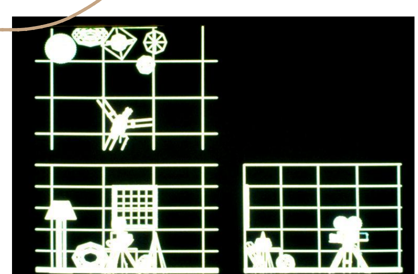
Course Topics

- *geometric transformations*
- *Javascript and WebGL*
- *graphics pipeline*
- *view volumes and projections*
- *scan conversion*
- *clipping and culling*
- *local illumination*
- *texture mapping*
- **ray tracing, global illumination**
- **colour perception**
- **parametric curves & surfaces**
- **character skinning**
- **animation**

Geometric Transformations

the work horse: 4x4 matrices

$$P_{WCS} = M_{OCS \to WCS} \; P_{OCS}$$



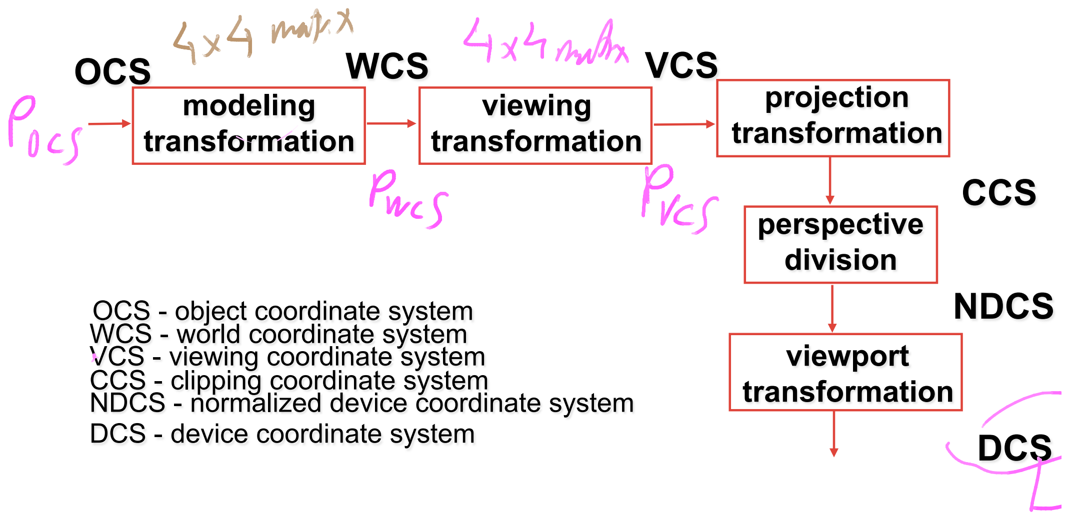
$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_{WCS} = \begin{bmatrix} 4x4 \\ \text{matrix} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_{OCS}$$

homogenous coordi

P_{OCS}

↑ inverse takes you from WCS → OCS

Projective Rendering Pipeline



OCS - object coordinate system
WCS - world coordinate system
VCS - viewing coordinate system
CCS - clipping coordinate system
NDCS - normalized device coordinate system
DCS - device coordinate system

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Geometric Transformations

- *review of relevant math*
- *4x4 transformation matrices*

Math Review

matrix vector multiplication

- points as column vectors

$$\begin{bmatrix} x' \\ y' \\ z' \\ h' \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ h \end{bmatrix}$$

- points as row vectors

$$\begin{bmatrix} x' & y' & z' & h' \end{bmatrix} = \begin{bmatrix} x & y & z & h \end{bmatrix} \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix}^T$$

Math Review

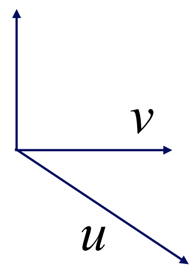
dot product

- also called *inner product*

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \bullet \begin{bmatrix} a \\ b \\ c \end{bmatrix} = x * a + y * b + z * c$$

Math Review

Cross Product



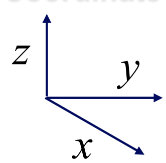
Right Handed Coordinate Syste

(curl fingers from u to v; thumb points to u x v)

