CPSC 314

Assignment 3

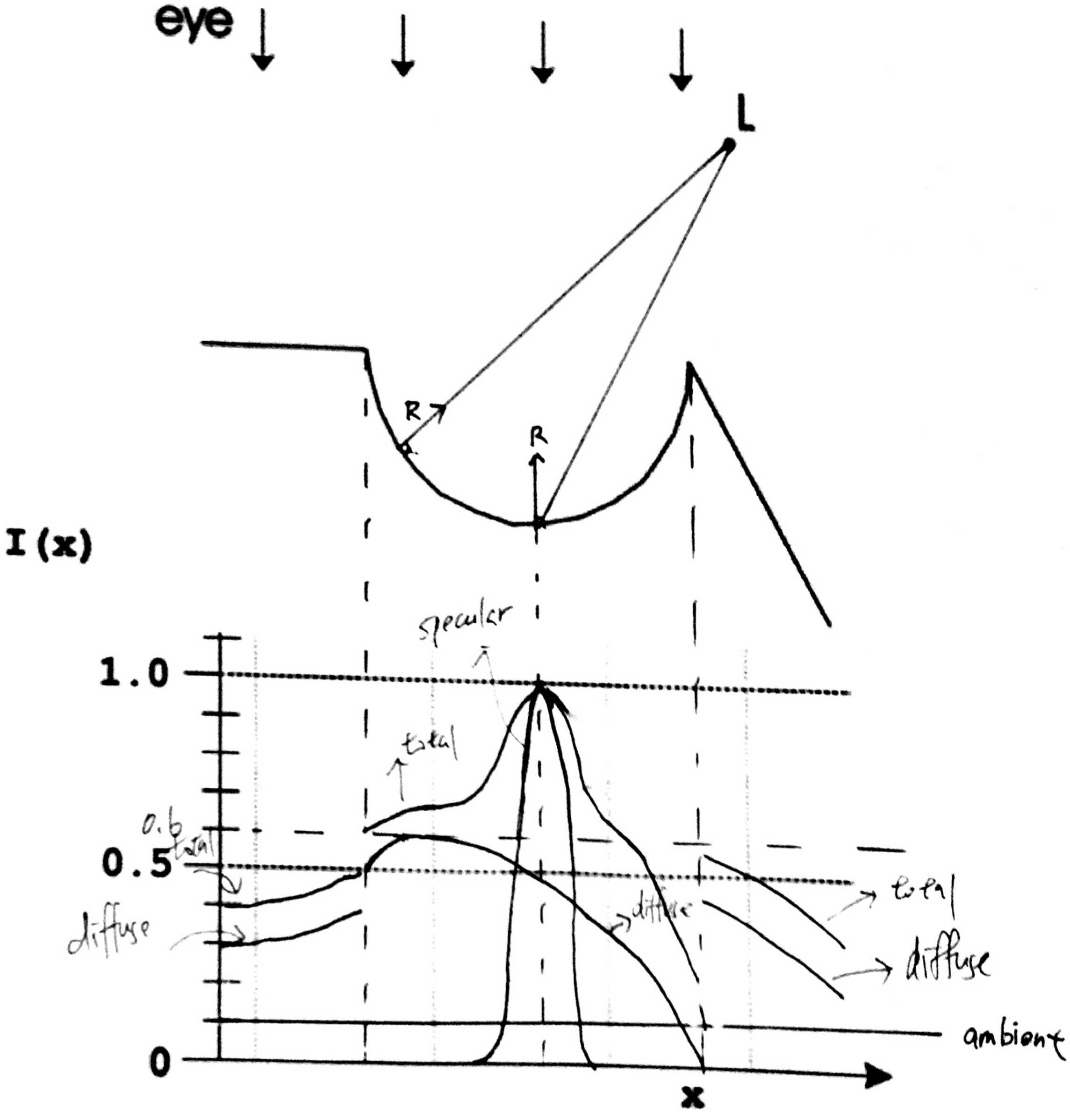
1. (6 points) Local Illumination

Sketch the illumination for the following scene when computed using the Phong illumination model. The scene is viewed from above using an orthographic projection and is lit by the single light source L. Draw 4 sketches, one for each of ambient, diffuse, specular, and total illumination. The Phong illumination model is given by:

$$I = k_a I_a + k_d I_d (N \cdot L) + k_s I_s (R \cdot V)^n$$

with the following values:

$$I_a = I_d = I_s = 1.0, k_a = 0.1, k_d = 0.6, k_s = 1.0, n = 100$$

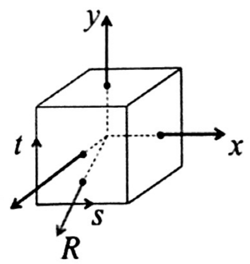


2. (4 points) Cube Map

An environment map uses texture maps to model what is seen in the distance in any given direction. A cube map is one type of environment map – it models the surrounding environment using a large cube that surrounds the scene. One of it's main purposes is being able to efficiently model what should be seen in the reflection of a specular object, i.e., the polished surface of a car or other objects with specular surfaces.

