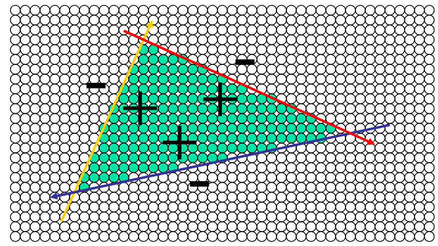
Chapter 9

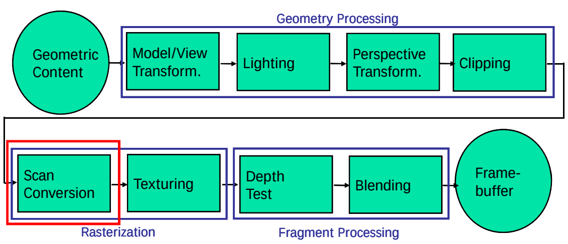
Scan Conversion (part 2)– Drawing Polygons on Raster Display

Implicit Formulation

- Triangle (convex polygon) = intersection of edge half-spaces
  - Defined by set of implicit line equations



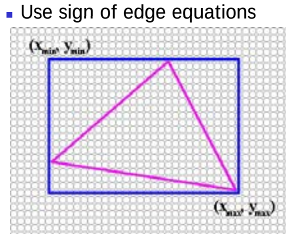
Rendering Pipeline



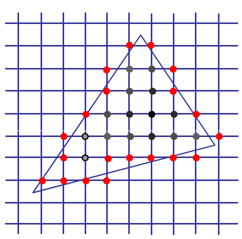
Using Implicit Edge Equations

Usage:

- Go over each pixel on screen
  - To be efficient restrict to bounding rectangle
- Check if pixel is inside/outside of triangle
  - Use sign of edge equations



Triangle/Polygon Rasterization



Computing Edge Equations

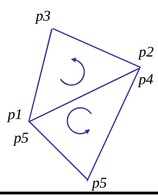
- Implicit equation of a triangle edge:

$$L(x,y) = (y_e - y_s)(x - x_s) - (x_e - x_s)(y - y_s) = 0$$

  - see Bresenham algorithm
  - L(x,y) positive on one side of edge, negative on the other
- What about the sign?
  - Which side is in, which is out?

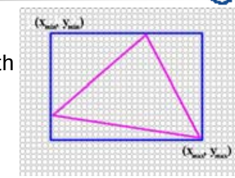
Edge Equations

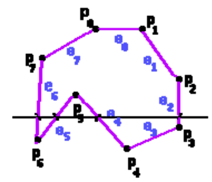
- Determining the sign
  - Which side is “in” and which is “out” depends on order of start/end vertices…
  - Convention: specify vertices in counter-clockwise order



Scan Conversion of Polygons

- General Algorithm
  - Intersect each scanline with all edges
  - Sort intersections in x
  - Calculate parity to determine in/out
  - Fill the ‘in’ pixels
  - Efficiency improvement:
    - Exploit row-to-row coherence using “edge table”





Edge Equations

- Counter-Clockwise Triangles
  - The equation L(x,y) as specified above is *negative inside, positive outside*
    - *Flip sign:*

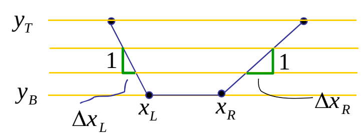
$$L(x,y) = -(y_e - y_s)(x - x_s) + (y - y_s)(x_e - x_s) = 0$$

- *Clockwise triangles*
  - *Use original formula*

$$L(x,y) = (y_e - y_s)(x - x_s) - (y - y_s)(x_e - x_s) = 0$$

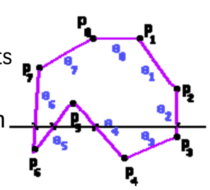
Edge Walking

- Next intersection along edge determined from previous



Scan Conversion of Polygons

- Implicit formulation works for any convex polygon
  - Doesn’t work for non-convex polygons
- Observation:
  - Straight line intersection with polygon = set of segments
- Alternative: algorithm based on scan-line/edge intersections
  - Works for **general** polygons
  - Less per pixel computations

