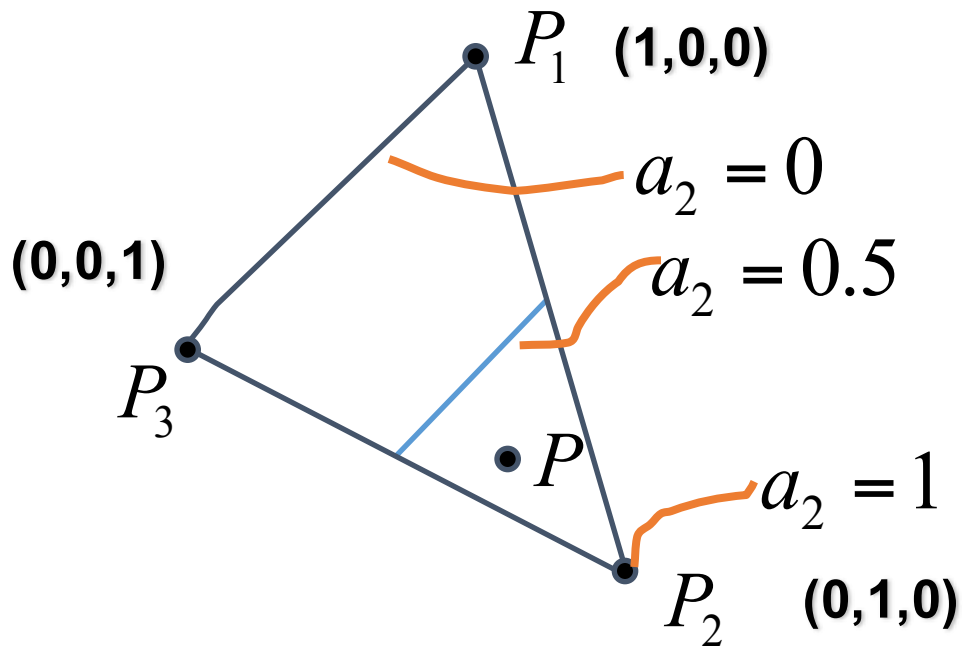
BARYCENTRIC COORDINATES

- weighted (affine) combination of vertices

$$P = a_1 \cdot P_1 + a_2 \cdot P_2 + a_3 \cdot P_3$$

$$a_1 + a_2 + a_3 = 1$$

$$0 \le a_1, a_2, a_3 \le 1$$



BARYCENTRIC COORDINATES

- Positive inside the triangle
- Always sum up to 1
- If we extend barycentric coordinates outside the triangle,

$$A = \frac{1}{2}\left\|\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}\right\| \quad \Longrightarrow \quad A = \frac{1}{2}(P_1P_2 \times P_1P_3)_z$$

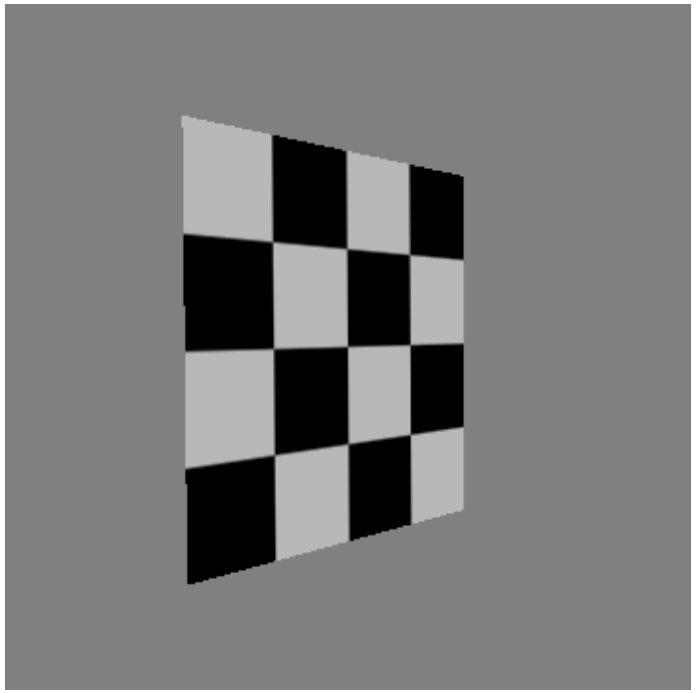
- We have signed areas
- Outside the triangle at least one coordinate < 0

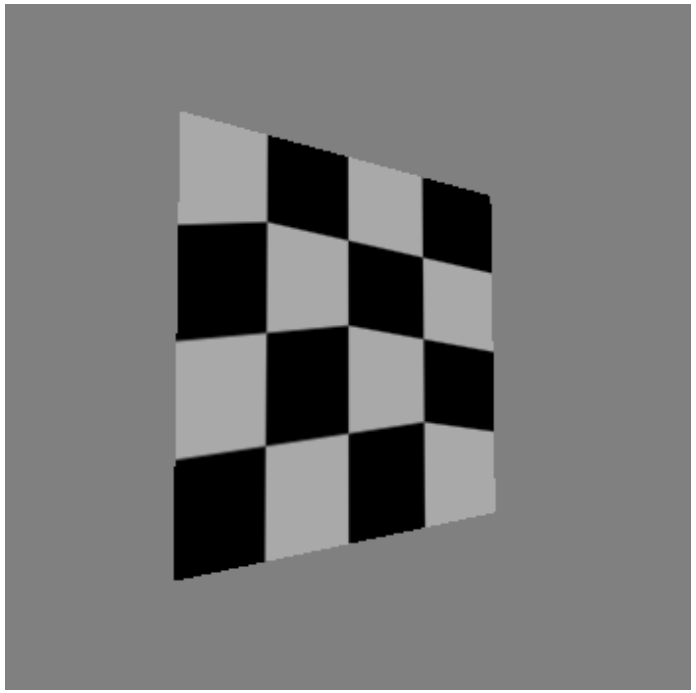
ISSUE WITH INTERPOLATION UNDER PERSPECTIVE PROJECTION

TEXTURE MAPPING

Texture coordinate interpolation

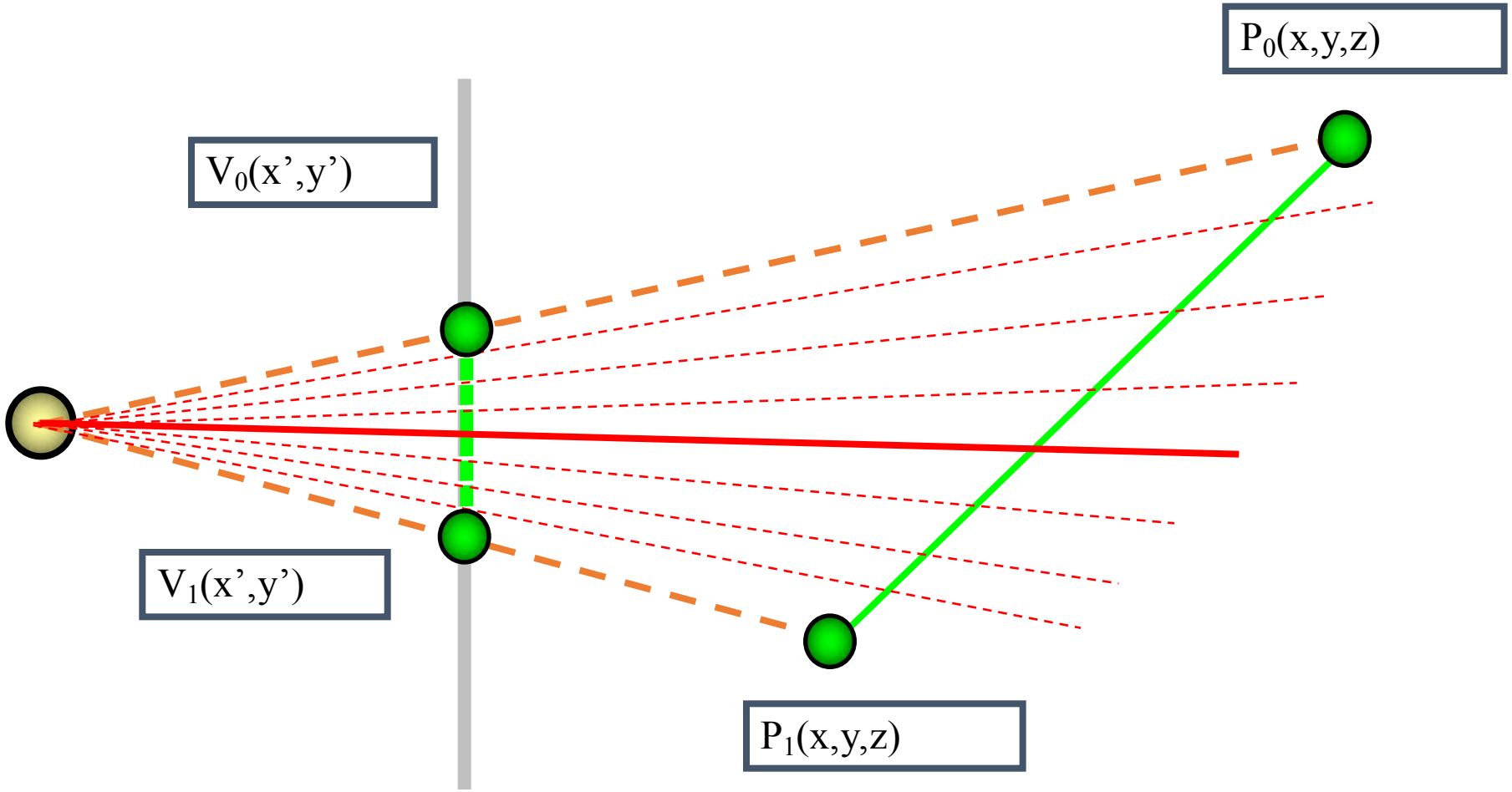
- Perspective foreshortening problem
- Also problematic for color interpolation, etc.





