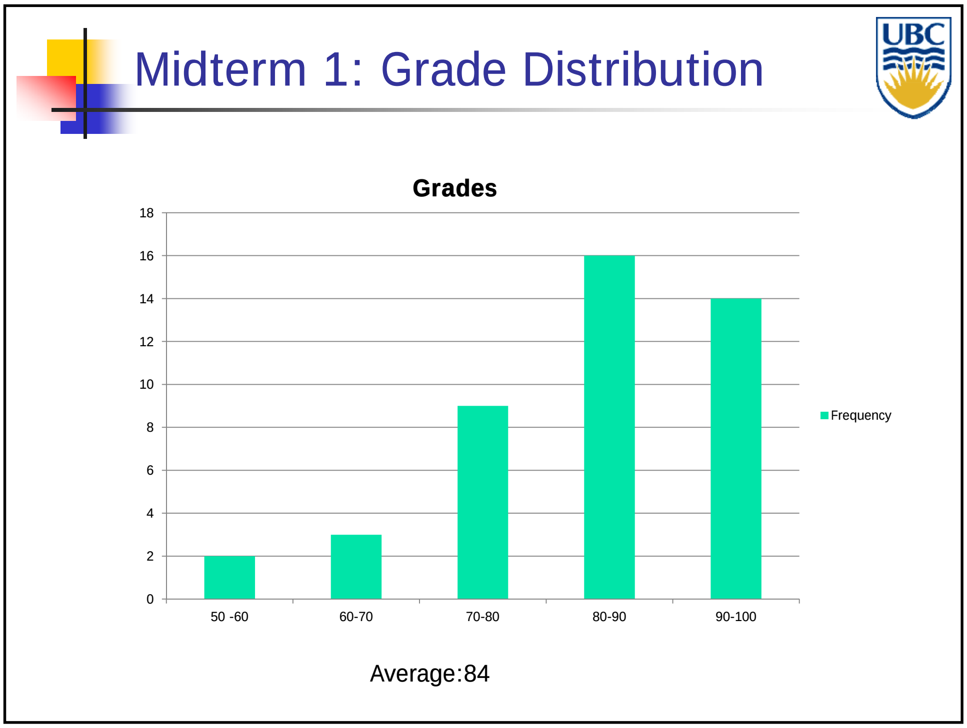
Chapter 8

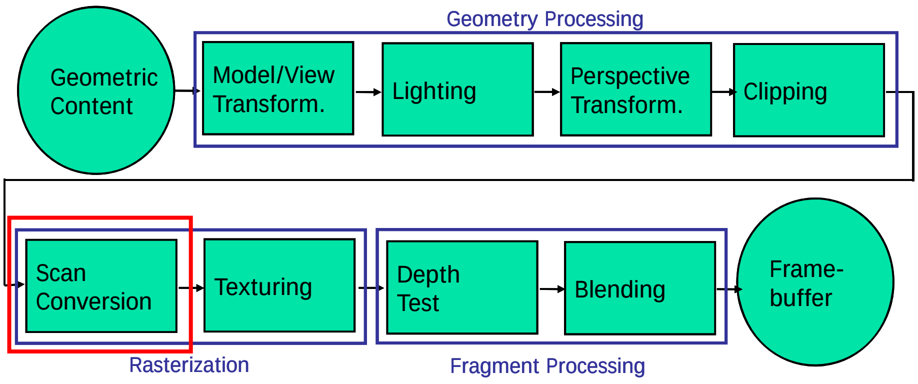
Scan Conversion – Drawing on Raster
Display (part 1 – Lines)