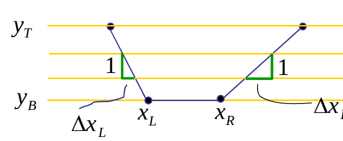


Edge Walking

- Special case: Scan-converting a trapezoid
  - Exploit continuous L and R edges
    - Predict intersections from one line to next

scanTrapezoid($x_L, x_R, y_B, y_T, \Delta x_L, \Delta x_R$)

```
for (y=yB; y<=yT; y++) {
  for (x=xL; x<=xR; x++)
   setPixel(x,y);
  xL += DxL;
  xR += DxR;
}
```

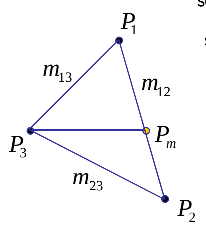


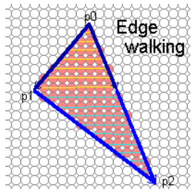
Edge Walking Triangles

- Split triangles into two “trapezoids” with continuous left and right edges

scanTrapezoid($x_3, x_m, y_3, y_1, \frac{1}{m_{13}}, \frac{1}{m_{12}}$)

scanTrapezoid($x_2, x_2, y_2, y_3, \frac{1}{m_{23}}, \frac{1}{m_{12}}$)





Discussion:

- Modern GPUs:
  - Use edge equations
    - Plus plane equations for attribute interpolation
    - No clipping of primitives required
  - Faster with many small triangles

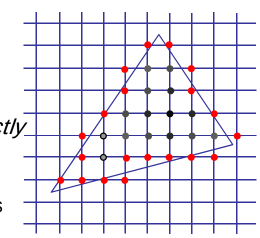
Edge Walking Triangles

Issues

- Many applications have small triangles
  - Setup cost is non-trivial
- Clipping triangles produces non-triangles
  - Can be avoided through re-triangulation

Rasterization Issues (Independent of Algorithm)

- Exactly which pixels should be lit?
  - Those pixels inside the triangle edge (of course)
  - *But what about pixels exactly on the edge?*
    - Don’t draw them: gaps possible between triangles
    - Draw them: order of triangles matters

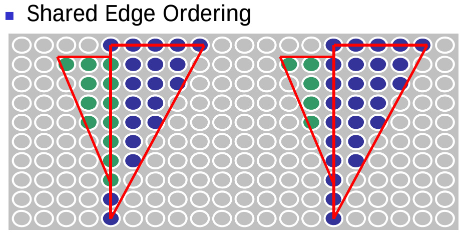


Discussion

- Old hardware:
  - Use edge-walking algorithm
    - Scan-convert edges, then fill in scanlines
    - Compute interpolated values by interpolating along edges, then scanlines
  - Requires clipping of polygons against viewing volume
  - Faster if you have a few, large polygons
  - Possibly faster in software

Triangle Rasterization Issues

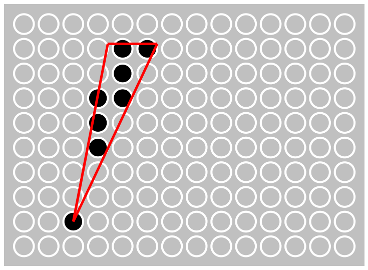
- Shared Edge Ordering



- Need a consistent (if arbitrary) rule
  - Example: draw pixels on left or top edge, but not on right or bottom edge

Triangle Rasterization Issues

- Sliver

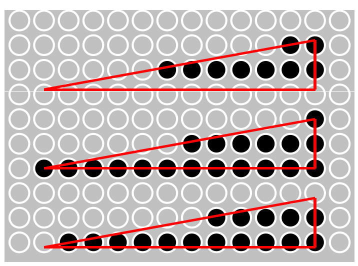


Shading

Assigning colors inside triangle interior

Triangle Rasterization Issues

- Moving Slivers



Shading

- Input to Scan Conversion:
  - Vertices of triangles (lines, quadrilaterals...)
  - Color (per vertex)
    - Specified with glColor
    - Or: computed with lighting
  - World-space normal (per vertex)
    - Left over from lighting stage
- Shading Task:
  - Determine color of every pixel in the triangle

Triangle Rasterization Issues

- These are ALIASING Problems
  - Problems associated with representing continuous functions (triangles) with finite resolution (pixels)
  - More on this problem when we talk about sampling...