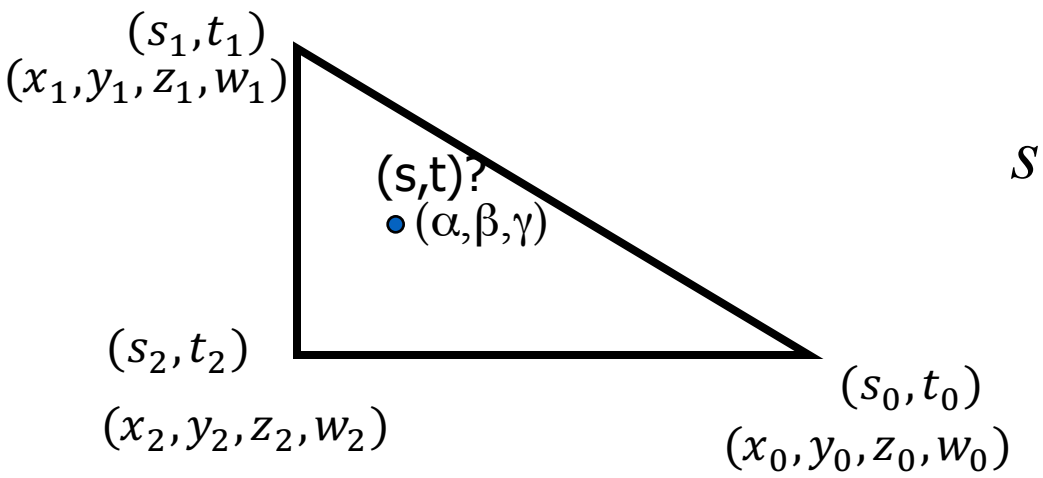
INTERPOLATION: SCREEN VS. WORLD SPACE

- Screen space interpolation incorrect under perspective
  - Problem ignored with shading, but artifacts more visible with texturing



TEXTURE COORDINATE INTERPOLATION

- Perspective Correct Interpolation
  - α, β, γ : Barycentric coordinates (2D) of point **P**
  - s_0, s_1, s_2 : texture coordinates of vertices
  - w_0, w_1, w_2 : homogenous coordinate of vertices



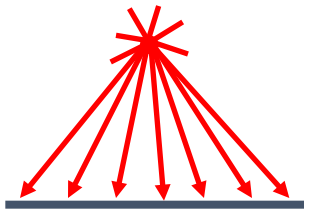
$$s = \frac{\alpha \cdot s_0 / w_0 + \beta \cdot s_1 / w_1 + \gamma \cdot s_2 / w_2}{\alpha / w_0 + \beta / w_1 + \gamma / w_2}$$

- Similarly for t

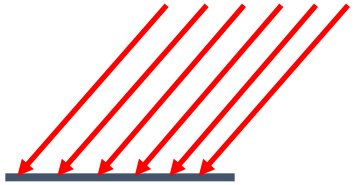
Derivation (similar triangles):
https://www.comp.nus.edu.sg/~lowkl/publications/lowk_persp_interp_techrep.pdf

LIGHT SOURCES

- Point source
  - light originates at a point
  - Rays hit planar surface at different angles



- Parallel source
  - light rays are parallel
  - Rays hit a planar surface at identical angles
  - May be modeled as point source at infinity
  - *Directional light*



LIGHT

- Light has color
- Interacts with object color (r,g,b)

$$I = I_a k_a$$

$$I_a = (I_{ar}, I_{ag}, I_{ab})$$

$$k_a = (k_{ar}, k_{ag}, k_{ab})$$

$$I = (I_r, I_g, I_b) = (I_{ar}k_{ar}, I_{ag}k_{ag}, I_{ab}k_{ab})$$

- Blue light on white surface?
- Blue light on red surface?

DIFFUSE REFLECTION

- Illumination equation is now:

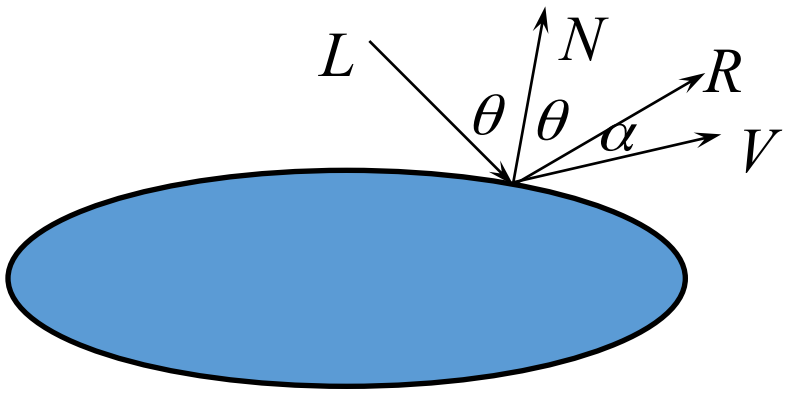
$$I = I_a k_a + I_p k_d (N \cdot L) = I_a k_a + I_p k_d \cos\theta$$

- I_p - point/parallel source's intensity
- k_d - surface diffuse reflection coefficient

- Can we locate light source from shading?

SPECULAR REFLECTION

- Shiny objects (e.g. metallic) reflect light in preferred direction R determined by surface normal N.



- Most objects are not ideal mirrors - reflect in the immediate vicinity of R

ILLUMINATION EQUATION

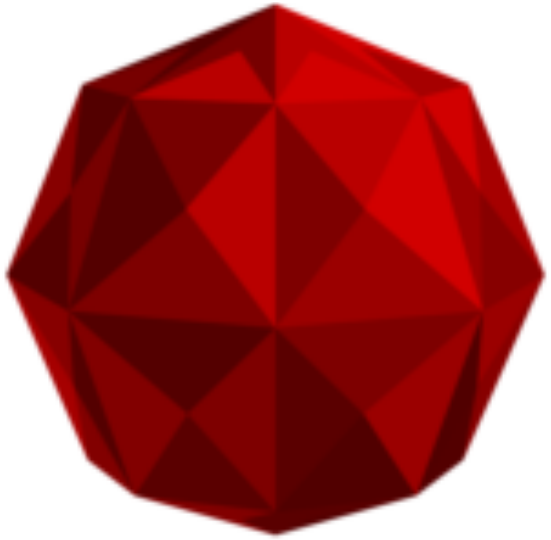
- For multiple light sources:

$$I = I_a k_a + \sum_p \frac{I_p}{d_p^{\,2}} (k_d (N \cdot L_p) + k_s (R_p \cdot V)^n)$$

