Composition of Transformations

Transformations

translate(a,b,c)

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & & & a \\ & 1 & & b \\ & & 1 & c \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

scale(a,b,c)

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} a & & & \\ & b & & \\ & & c & \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$Rotate(z,\theta)$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & & \\ \sin\theta & \cos\theta & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$P_A = \begin{bmatrix} i & j & k & o \\ 0 & 0 & 0 & 1 \end{bmatrix} P_B$$

$c_{12} = \cos(\theta_1 + \theta_2)$

$s_{12} = \sin(\theta_1 + \theta_2)$

cos

Simple Compositions

translate(a,b,c) translate(d,e,f)

$$\begin{bmatrix} 1 & & & a \\ & 1 & & b \\ & & 1 & c \\ & & & 1 \end{bmatrix} \begin{bmatrix} 1 & & & d \\ & 1 & & e \\ & & 1 & f \\ & & & 1 \end{bmatrix} = \begin{bmatrix} 1 & & & a+d \\ & 1 & & b+e \\ & & 1 & c+f \\ & & & 1 \end{bmatrix}$$

translate

scale(a,b,c) scale(d,e,f)

$$\begin{bmatrix} a & & & \\ & b & & \\ & & c & \\ & & & 1 \end{bmatrix} \begin{bmatrix} d & & & \\ & c & & \\ & & f & \\ & & & 1 \end{bmatrix} = \begin{bmatrix} ad & & & \\ & be & & \\ & & cf & \\ & & & 1 \end{bmatrix}$$

scale(ad, b

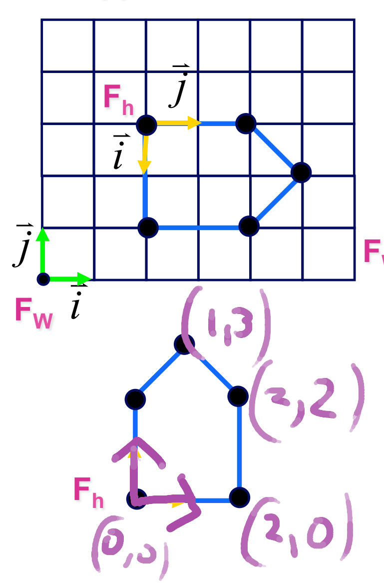
$Rotate(z,\theta_1)\ Rotate(z,\theta_2)$

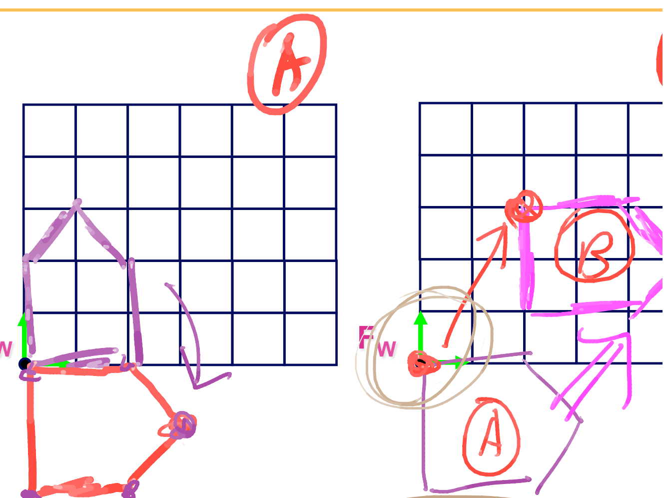
$$\begin{bmatrix} c_1 & -s_1 & & \\ s_1 & c_1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & & \\ s_2 & c_2 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} = \begin{bmatrix} c_1c_2 - s_1s_2 & -s_1 \\ s_1c_2 + c_1s_2 & c_1c \end{bmatrix}$$

$P_B = Translate(2,3,0)\ Rotate(z, -90°)$

Composing Transformations

suppose we want





$P_A = Rotate(z, -90°)\ P_h$

$P_B = Translate(2,3,0)\ P_A$

Composing Transformations

local coords,

$$P_W = Translate(2,3,0)\,Rotate(z,-90)\,P_h$$

fixed coords

- R-to-L: interpret operations wrt fixed coords
- L-to-R: interpret operations wrt local coords
- most GL utility packages: L-to-R, local coords

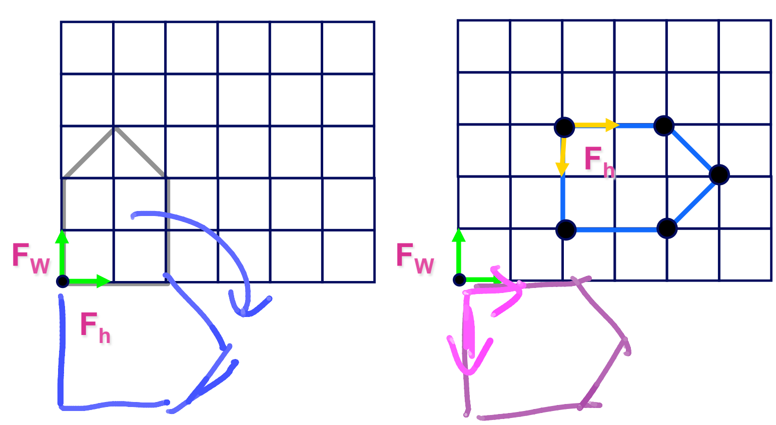
Translate(2,3,0)