Midterm 1: Grade Distribution

Average:84

Rendering Pipeline

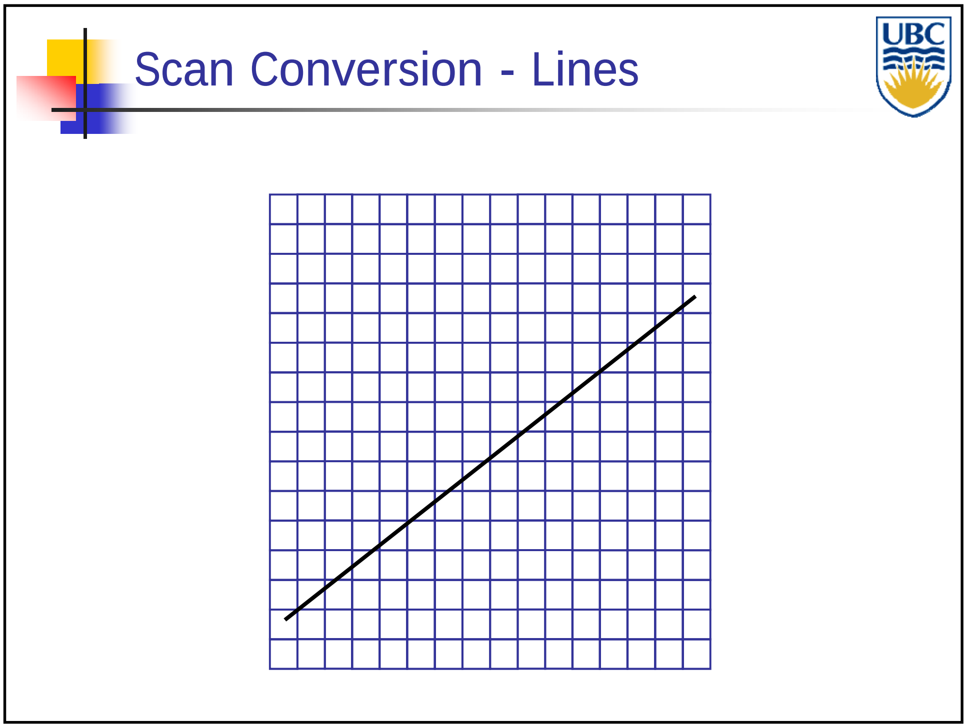


- Discard geometry outside viewport window

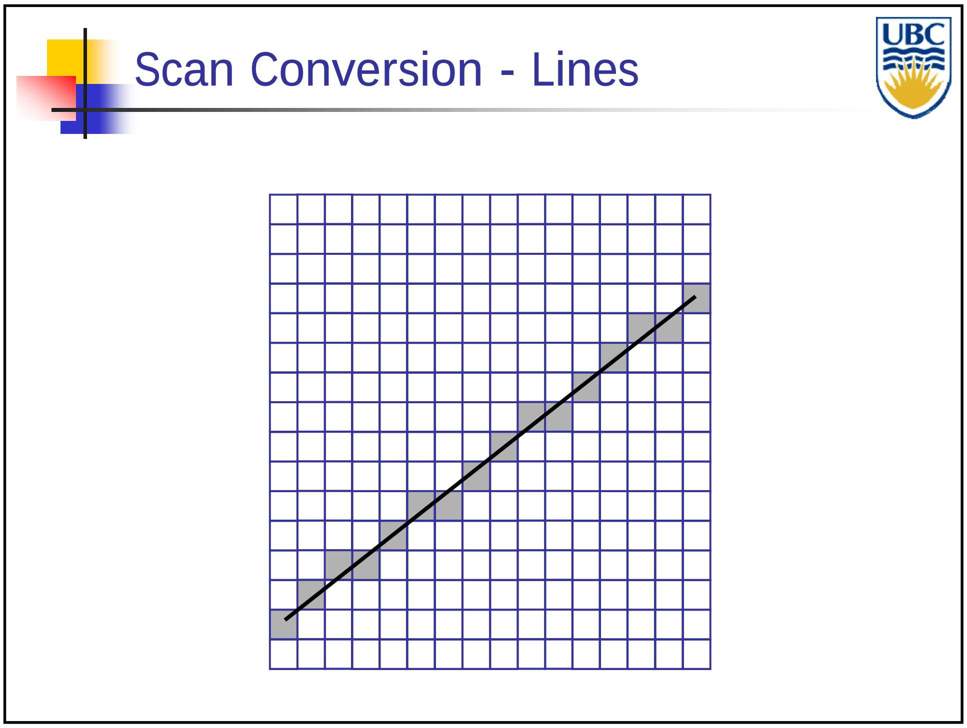
Scan Conversion - Rasterization

Convert continuous rendering primitives into discrete fragments/pixels

- Lines
  - Bresenham (Midpoint)
- Triangles
  - Implicit formulation
  - Scanline
- Interpolation