- d_p- distance between surface and light source + distance between surface and viewer (Heuristic atmospheric attenuation)

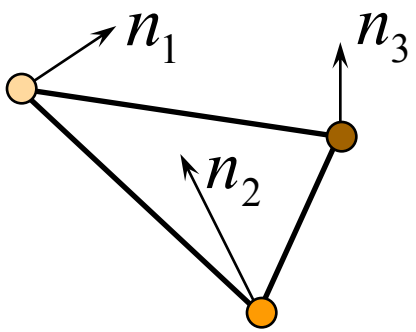
FLAT SHADING

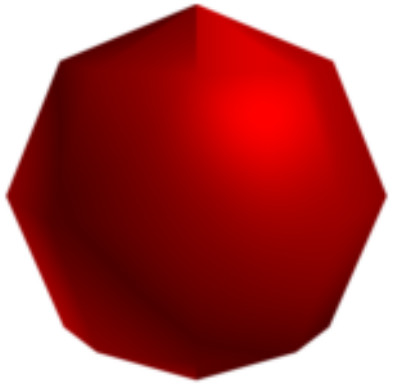
- Illumination value depends only on polygon normal
  - each polygon colored with uniform intensity
- Not adequate for polygons approximating smooth surface
- Looks non-smooth
  - worsened by Mach bands effect



GOURARD SHADING

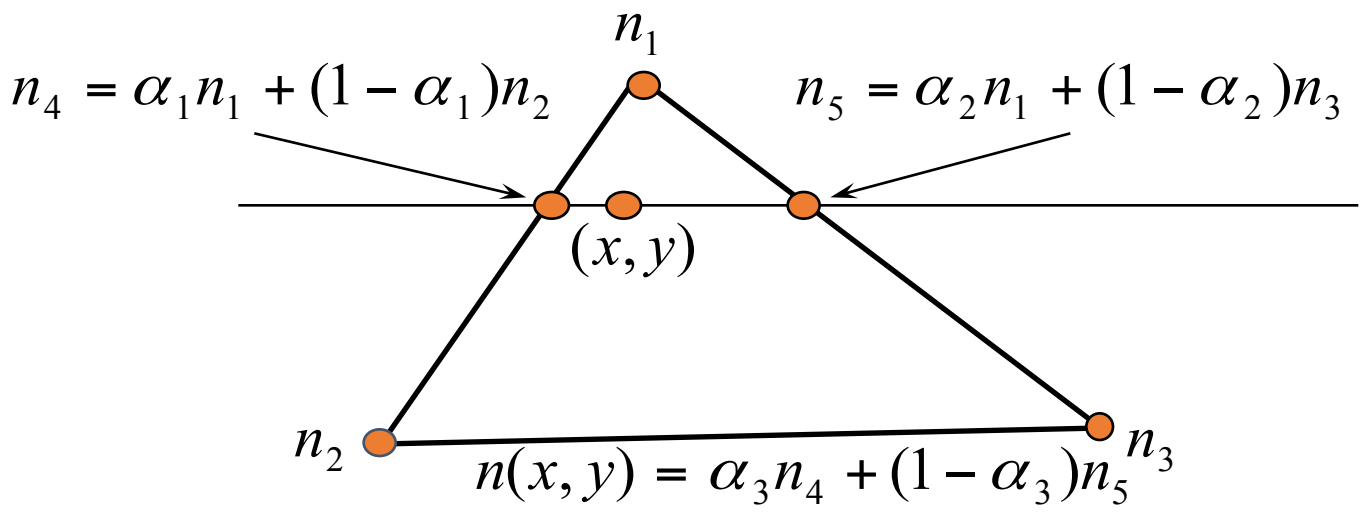
- Polyhedron - approximation of smooth surface
  - Assign to each vertex normal of original surface at point
  - If surface not available use estimate normal
- Compute illumination intensity at vertices using those normals
- Linearly interpolate vertex intensities over interior pixels of polygon projection





PHONG SHADING

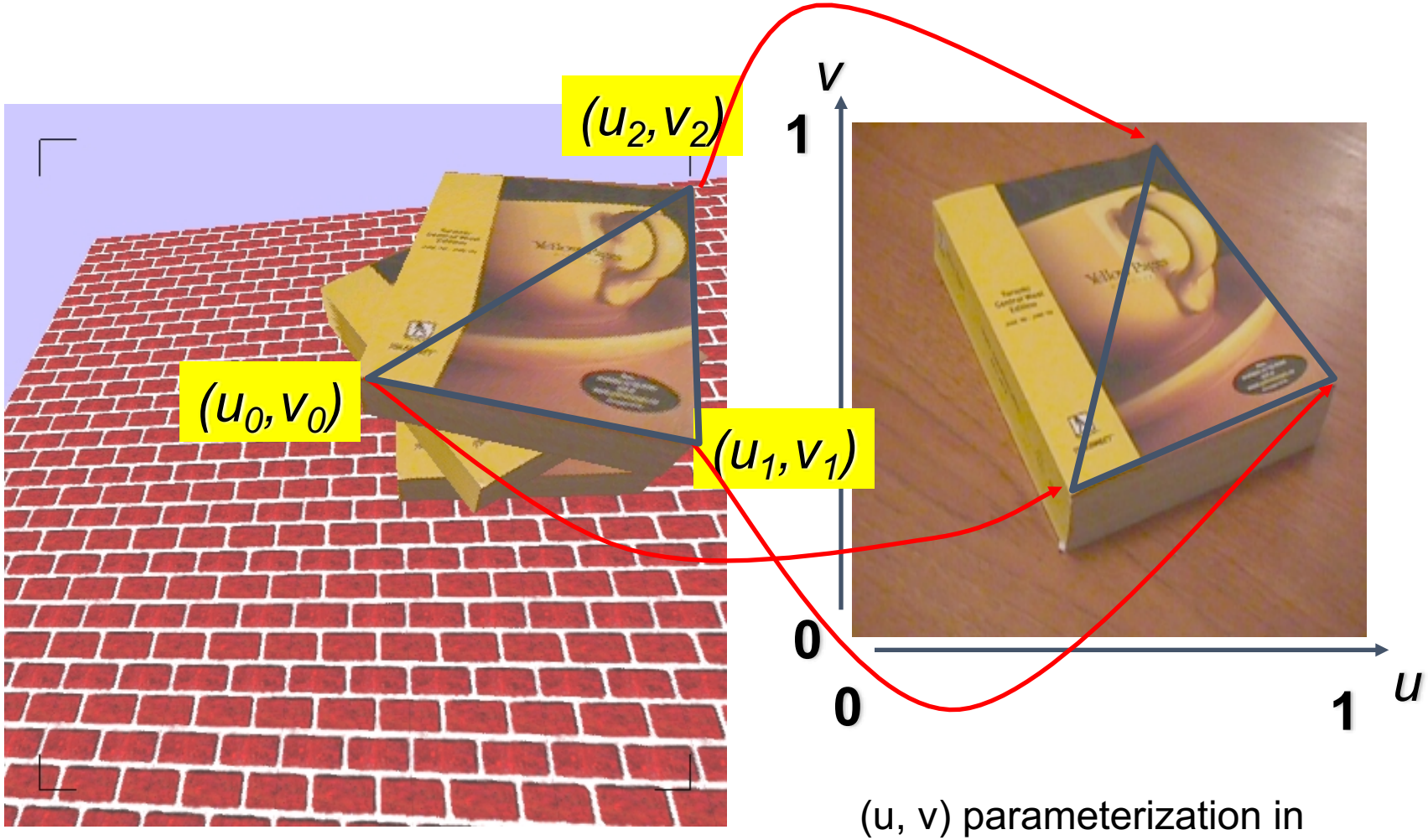
- Interpolate (in image space) normal vectors instead of intensities
- Apply illumination equation for each interior pixel with its own normal