Shading

- How can we assign pixel colors using this information?
  - Easiest: flat shading
    - Whole triangle gets one color (color of 1st vertex)
  - Better: Gouraud shading
    - Linearly interpolate color across triangle
  - Even better: Phong shading
    - Linearly interpolate the normal vector
    - Compute lighting for every pixel
    - Note: not supported by rendering pipeline as discussed so far

Flat Shading

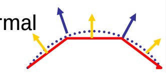
- Simplest approach: calculate illumination at one point per polygon (e.g. center)

- Obviously inaccurate for smooth surfaces

Improving Flat Shading

- What if we evaluate Phong lighting model at each pixel of the polygon?
  - Better, but result still clearly faceted

- Gouraud Shading: For smoother-looking surfaces introduce vertex normals at each vertex
  - Usually different from facet normal
  - Used only for shading
  - Think of as a better approximation of the real surface that the polygons approximate



Flat Shading Approximations

- If an object really is faceted, is this accurate?

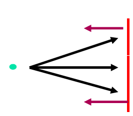
Vertex Normals

- Vertex normals may be
  - Provided with the model
  - Computed from first principles
  - Approximated by averaging the normals of the facets that share the vertex

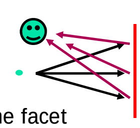
Flat Shading Approximations

- If an object really is faceted, is this accurate?

- no!
  - For point sources, direction to light varies across the facet

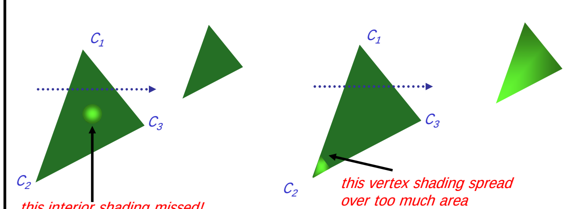


  - For specular reflectance, direction to eye varies across the facet



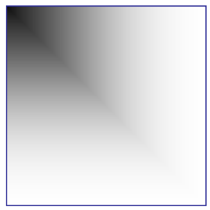
Gouraud Shading Artifacts

- Often appears dull, chalky
- Lacks accurate specular component
  - if included, will be averaged over entire polygon



Gouraud Shading Artifacts

- Mach bands
  - Eye enhances discontinuity in first derivative
  - Very disturbing, especially for highlights



Phong Shading Difficulties

- Computationally expensive
  - Per-pixel vector normalization and lighting computation!
  - Floating point operations required
- Lighting after perspective projection
  - Messes up the angles between vectors
  - Have to keep eye-space vectors around
- No direct support in standard rendering pipeline
  - But can be simulated with texture mapping, procedural shading hardware

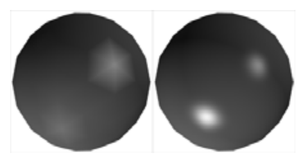
Phong Shading

- linearly interpolating surface normal across the facet, applying Phong lighting model at every pixel
  - Same input as Gouraud shading
  - Pro: much smoother results
  - Con: considerably more expensive

- Not the same as Phong lighting
  - Common confusion
  - Phong lighting: empirical model to calculate illumination at a point on a surface

Shading Artifacts: Silhouettes

- Polygonal silhouettes remain

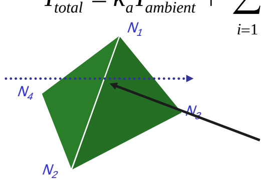


Gouraud *Phong*

Phong Shading

- Linearly interpolate the vertex normals
  - Compute lighting equations at each pixel
  - Can use specular component

$$I_{total} = k_a I_{ambient} + \sum_{i=1}^{\#lights} I_i \left(k_d (\mathbf{n} \cdot \mathbf{l}_i) + k_s (\mathbf{v} \cdot \mathbf{r}_i)^{n_{shiny}} \right)$$



remember: normals used in diffuse and specular terms

discontinuity in normal's rate of change harder to detect

Interpolation – access triangle interior

- Interpolate between vertices:
  - z
  - r,g,b - colour components
  - u,v - texture coordinates
  - N_x, N_y, N_z - surface normals
- Equivalent
  - Barycentric coordinates
  - Bilinear interpolation
  - Plane Interpolation