Scan Conversion - Lines



Scan Conversion - Lines

Idea: Use Explicit Line Formula

Explicit - one coordinate as function of the others

$$y = f(x)$$

$$z = f(x, y)$$

line

$$y = mx + b$$

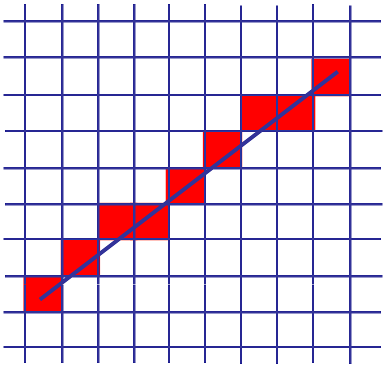
$$y = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1) + y_1$$

Typically separate into 4 (or 8) cases (why?)

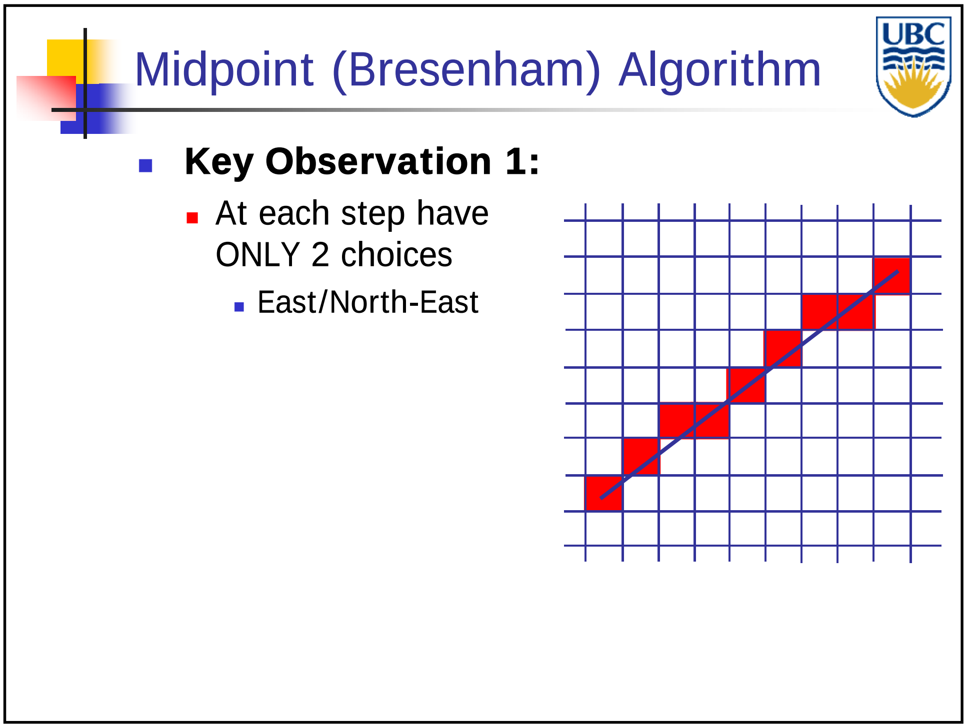
Basic Line Drawing

Assume $x_1 < x_2$ & line slope absolute value is ≤ 1

```
Line ( $x_1, y_1, x_2, y_2$ )
begin
float $dx, dy, x, y, slope$ ;
$dx \Leftarrow x_2 - x_1$;
$dy \Leftarrow y_2 - y_1$;
$slope \Leftarrow dy / dx$ ;
$y \Leftarrow y_1$
for $x$ from $x_1$ to $x_2$ do
begin
    PlotPixel ( $x$, Round ( $y$ ) ) ;
    $y \Leftarrow y + slope$ ;
end ;
end ;
```