$$n(x, y) = \alpha_3 n_4 + (1 - \alpha_3) n_5$$

$$c(x, y) = Ill(n(x, y))$$

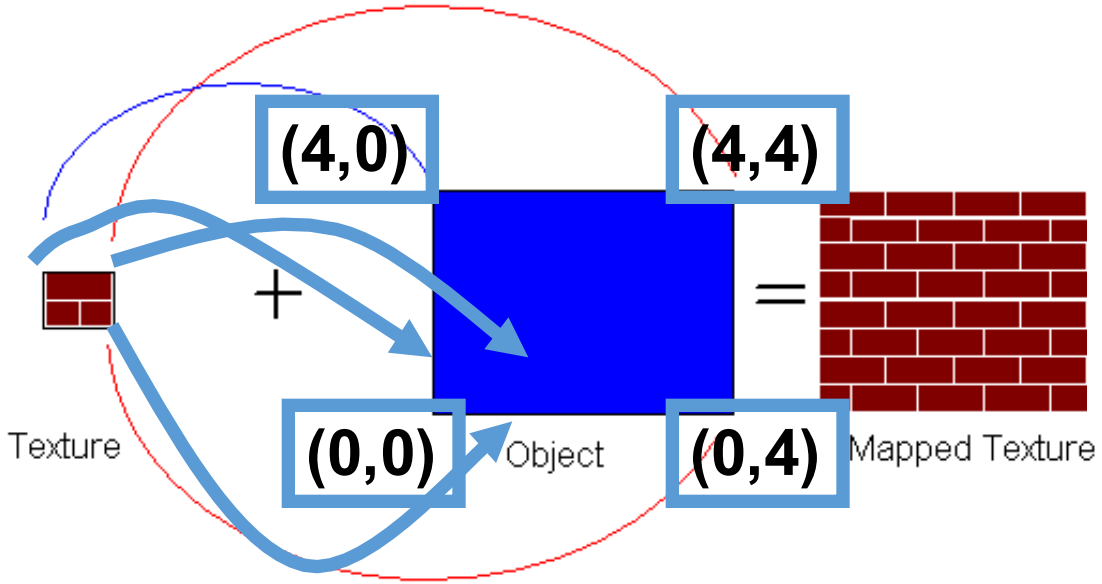
Texture Mapping



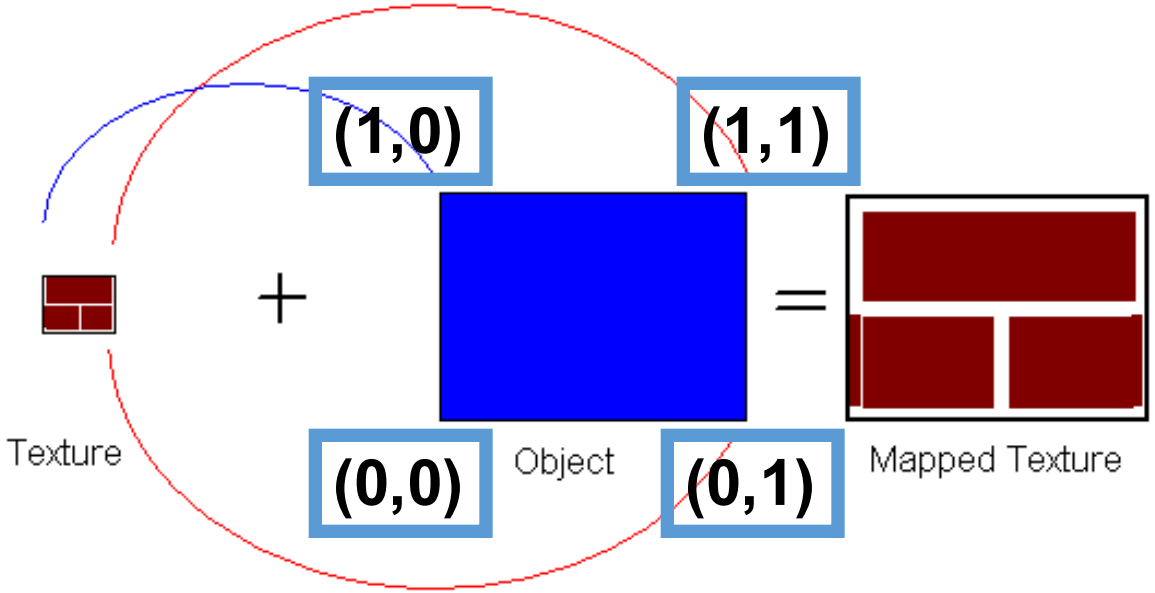
(u, v) parameterization in OpenGL

EXAMPLE TEXTURE MAP

```
glTexCoord2d(4, 4);
glVertex3d (x, y, z);
```

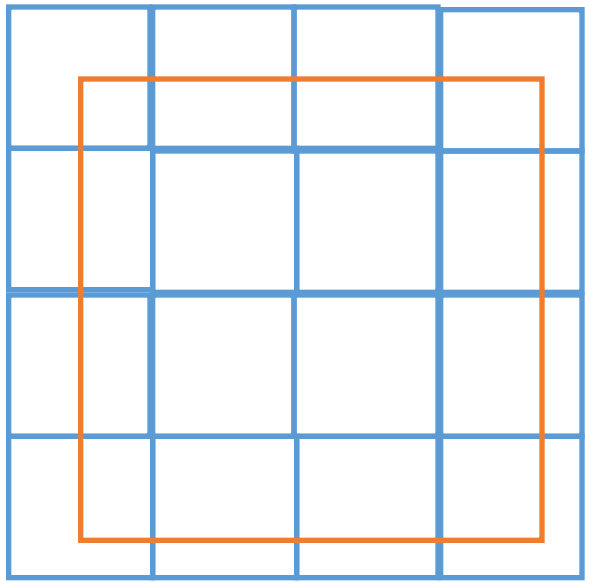



```
glTexCoord2d(1, 1);
glVertex3d (x, y, z);
```

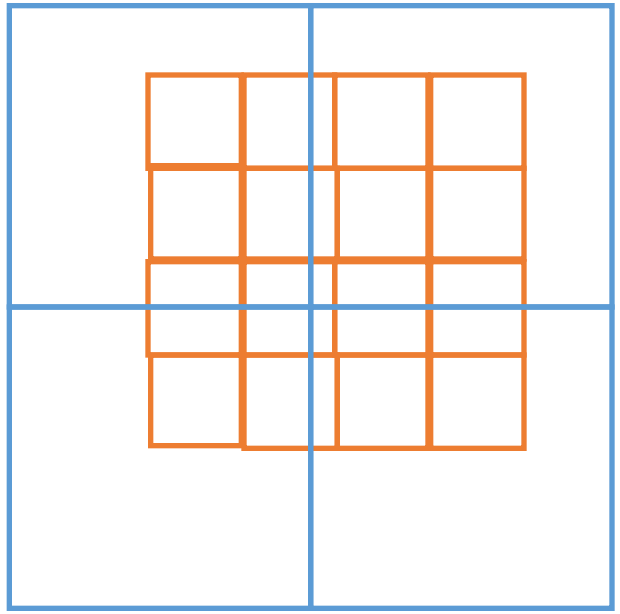


RECONSTRUCTION

- how to deal with:
  - pixels that are much larger than texels?
    - minification

