Curves and Surfaces

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- how to model curves?
- how to model surfaces ?

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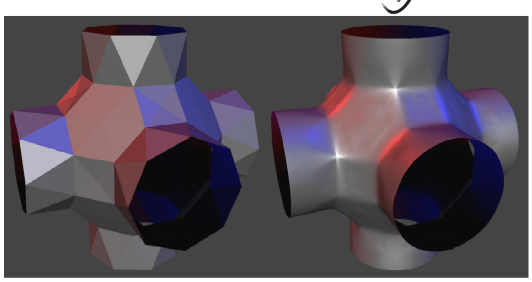
1. Subdivision curves and surfaces

– repeated corner cutting



Initial mesh   Butterfly scheme interpolation

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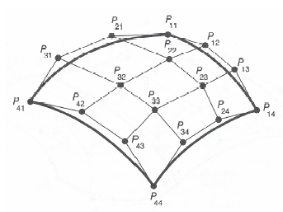
2. Parametric curves and surfaces



http://www.sjbaker.org/teapot/NewellTeaset.jpg

Parametric Curve examples

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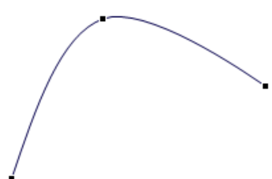
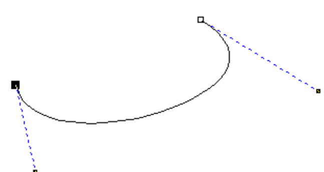


Powerpoint



CorelDraw

Test yourself... $\boxed{\text{Curve}}$

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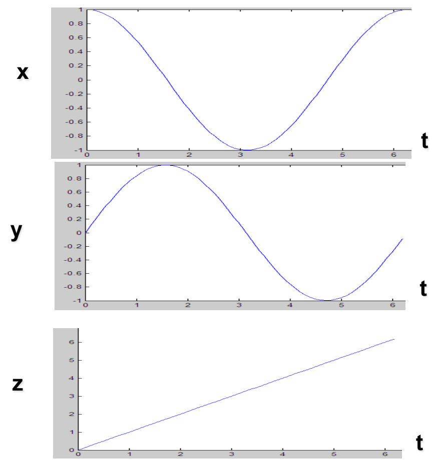
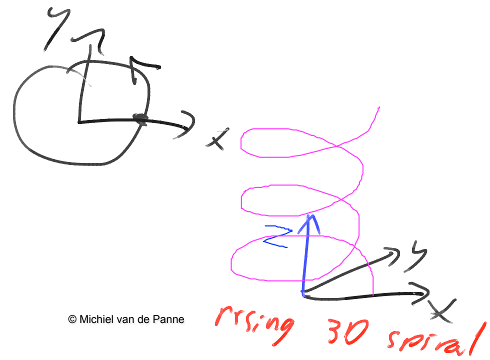


Typically we'll use cubic functions of t.



rising 3D spiral

Cubic Parametric Curves

$$x(t) = a_3 t^3 + a_2 t^2 + a_1 t + a_0$$

$$y(t) = b_3 t^3 + b_2 t^2 + b_1 t + b_0$$

$$z(t) = \dots$$

$$t \in [0, 1]$$

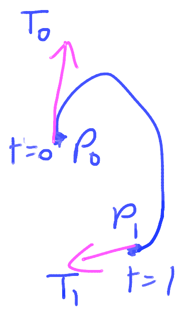
polynomial basis

basis coefficients

$$x(t) = \underbrace{\begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix}}_{\text{polynomial basis}} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix}$$

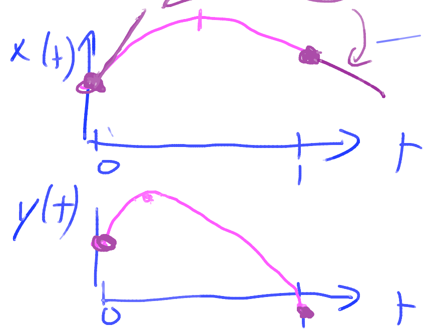
$$= T \cdot A$$

Hermite Curves (~~see~~ also: Bézier)



user input

$$\begin{bmatrix}P_0\\P_1\\T_0\\T_1\end{bmatrix} \xrightarrow{x} \begin{bmatrix}x(0)\\x(1)\\x'(0)\\x'(1)\end{bmatrix} \xrightarrow{M} \begin{bmatrix}a_3\\a_2\\a_1\\a_0\end{bmatrix}$$



$$\Longrightarrow x(t) = [t^3 \quad t^2 \quad t \quad 1] \underbrace{\begin{bmatrix}a_3\\a_2\\a_1\\a_0\end{bmatrix}}_{A}$$

$$\frac{dx}{dt} = x'(t) = [3t^2 \quad 2t \quad 1 \quad 0]\, A$$

$$\begin{aligned}
x(0) &= [0 \quad 0 \quad 0 \quad 1] \cdot A \\
x(1) &= [1 \quad 1 \quad 1 \quad 1] \cdot A \\
x'(0) &= [0 \quad 0 \quad 1 \quad 0] \cdot A \\
x'(1) &= [3 \quad 2 \quad 1 \quad 0] \cdot A
\end{aligned}$$

$$\begin{bmatrix} x(0) \\ x(1) \\ x'(0) \\ x'(1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix}$$

$$G_x = C \cdot A$$

$$\Longrightarrow \underline{A = C^{-1} G_x}$$

$$x(t) = [t^3 \quad t^2 \quad t \quad 1] \cdot A$$

$$= [t^3 \quad t^2 \quad t \quad 1] \cdot C^{-1} \, G_x$$

Hermite basis matrix

└ same for $y(t)$, $z(t)$

draw curve by evaluating $x(t)$ for $t \in [0, 1]$