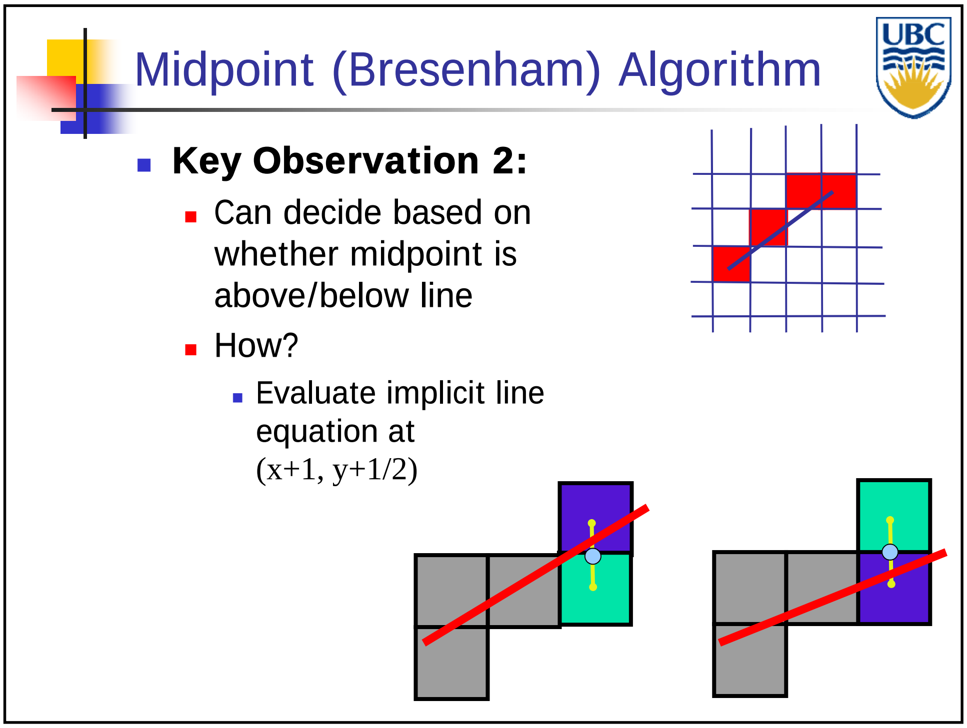


Questions:
Can this algorithm use integer arithmetic ?



Midpoint (Bresenham) Algorithm

- **Key Observation 1:**
  - At each step have ONLY 2 choices
    - East/North-East



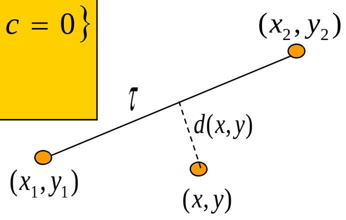
Midpoint (Bresenham) Algorithm

- **Key Observation 2:**
  - Can decide based on whether midpoint is above/below line
  - How?
    - Evaluate implicit line equation at (x+1, y+1/2)

Bresenham Algorithm

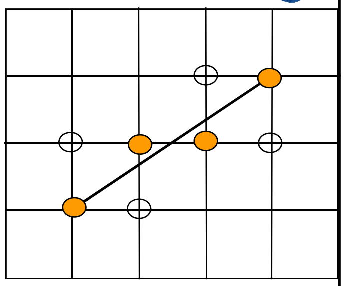
Implicit formulation = distance (up to scale)

$$\tau = \{(x, y) : ax + by + c = xdy - ydx + c = 0\}$$
$$d(x, y) = 2(xdy - ydx + c)$$

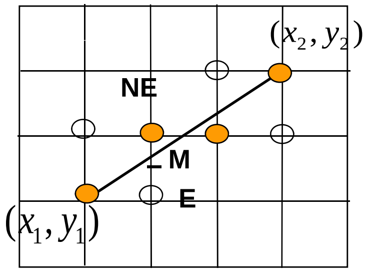


- Given point $P = (x, y)$, $d(x, y)$ is signed distance of p to τ (up to scale)
- d is zero for $P \in \tau$

Bresenham (Midpoint) Algorithm

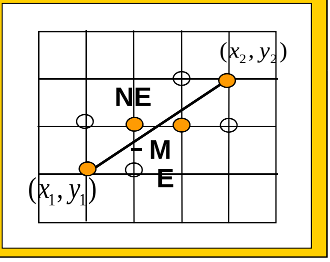


- Starting point satisfies $d(x_1, y_1) = 0$
- Each step moves right (east) or upper right (northeast)
- Sign of $d(x + 1; y + \frac{1}{2})$ indicates if to move east or northeast