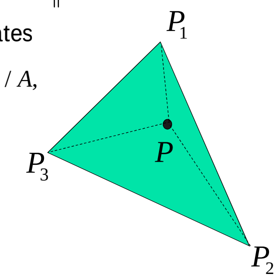
Barycentric Coordinates

- Area

$$A = \frac{1}{2}\left\| \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} \right\|$$

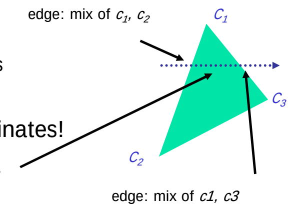
- Barycentric coordinates

$$a_1 = A_{P_2P_3P} / A, a_2 = A_{P_3P_1P} / A,$$
$$a_3 = A_{P_1P_2P} / A,$$
$$P = a_1P_1 + a_2P_2 + a_3P_3$$



Bi-Linear Interpolation

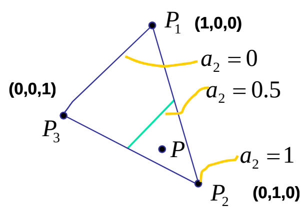
- Most common approach, and what OpenGL does
  - Perform Phong lighting at the vertices
  - Linearly interpolate the resulting colors over faces
    - Along edges
    - Along scanlines
- Equivalent to Barycentric Coordinates!



Barycentric Coordinates

- weighted combination of vertices

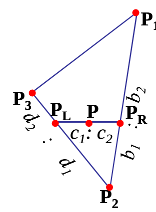
$$P = a_1 \cdot P_1 + a_2 \cdot P_2 + a_3 \cdot P_3$$
$$a_1 + a_2 + a_3 = 1$$
$$0 \le a_1, a_2, a_3 \le 1$$



Bi-Linear interpolation

- Formulation

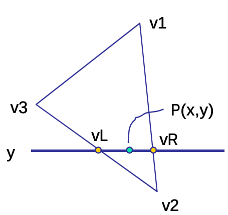
$$P = \frac{c_2}{c_1 + c_2} \cdot P_L + \frac{c_1}{c_1 + c_2} \cdot P_R$$
$$P_L = \frac{d_2}{d_1 + d_2} P_2 + \frac{d_1}{d_1 + d_2} P_3$$
$$P_R = \frac{b_2}{b_1 + b_2} P_2 + \frac{b_1}{b_1 + b_2} P_1$$



$$P = \frac{c_2}{c_1 + c_2}\left(\frac{d_2}{d_1 + d_2} P_2 + \frac{d_1}{d_1 + d_2} P_3\right) + \frac{c_1}{c_1 + c_2}\left(\frac{b_2}{b_1 + b_2} P_2 + \frac{b_1}{b_1 + b_2} P_1\right)$$

Alternative formula: Bi-Linear Interpolation

- Interpolate quantity along L and R edges
  - (as a function of y)
  - Then interpolate quantity as a function of x

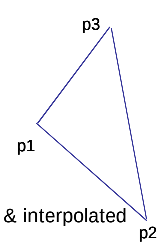


Another Alternative: Plane Equation

- Observation: Values vary linearly in image plane
  - E.g.: r = Ax + By + C
    - r= red channel of the color
    - Same for g, b, Nx, Ny, Nz, z...
  - From info at vertices we know:

$$r_1 = Ax_1 + By_1 + C$$
$$r_2 = Ax_2 + By_2 + C$$
$$r_3 = Ax_3 + By_3 + C$$

    - Solve for A, B, C
    - One-time set-up cost per triangle & interpolated value



Discussion

- Which algorithm (formula) to use when?
  - Bi-linear interpolation
    - Together with trapezoid scan conversion
  - Plane equations
    - Together with implicit (edge equation) scan conversion
  - Barycentric coordinates
    - Too expensive in current context
    - But: method of choice for ray-tracing
      - Whenever you only need to compute the value for a single pixel

Validation

- All formulations should provide same value
- Can verify barycentric properties

$$a_1 + a_2 + a_3 = 1$$
$$0 \le a_1, a_2, a_3 \le 1$$