Rotate(z, -90°)

Translate: M ←

Rotate: M ←

Composing Transformations



local coords, rotate, then translate

$P_w = Rot(z, -90°)\,Translate(-3, 2, 0)$ |

Composing Transformations

Equivalence

$$P_W = Trans(2,3,0)\,Rot(z,-90)\,P_h$$

$$P_W = Rot(z,-90)\,Trans(-3,2,0)\,P_h$$

Undoing Transformations

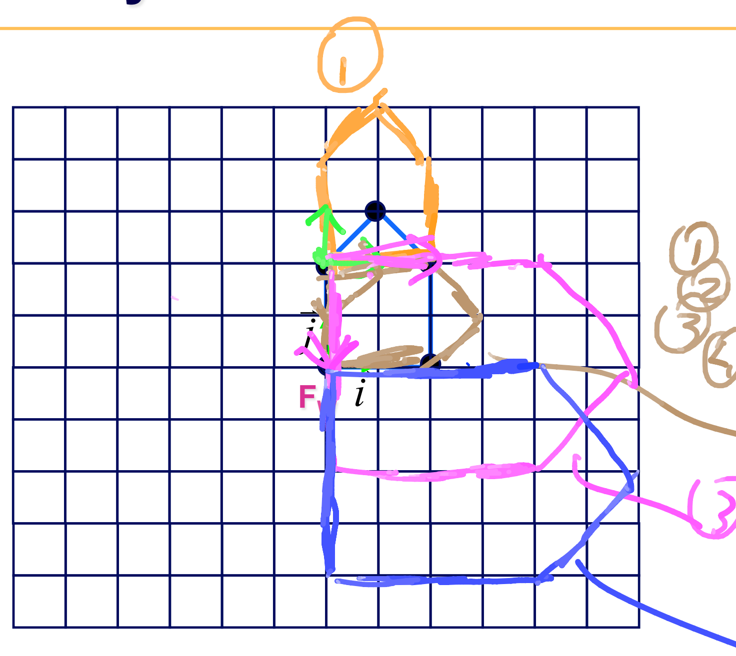
Trans(a, b, c) Trans(-a, -b, -c) .

Rotate(θ, z) Rotate(-θ, z) = .

Rotate(θ, -z) = .

Scale(a, b, c) Scale(1/a, 1/b, 1/c) =

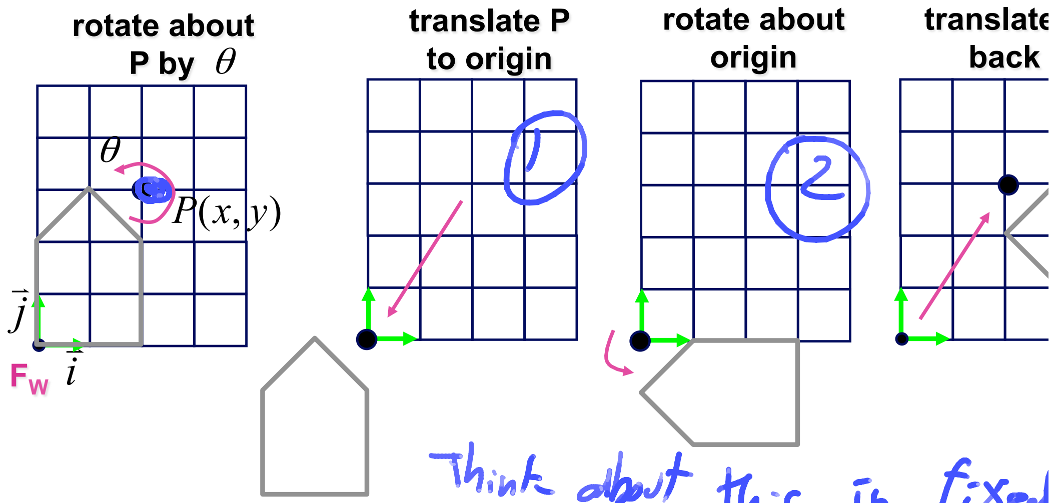
Test yourself ...




```
translate(0,2,0);
rotate(-90,0,0,1);
scale(2,2,2);
translate(1,0,0);
drawHouse();
```

Rotation about a point

Rotate z (



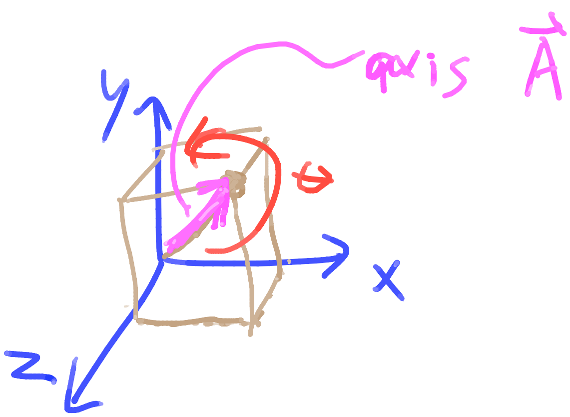
Think about this in fixed

$M = Trans(x,y,0)\ Rot(z,\theta)\ T$

3 2

Rotation about an arbitrary axis

glRotatef(angle, x, y, z);



plan:

A align the

B rotate ar

C undo ste