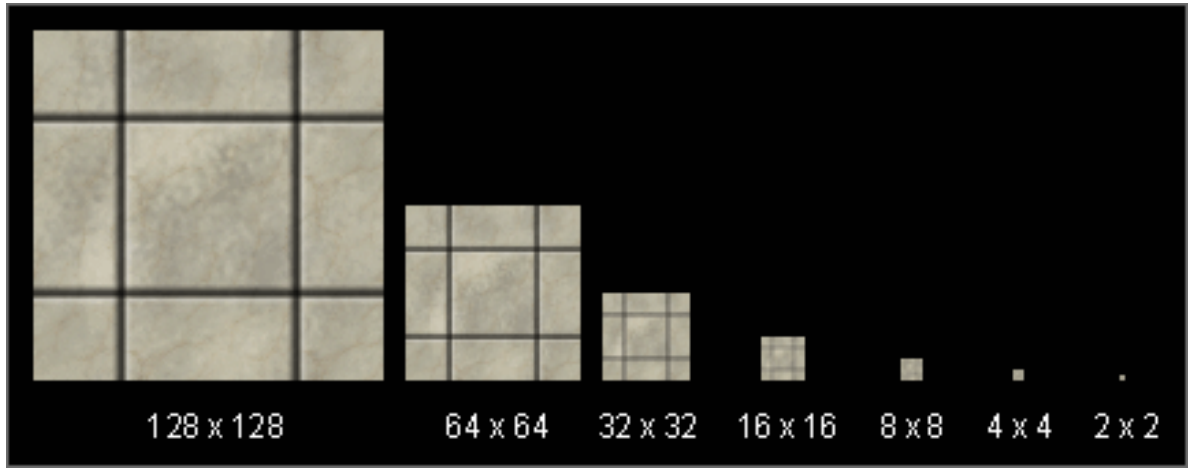


  - pixels that are much smaller than texels ?
    - magnification

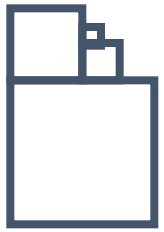


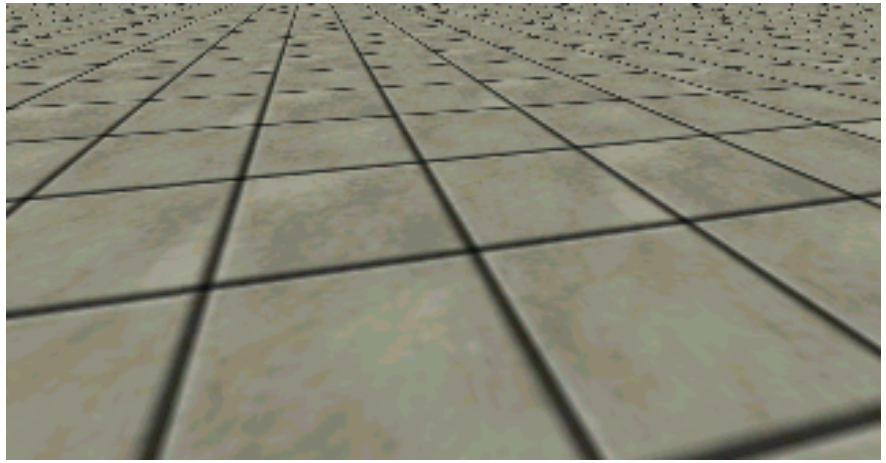
MIPMAPPING

use “image pyramid” to precompute averaged versions of the texture

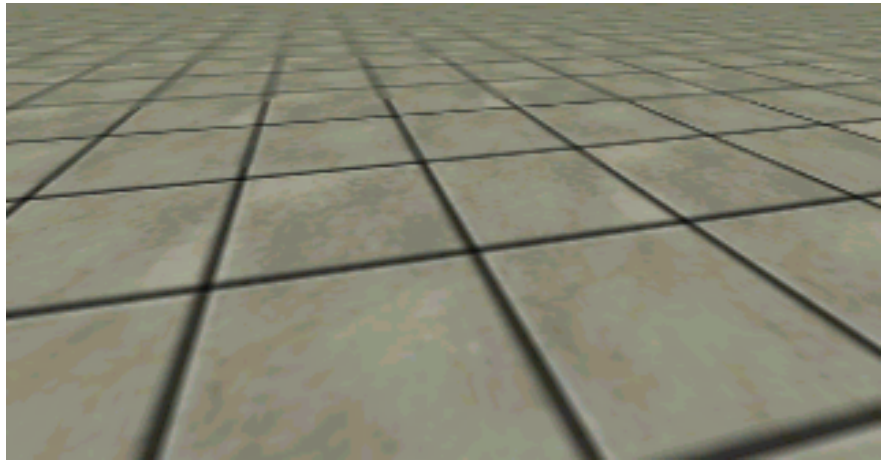


store whole pyramid in single block of memory





Without MIP-mapping

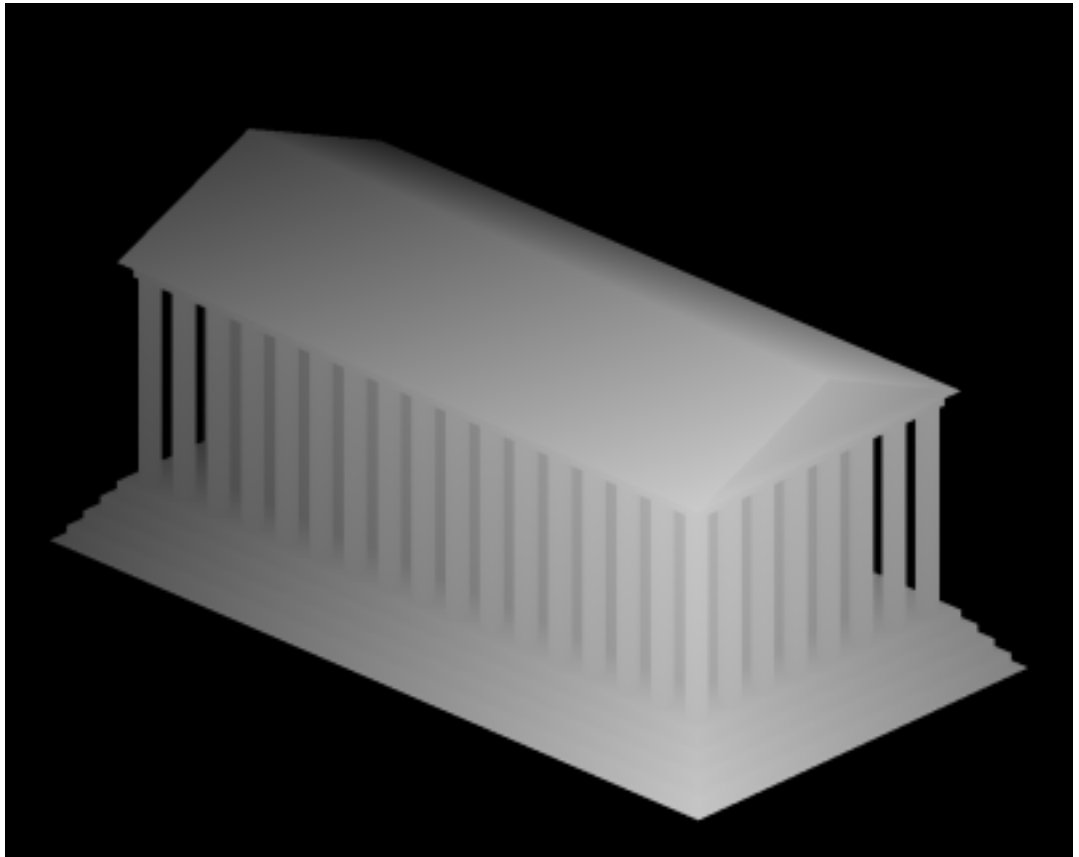


With MIP-mapping

SHADOWS

Need at least 2 shader passes:

1. Draw everything as it's viewed from the LIGHT SOURCE
   Depth per pixel ('depth map')



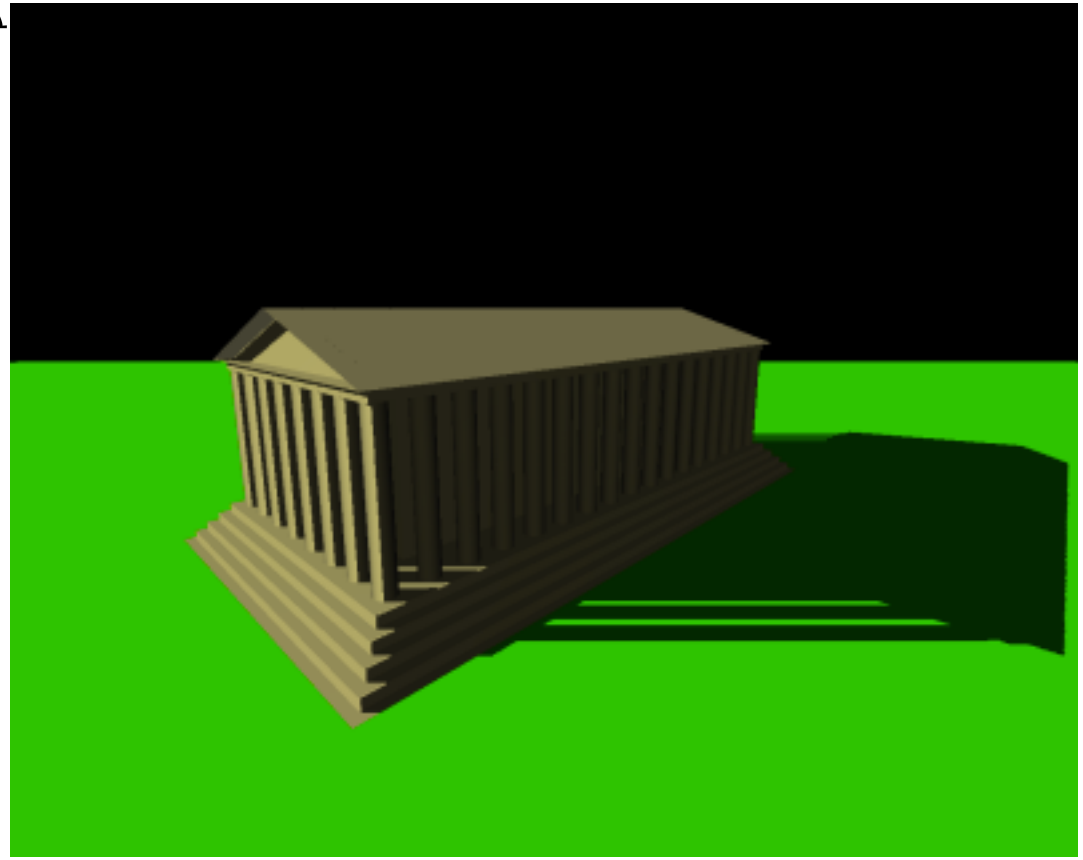
SHADOW MAPS

Need at least 2 shader passes:

1. Draw everything as it's viewed from the LIGHT SOURCE
   Depth per pixel ('depth map').
2. Now draw everything from CAMERA
   When computing color per pixel:

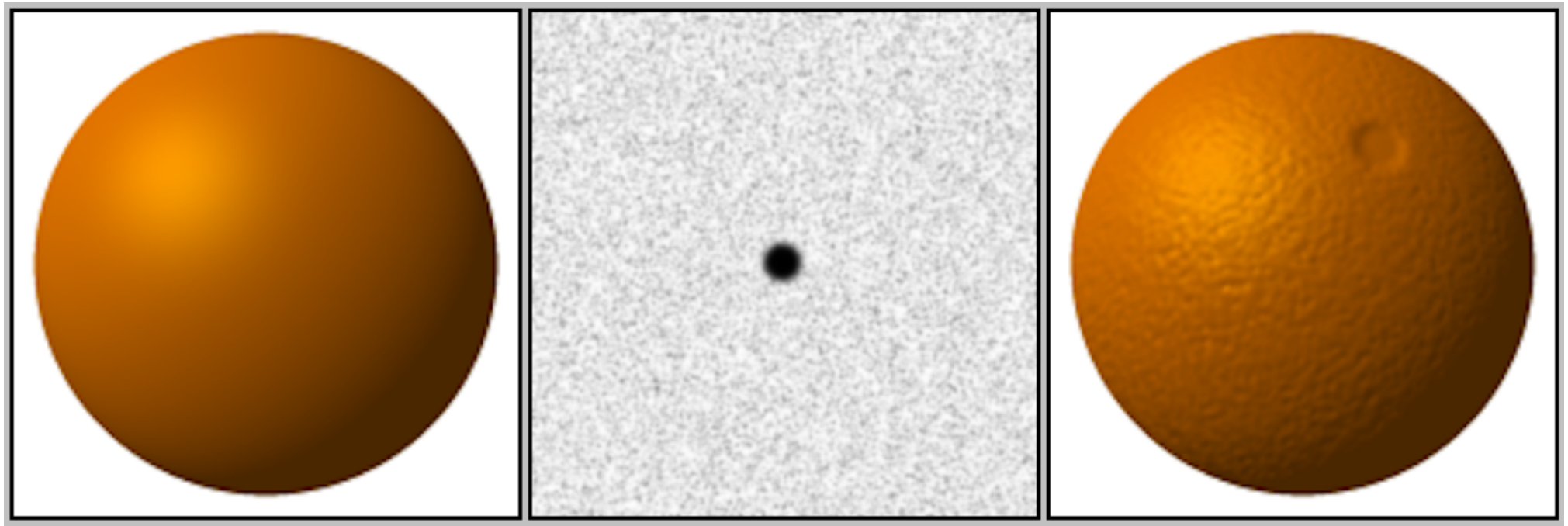
   - Find corresponding depth map pixel:
     D - distance from light source

   - Is distance from me to the camera > D?
     - Yes: I am occluded! I'm in SHADOW.
     - No: I'm LIT!



BUMP MAPPING: NORMALS AS TEXTURE

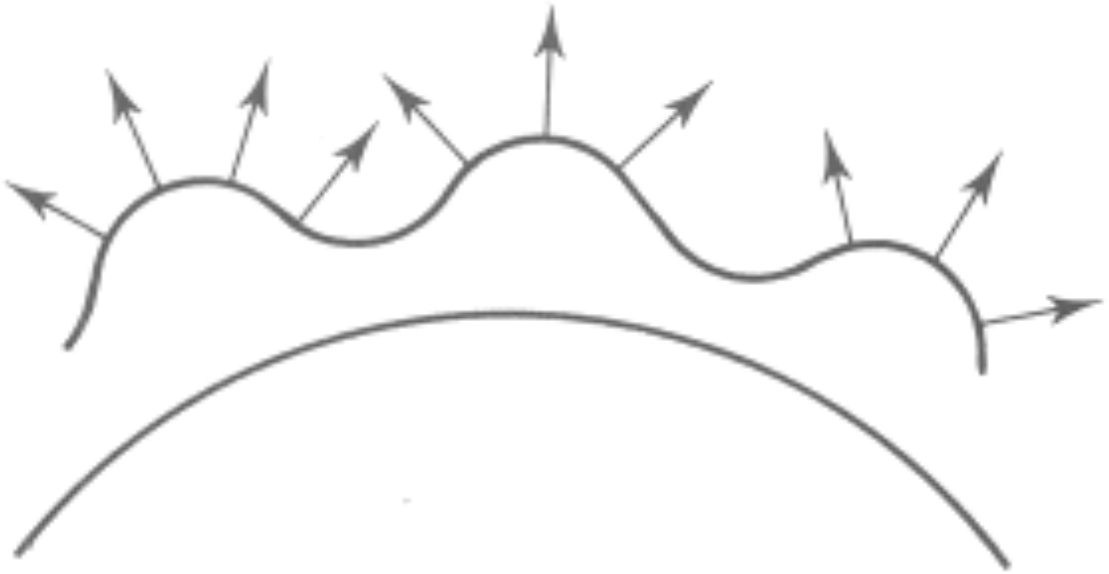
- object surface often not smooth – to recreate correctly need complex geometry model
- can control shape "effect" by locally perturbing surface normal
  - random perturbation
  - directional change over region