Bresenham (Midpoint) Algorithm

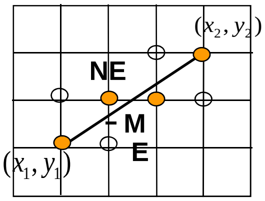
```
Line ( $x_1, y_1, x_2, y_2$ )
begin
int $x, y, dx, dy, d$;
x ⇐ $x_1$;          $y \Leftarrow y_1$;
$dx \Leftarrow x_2 - x_1$;     $dy \Leftarrow y_2 - y_1$;
PlotPixel ( $x, y$ );
while ( $x < x_2$ ) do
    $d = (2x+2)dy - (2y+1)dx + 2c$; // $2((x+1)dy - (y+.5)dx + c)$
    if ( $d < 0$ ) then
    begin
        $x \Leftarrow x+1$;
    end ;
    else begin
        $x \Leftarrow x+1$;
        $y \Leftarrow y+1$;
    end ;
    PlotPixel ( $x, y$ );
end ;
end ;
```



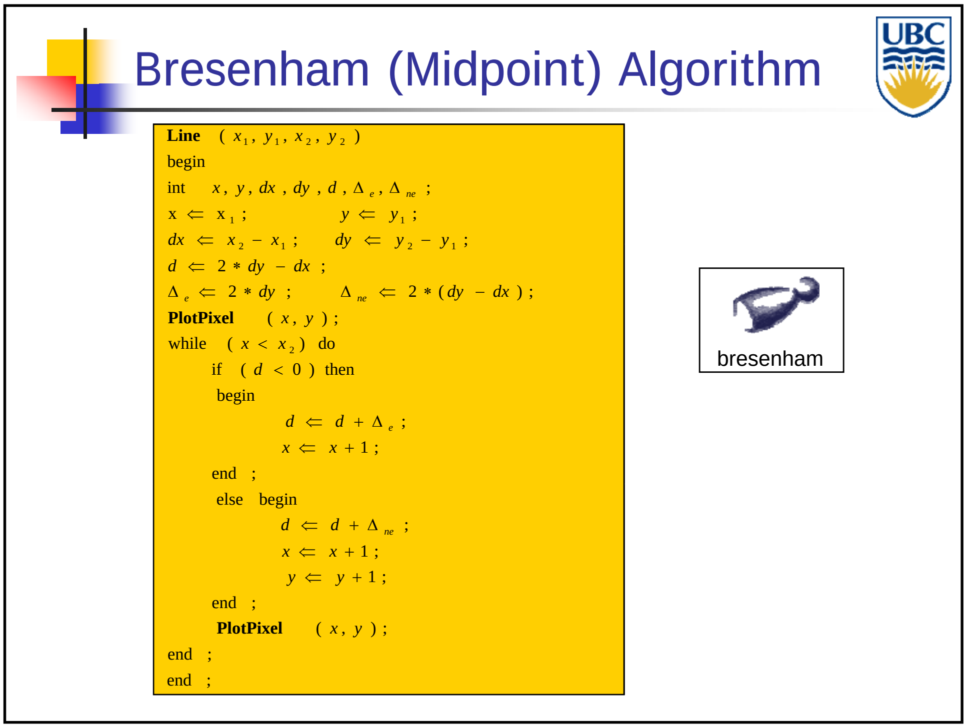
bresenham

Bresenham (Midpoint) Algorithm

- Insanely efficient version (less computations inside the loop)
  – compute d incrementally



