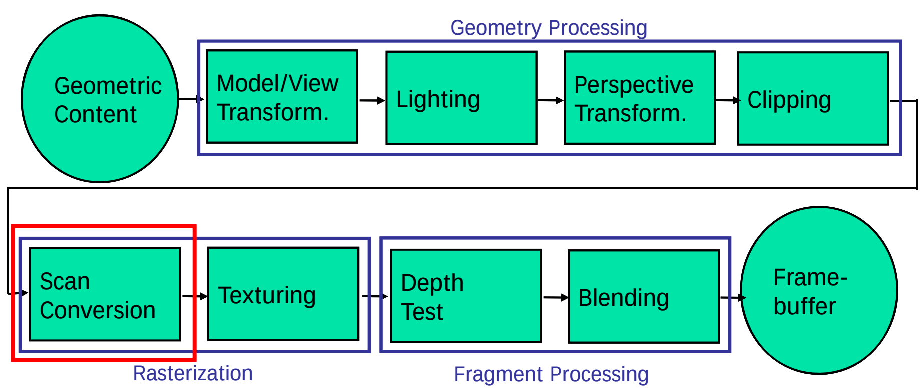
Chapter 8

Scan Conversion – Drawing on Raster Display (part 1 – Lines)

Rendering Pipeline



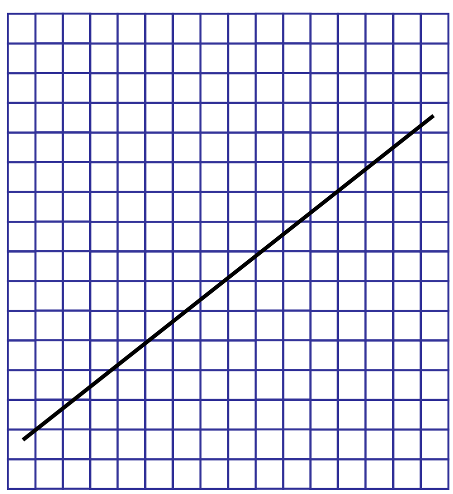
- Discard geometry outside viewport window

Scan Conversion - Rasterization

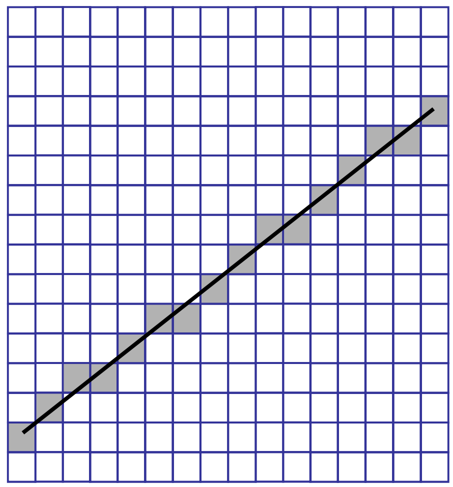
Convert continuous rendering primitives into discrete fragments/pixels

- Lines
  - Basic (explicit)
  - Bresenham (Midpoint)
- Triangles
  - Implicit formulation
  - Scanline
- Interpolation

Scan Conversion - Lines



Scan Conversion - Lines



Idea: Use Explicit Line Formula

Explicit - one coordinate as function of the others

$$y = f(x)$$

$$z = f(x, y)$$

line

$$y = mx + b$$

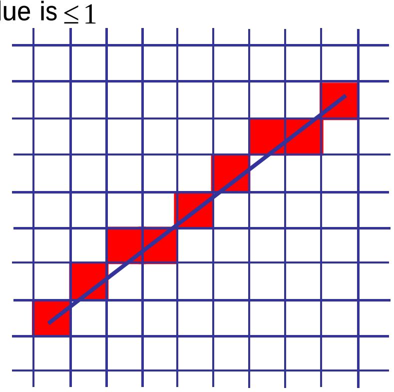
$$y = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1) + y_1$$

Typically separate into 4 (or 8) cases (why?)