BUMP MAPPING

$\boldsymbol{O}'(u)$

Lengthening or shortening $\boldsymbol{O}(u)$ using $\boldsymbol{B}(u)$



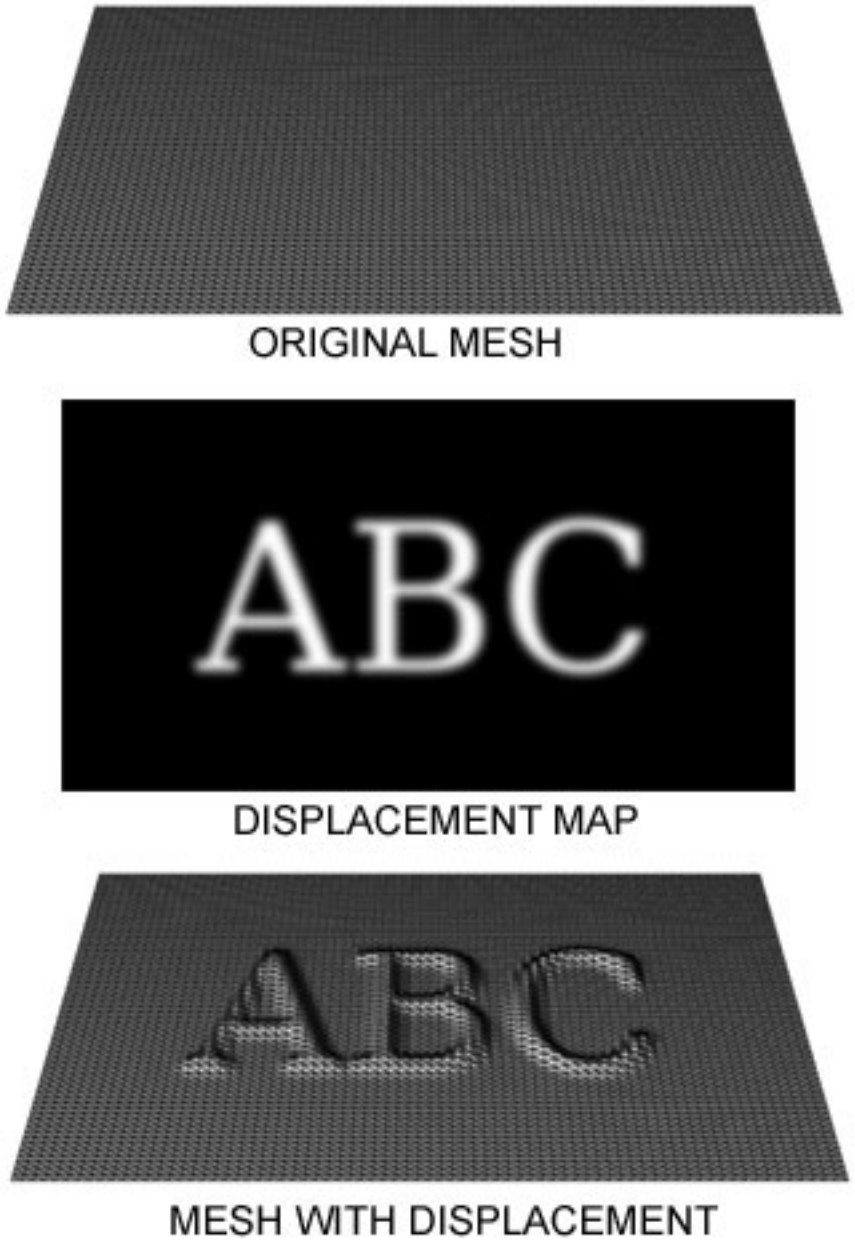
$\boldsymbol{N}'(u)$

The vectors to the 'new' surface

DISPLACEMENT MAPPING

- bump mapping gets silhouettes wrong
  - shadows wrong too

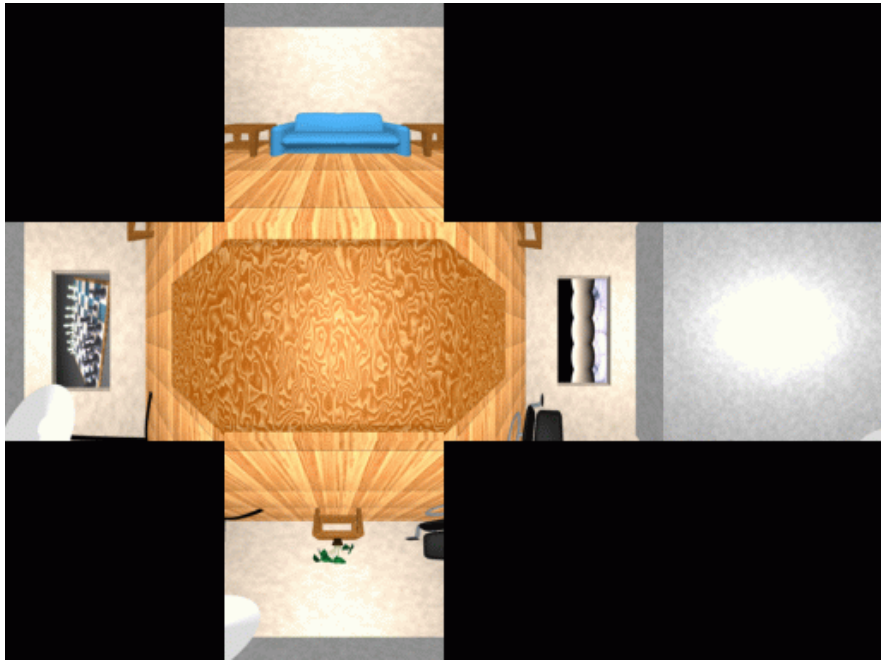
- change surface geometry instead
  - only recently available with realtime graphics
  - need to subdivide surface



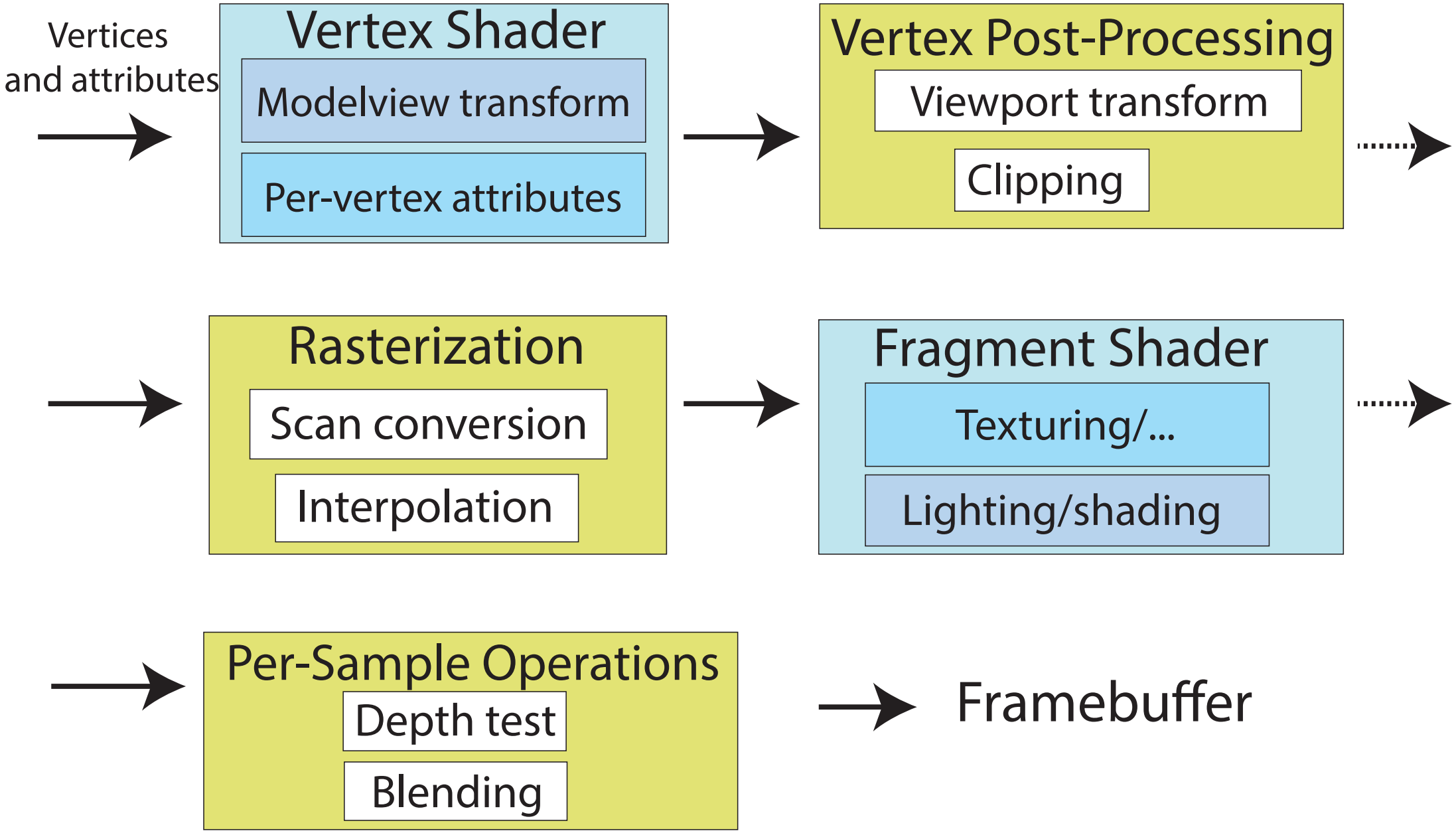
https://en.wikipedia.org/wiki/Displacement_mapping#/media/File:Displacement.jpg