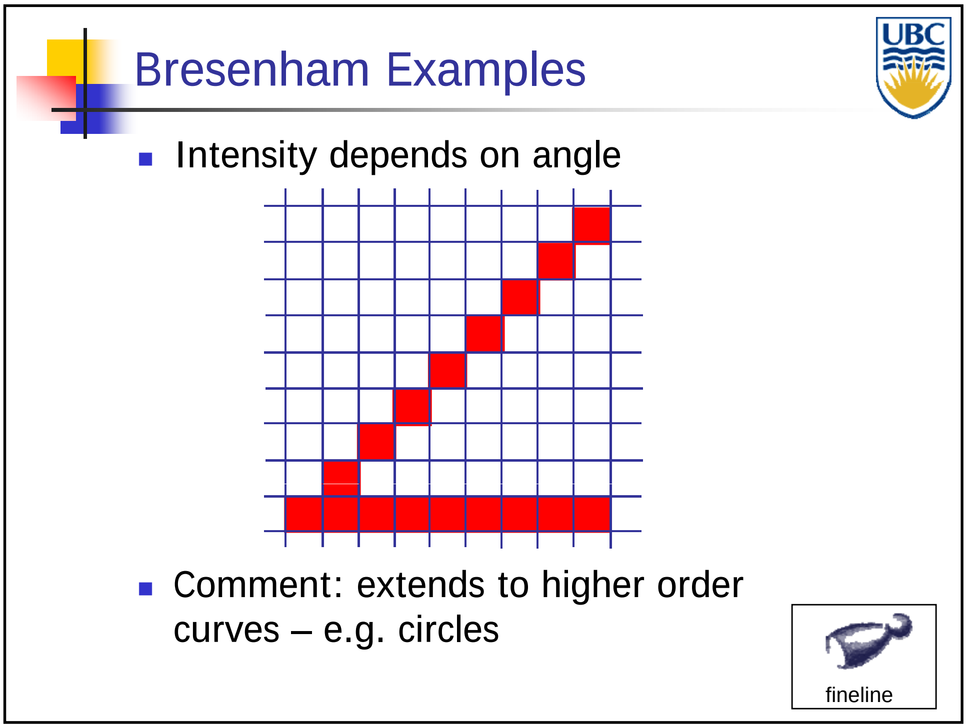
- At (x_1, y_1)
  $d_{start} = d(x_1+1, y_1+\frac{1}{2}) = 2dy - dx$
- Increment in *d* (after each step)
  - If move east $\Delta_e = d(x+2, y+\frac{1}{2}) - d(x+1, y+\frac{1}{2}) =$
    $2((x+2)dy - (y+\frac{1}{2})dx + c)) - 2((x+1)dy - (y+\frac{1}{2})dx + c) = 2dy$
  - If move northeast $\Delta_{ne} = d(x_1+2, y_1+\frac{3}{2}) - d(x_1+1, y_1+\frac{1}{2}) =$
    $2((x+2)dy - (y+\frac{3}{2})dx + c)) - 2((x+1)dy - (y+\frac{1}{2})dx + c) = 2(dy - dx)$



Bresenham (Midpoint) Algorithm

```
Line ($x_1, y_1, x_2, y_2$)
begin
int $x, y, dx, dy, d, \Delta_e, \Delta_{ne}$;
$x \Leftarrow x_1$;          $y \Leftarrow y_1$;
$dx \Leftarrow x_2 - x_1$;     $dy \Leftarrow y_2 - y_1$;
$d \Leftarrow 2 * dy - dx$;
$\Delta_e \Leftarrow 2 * dy$;     $\Delta_{ne} \Leftarrow 2 * (dy - dx)$;
PlotPixel ($x, y$);
while ($x < x_2$) do
    if ($d < 0$) then
    begin
        $d \Leftarrow d + \Delta_e$;
        $x \Leftarrow x + 1$;
    end ;
    else begin
        $d \Leftarrow d + \Delta_{ne}$;
        $x \Leftarrow x + 1$;
        $y \Leftarrow y + 1$;
    end ;
    PlotPixel ($x, y$);
end ;
end ;
```