Basic Line Drawing

Assume $x_1 < x_2$ & line slope absolute value is ≤ 1

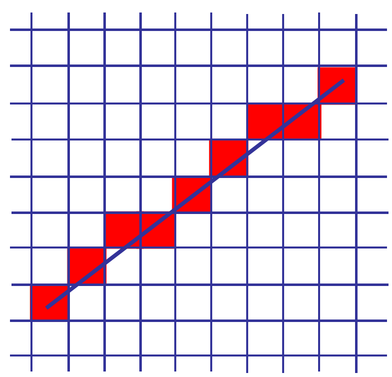
Line (x_1, y_1, x_2, y_2)
begin
float $dx, dy, x, y, slope$;
$dx \Leftarrow x_2 - x_1$;
$dy \Leftarrow y_2 - y_1$;
$slope \Leftarrow {}^{dy}/_{dx}$;
$y \Leftarrow y_1$
for x from x_1 to x_2 do
begin
PlotPixel $(x,$ **Round** $(y))$;
$y \Leftarrow y + slope$;
end ;
end ;



Questions:
Can this algorithm use integer arithmetic ?

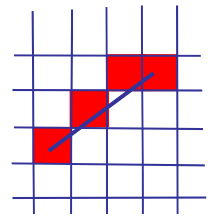
Midpoint (Bresenham) Algorithm

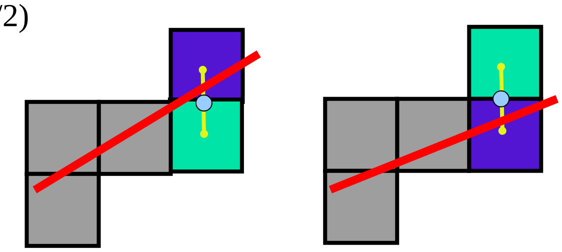
- **Key Observation 1:**
  - At each step have ONLY 2 choices
    - East/North-East



Midpoint (Bresenham) Algorithm

- **Key Observation 2:**
  - Can decide based on whether midpoint is above/below line
  - How?
    - Evaluate implicit line equation at (x+1, y+1/2)

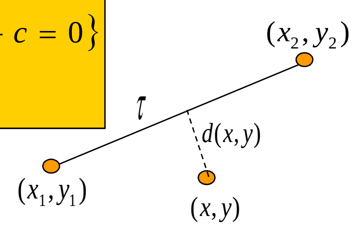




Bresenham Algorithm

Implicit formulation = distance (up to scale)

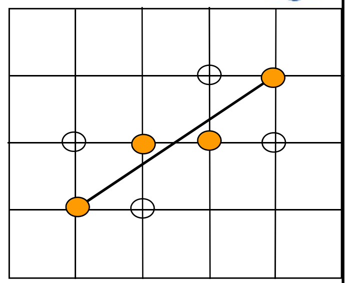
$$\tau = \{(x, y) : ax + by + c = xdy - ydx + c = 0\}$$
$$d(x, y) = 2(xdy - ydx + c)$$

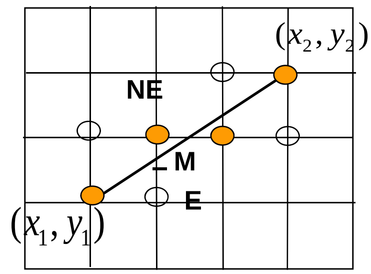


- Given point $P = (x, y)$, $d(x, y)$ is signed distance of p to τ (up to scale)
- d is zero for $P \in \tau$

Bresenham (Midpoint) Algorithm

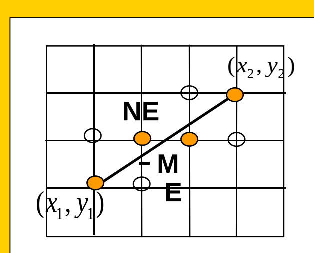
- Starting point satisfies $d(x_1, y_1) = 0$
- Each step moves right (east) or upper right (northeast)
- Sign of $d(x+1; y+\frac{1}{2})$ indicates if to move east or northeast





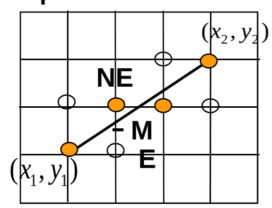
Bresenham (Midpoint) Algorithm

```
Line (x1, y1, x2, y2)
begin
int x, y, dx, dy, d;
x ⇐ x1;            y ⇐ y1;
dx ⇐ x2 − x1;      dy ⇐ y2 − y1;
PlotPixel (x, y);
while (x < x2) do
      d = (2x + 2)dy - (2y + 1)dx + 2c; // 2((x + 1)dy - (y + .5)dx + c)
      if (d < 0) then
      begin
            x ⇐ x + 1;
      end ;
      else begin
            x ⇐ x + 1;
            y ⇐ y + 1;
      end ;
      PlotPixel (x, y);
end ;
end ;
```