Math Review

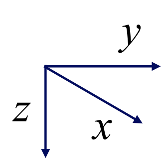
Coordinate Systems

Right-handed Coordinate System



using right-hand rule

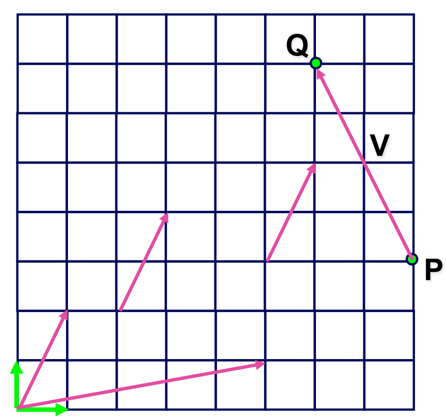
Left-handed Coordinate System



using left-hand rule

Math Reviw

Points and Vectors

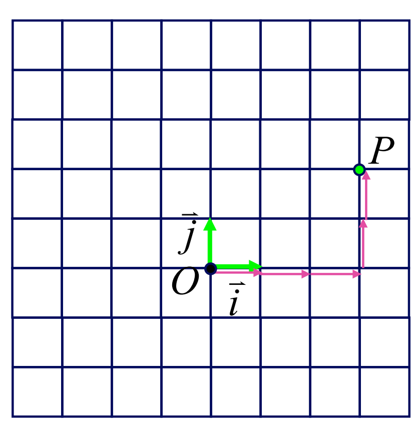


vector space
vectors are invariant
under translation

affine space:
allows vector-to-point additior

Math Reviw

Coordinate System vs Frame



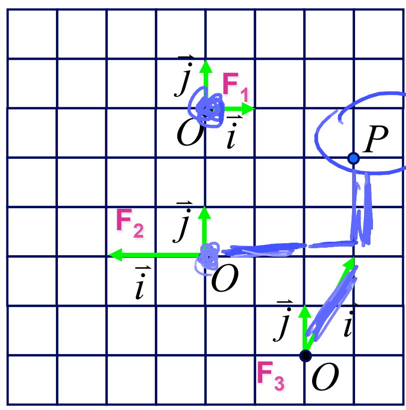
coordinate system: basis
frame: basis vectors +

$P = O + x\vec{i}$

origin

Math Review

Working with Frames



$P = O + x\vec{i} + y\vec{j}$

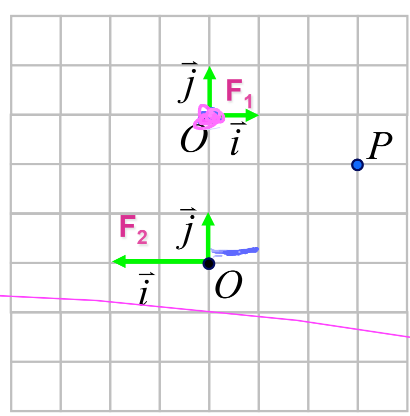
F1 $P_1(3,-1)$

F2 $P_2(-1.5,2)$

F3 $P_3(1,2)$

Transformations

Transformations as a change of frame



$P = O + x\vec{i} + y\vec{j}$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}_1 + x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix}_1 + y_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix}_1$$

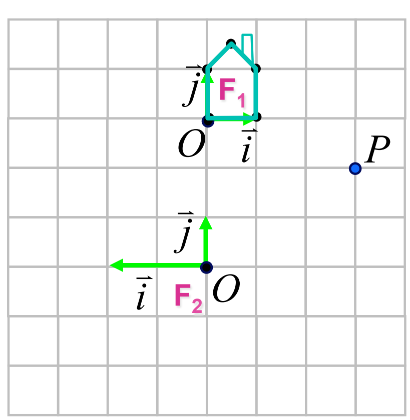
$$\begin{bmatrix} x \\ y \end{bmatrix}_2 = \begin{bmatrix} 0 \\ 3 \end{bmatrix}_2 + x_1 \begin{bmatrix} -0.5 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}_2 = \begin{bmatrix} -0.5 & 0 \\ 0 & 1 \end{bmatrix}$$

check:

Transformations

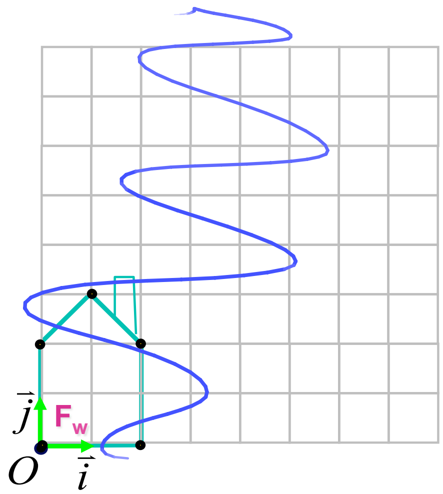
change of basis expressed using a matrix



$$\begin{bmatrix} x \\ y \end{bmatrix}_2 =$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_2 =$$