Given a 3D direction for a reflected vector, $R(R_x, R_y, R_z)$, develop that first determines whether R hits the given front face of the cube map. Then if it does, give the expressions for computing the texture coordinates s and t within that front face. Assume $s \in [0, 1], t \in [0, 1]$.

You can assume that the cube map is symmetrically centred about the origin, which therefore also means that the size of the cube in the diagram does not matter. For example, a ray that has a reflected direction $R(k, k, k)$ will always pass through the $s = 1, t = 1$ corner of the face regardless of the scale of the cube. Note that the point from which the reflected ray comes does not matter, only its direction. The cube map thus models the environment *at infinity*.



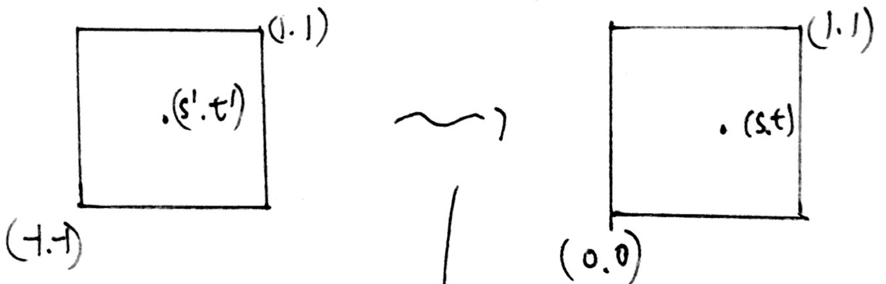
R hits the front face if Rz is the dominate (highest magnitute) component and $R_z > 0$, i.e.

$$R_z \ge |R_x| \,\&\&\, R_z \ge |R_y|$$

Front face is parallel to x.y-plane ⇒ $z = k$, where k can be any constant.

R_x and R_y is normalized by R_z ⇒ $s' = \frac{R_x}{R_z}$, $t' = \frac{R_y}{R_z}$

but the cube is defined as x.y. $z = \pm k$



scale from (s', t') to $(s.t)$ ⇒ $s = \frac{1}{2}\left(\frac{R_x}{R_z} + 1\right)$

$$t = \frac{1}{2}\left(\frac{R_y}{R_z} + 1\right)$$

3. (4 points) A ray R(t) begins at a known eyepoint, E, and is travels in a direction V towards a given screen pixel. In the scene, there is a cone of radius r with its circular base sitting on the xy-plane, centered on the point $(a, b, 0)$. The cone has radius r at its base and has its tip located at (a, b, h). Describe how to compute whether or not the ray intersects the cone, and, if so, at what point (x, y, z) the ray first encounters the cylinder.

Check both base surface and the cone.

$R(t) = E + t\vec{V} \Rightarrow \begin{cases} x(t) = e_x + tV_x \\ y(t) = e_y + tV_y \\ z(t) = e_z + tV_z \end{cases}$

$E = (e_x, e_y, e_z)$

$\vec{V} = (V_x, V_y, V_z)$

※ Check the hit to the base

⇒ find where R(t) hit xy-plane

$\Rightarrow 0 = e_z + tV_z \Rightarrow \left(t = \frac{-e_z}{V_z}\right)$

⇒ the ray cross the xy-plane at

$\begin{cases} x = e_x - \frac{e_z}{V_z} V_x \\ y = e_y - \frac{e_z}{V_z} V_y \\ z = 0 \end{cases}$

⇒ check if the point is within circle.

$F(x, y) = r^2 - (x-a)^2 - (y-b)^2$

⇒ plug in (x, y) ⇒ check if $F(x, y) \geq 0$

⇒ if so, R intersect the base at $R\left(\frac{-e_z}{V_z}\right)$.

※ Check the hit to the cone.

The cone equation:

$G(x, y, z) = (x-a)^2 + (y-b)^2 - (z-h)^2 \left(\frac{r^2}{h^2}\right)$

⇒ if $G(x, y, z) = 0$ and $0 \leq z \leq h$

⇒ point (x, y, z) is on the cone.

⇒ So let $G(x, y, z) = 0$, and plug in

$\begin{cases} x(t) = e_x + tV_x \\ y(t) = e_y + tV_y \\ z(t) = e_z + tV_z \end{cases}$ ⇒ Solve for t.

⇒ discard any solution that $\begin{cases} z(t) < 0 \\ z(t) > h \end{cases}$

⇒ To find out the first intersection

⇒ choose the smallest non-negative t value.

⇒ and use this t ⇒ $R(t) = E + t\vec{V}$