CUBE MAPPING

- 6 planar textures, sides of cube
  - point camera in 6 different directions, facing out from origin



THE RENDERING PIPELINE



PAINTER'S ALGORITHM: PROBLEMS

- Intersecting polygons present a problem
- Even non-intersecting polygons can form a cycle with no valid visibility order:

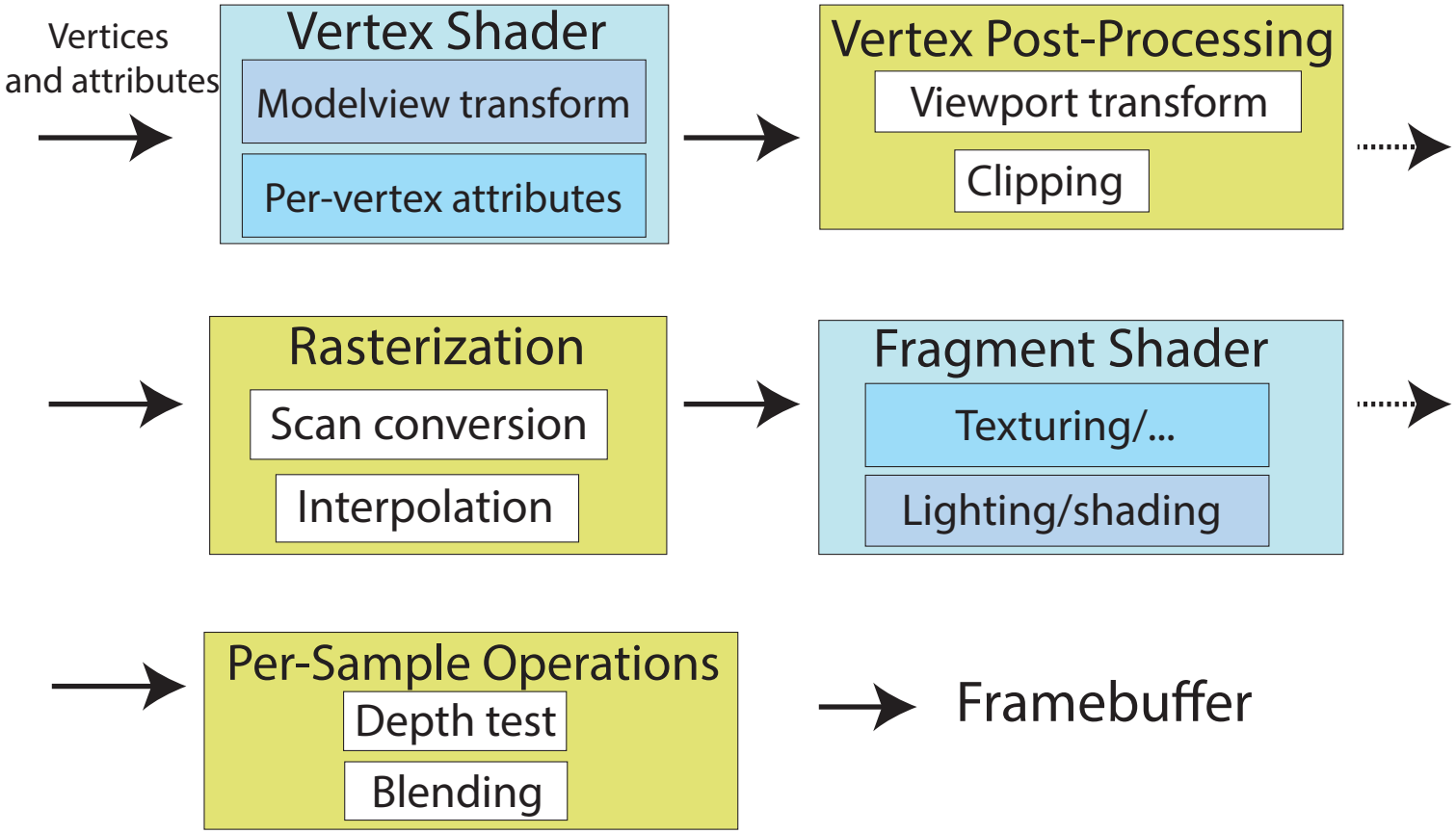
Z-BUFFER

- Store (r,g,b,z) for each pixel
  - typically 8+8+8+24 bits, can be more

```
for all i,j {
 Depth[i,j] = MAX_DEPTH
 Image[i,j] = BACKGROUND_COLOUR
}
for all polygons P {
  for all pixels in P {
    if (Z_pixel < Depth[i,j]) {
      Image[i,j] = C_pixel
      Depth[i,j] = Z_pixel
    }
  }
}
```

DEPTH TEST

- Why is it after FS?



DEPTH TEST PRECISION

- Reminder: projective transformation maps eye-space z to generic z-range (NDC)
- Simple example:

$$T\left(\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

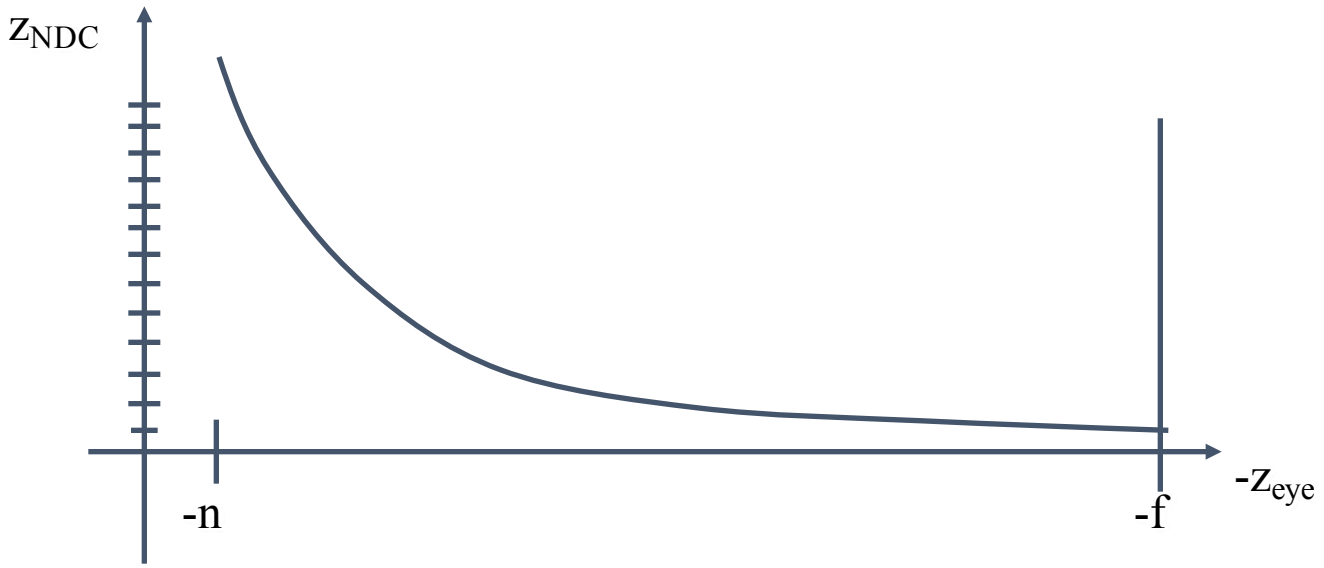
- Thus:

$$z_{NDC} = \frac{a z_{eye} + b}{-z_{eye}} = -a - \frac{b}{z_{eye}}$$

DEPTH TEST PRECISION

Therefore, depth-buffer essentially stores -1/z, rather than z!

- Issue with **integer** depth buffers
  - High precision for near objects
  - Low precision for far objects



DEPTH TEST PRECISION

- Low precision can lead to depth fighting for far objects
  - Two different depths in eye space get mapped to same depth in framebuffer
  - Which object "wins" depends on drawing order and scan-conversion

- Gets worse for larger ratios *f:n*
  - Rule of thumb: f:n < 1000 for 24 bit depth buffer

$$\frac{dz_{NDC}}{dz_{eye}} = \frac{-2fn}{(f-n)z_{eye}^2} = -\frac{2f}{\left(\frac{f}{n}-1\right)z_{eye}^2}$$