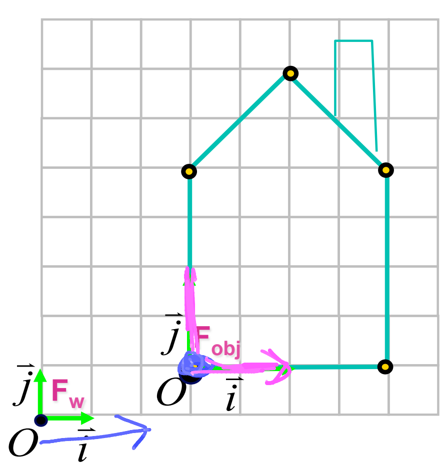
Usage of Transformations



set up the modeling matrix M

for each vertex v
v' = Mv

Usage of Transformations



$$P = O + x\vec{i} + y\vec{j}$$

$$\begin{bmatrix} x_{obj} \\ y_{obj} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}_{obj} + x_{obj}\begin{bmatrix} 1 \\ 0 \end{bmatrix}_{obj} + y_{obj}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}_w = \begin{bmatrix} 3 \\ 1 \end{bmatrix}_w + x_{obj} \cdot \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}_w = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 2 & 1 \end{bmatrix}$$

Using Transformations

$$\begin{bmatrix} x \\ y \end{bmatrix}_2 = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_1$$

2D

3D

3x3

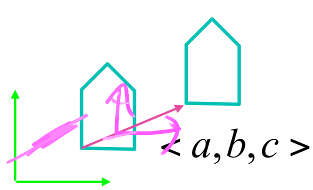
$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_W = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_W = \begin{bmatrix} 2 & 0 \\ 0 & 2 \\ 0 & 0 \\ 0 & \end{bmatrix}$$

Transformations

Translation

translate(a,b,c)



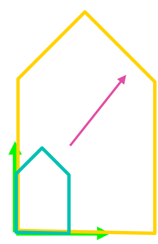
$$x' = x + a$$
$$y' = y + b$$
$$z' = z + c$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & & & a \\ & 1 & & b \\ & & 1 & c \\ & & & 1 \end{bmatrix}$$

Transformations

Scaling

scale(a,b,c)



$$x' = ax$$
$$y' = by$$
$$z' = cz$$

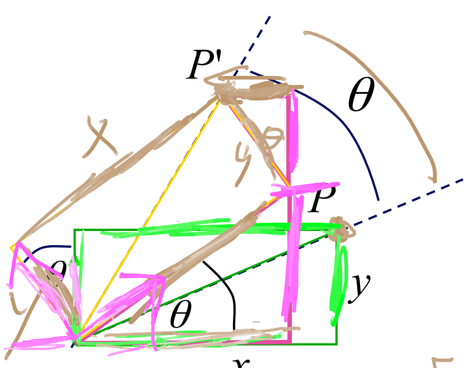
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} a & & & \\ & b & & \\ & & c & \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Transformations

Rotation

$Rotate(z,\theta)$

others

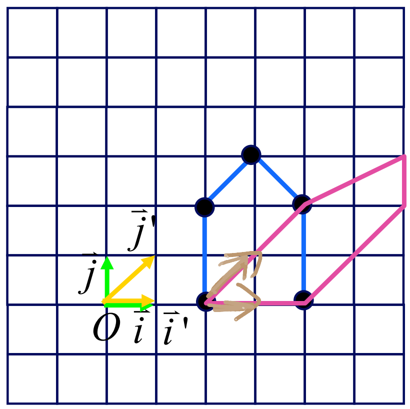


$$x' = x\cos\theta -$$
$$y' = x\sin\theta +$$
$$z' = z$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformations

Shear



~~shear~~

shear $\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Transformations

Affine transformations

- linear transformation + translations
- can be expressed as a 3x3 matrix + 3 vector

$$P' = M \cdot P + T$$

4x4 matrices

i j k

$$\underbrace{\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix}}_{w} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & T_x \\ m_{21} & m_{22} & m_{23} & T_y \\ m_{31} & m_{32} & m_{33} & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}}_{obj}$$

Origin } obj e

h=1