Bresenham (Midpoint) Algorithm

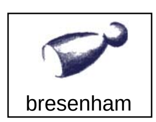
- Insanely efficient version (less computations inside the loop)
  – compute d incrementally
- At (x_1, y_1)
  $d_{start} = d(x_1 + 1, y_1 + \frac{1}{2}) = 2dy - dx$



- Increment in *d* (after each step)
  - If move east $\Delta_e = d(x+2, y+\frac{1}{2}) - d(x+1, y+\frac{1}{2}) =$
    $2((x+2)dy - (y+\frac{1}{2})dx + c)) - 2((x+1)dy - (y+\frac{1}{2})dx + c) = 2dy$
  - If move northeast $\Delta_{ne} = d(x_1+2, y_1+\frac{3}{2}) - d(x_1+1, y_1+\frac{1}{2}) =$
    $2((x+2)dy - (y+\frac{3}{2})dx + c)) - 2((x+1)dy - (y+\frac{1}{2})dx + c) = 2(dy - dx)$

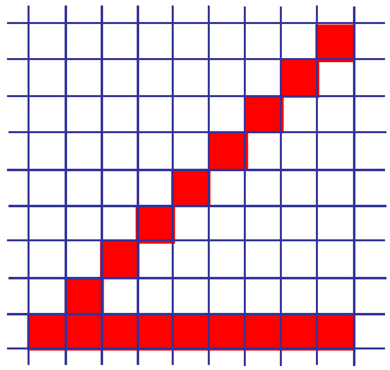
Bresenham (Midpoint) Algorithm

```
Line ( x1, y1, x2, y2 )
begin
int x, y, dx, dy, d, Δe, Δne ;
x ⇐ x1 ;            y ⇐ y1 ;
dx ⇐ x2 − x1 ;      dy ⇐ y2 − y1 ;
d ⇐ 2 * dy − dx ;
Δe ⇐ 2 * dy ;       Δne ⇐ 2 * (dy − dx) ;
PlotPixel ( x, y ) ;
while ( x < x2 ) do
      if ( d < 0 ) then
       begin
              d ⇐ d + Δe ;
              x ⇐ x + 1 ;
      end ;
       else begin
              d ⇐ d + Δne ;
              x ⇐ x + 1 ;
              y ⇐ y + 1 ;
      end ;
       PlotPixel ( x, y ) ;
end ;
end ;
```

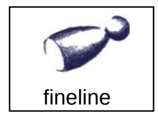


Bresenham Examples

- Intensity depends on angle



- Comment: extends to higher order curves – e.g. circles



Comparison: float/integer

Assume $x_1 < x_2$ & line slope is ≤ 1

```
Line (x1, y1, x2, y2)
begin
float dx, dy, x, y, slope ;
dx ⇐ x2 − x1;
dy ⇐ y2 − y1;
slope ⇐ dy/dx ;
y ⇐ y1
for x from x1 to x2 do
begin
    PlotPixel ( x, Round (y) );
    y ⇐ y + slope ;
end ;
end ;
```

```
Line ( x1, y1, x2, y2 )
begin
int  x, y, dx, dy, d, Δe, Δne ;
x ⇐ x1 ;        y ⇐ y1 ;
dx ⇐ x2 − x1 ;  dy ⇐ y2 − y1 ;
d ⇐ 2 * dy − dx ;
Δe ⇐ 2 * dy ;    Δne ⇐ 2 * (dy − dx) ;
PlotPixel ( x, y ) ;
while ( x < x2 ) do
    if ( d < 0 ) then
     begin
          d ⇐ d + Δe ;
     end ;
     else begin
          d ⇐ d + Δne ;
          y ⇐ y + 1 ;
     end ;
     x ⇐ x + 1 ;
     PlotPixel ( x, y ) ;
end ;
end ;
```

Implicit test

- Instead of clipping line in continuous space
  - For each integer value of (x,y) test if inside window just before drawing
  - Inefficient on CPU
  - On a parallel (GPU) processor can be surprisingly fast

```
Line (x1, y1, x2, y2)
begin
float dx, dy, x, y, slope;
dx ⇐ x2 − x1;
dy ⇐ y2 − y1;
slope ⇐ dy/dx;
y ⇐ y1
for x from x1 to x2 do
begin
    y_int = Round(y);
    if inside(x, y_int) PlotPixel(x, y_int);
    y ⇐ y + slope;
end;
end;
```

Scan Conversion of Lines

Discussion

- Integer: Bresenham
  - Good for hardware implementations (integer!)
- Floating Point
  - May be faster for software (depends on system)!
  - Easier to parallelize