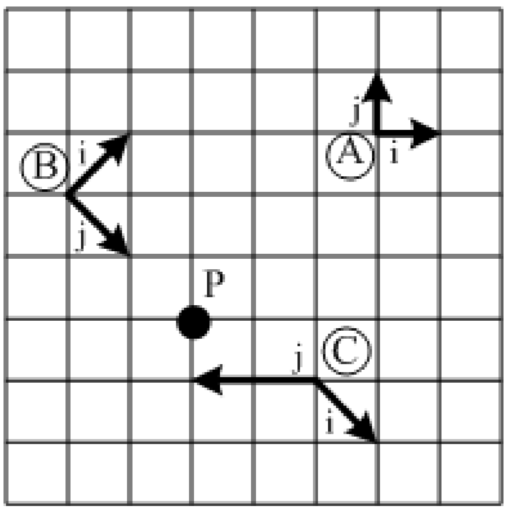
CPSC 314
Theory Assignment 2

1. Transformation as a Change of Coordinate Frame



Derive a transformation that takes a point from frame C to frame B, i.e., determine $M_{C \to B}$, where $P_B = M_{C \to B} P_C$. Verify your solution using the coordinates of P with respect to the different frames (see your answers in assignment 0).

2. Given a line segment $S = (P_0, P_1)$ in 2D and a point P, write an algorithm to find if the point is on the line segment. Note that your algorithm should operate in the Euclidean (not discrete) space.

3. Decompose the following complex transformations in homogeneous coordinates into a product of simple transformations (scaling, rotation, translation, shear). Pay attention to the order of transformations.

(a)

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

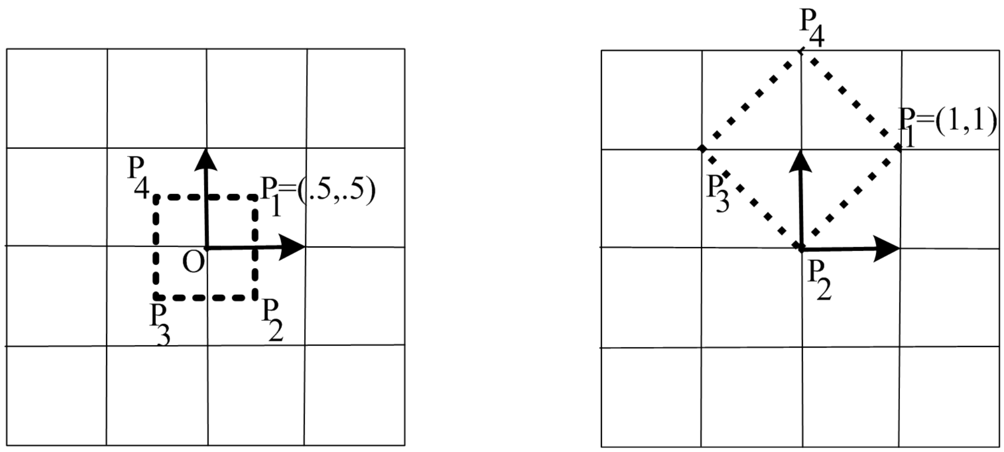
(b)

$$\begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(c) What is the inverse of the transformation matrix in part (b) of this question?

(d) Give the sequence of OpenGL transformations that would produce the same transformation matrix as in part (a) of this question.

4. Write down the 2D transformation matrix that maps the unit square centered at the origin as shown on the left to the square on the right in the figure below. Show your work.



5. Answer yes/no and provide a short explantion. All the transformations are in 3D.

   (a) Does perspective transformation preserve parallel lines?

   (b) Is shear * translate = translate * shear?

(c) Is shear1 * shear2 = shear2 * shear1?

(d) Does shear preserve lenghts?

6. Given the triangle T with vertices $P_1 = (1, 0, 0)$,$P_2 = (0, 2, 0)$, $P_3 = (0, 1, 1)$ and the transformation

$$S = \begin{pmatrix} 1 & -0.5 & 0 & 0 \\ 0.5 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(a) Compute the vertices of the triangle after applying the transformation S to it.

(b) Compute the normal of the triangle before and after applying the transformation S to it.

7. Prove that transforming a line segment using an affine transformation is equivalent to transforming its end points, or in other words prove that

$$T((1-u)P_1 + uP_2) = (1-u)T(P_1) + uT(P_2)$$

Why are the two claims above equivalent?