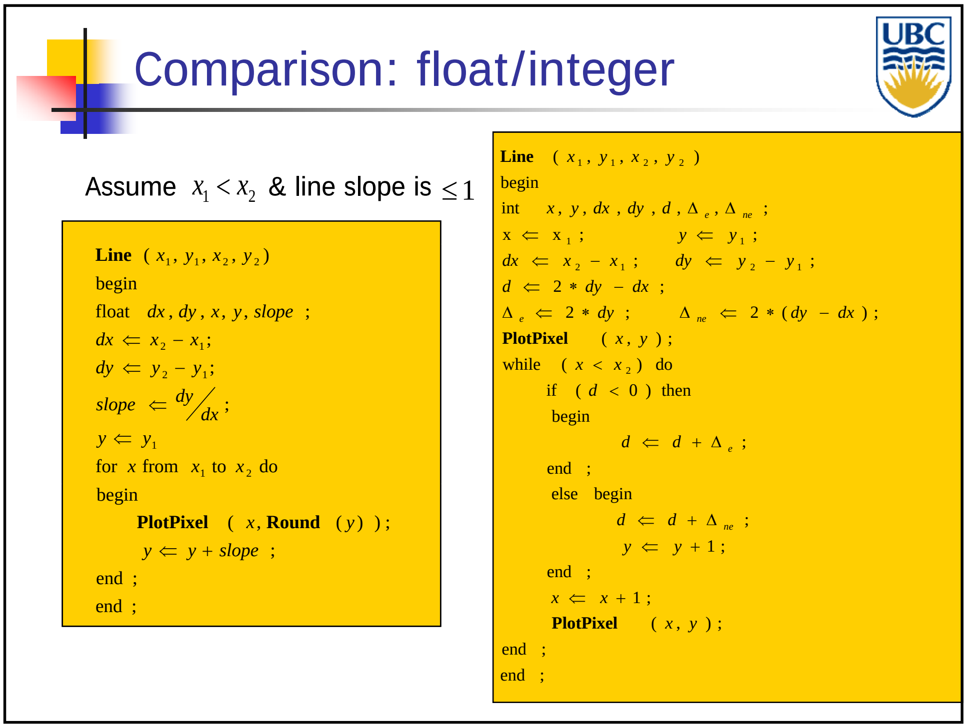
bresenham



Bresenham Examples

- Intensity depends on angle
- Comment: extends to higher order curves – e.g. circles

fineline

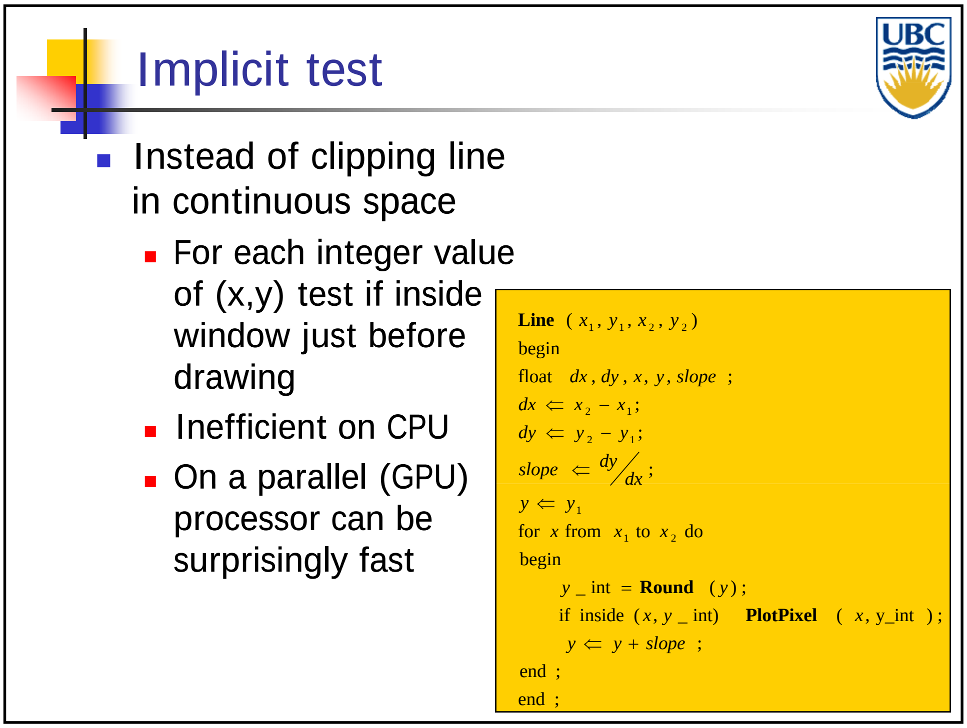


Comparison: float/integer

Assume $x_1 < x_2$ & line slope is ≤ 1

```
Line ($x_1, y_1, x_2, y_2$)
begin
float $dx, dy, x, y, slope$ ;
$dx \Leftarrow x_2 - x_1$;
$dy \Leftarrow y_2 - y_1$;
$slope \Leftarrow dy/dx$ ;
$y \Leftarrow y_1$
for $x$ from $x_1$ to $x_2$ do
begin
      PlotPixel ( $x$, Round ($y$) );
      $y \Leftarrow y + slope$ ;
end ;
end ;
```

```
Line ($x_1, y_1, x_2, y_2$)
begin
int $x, y, dx, dy, d, \Delta_e, \Delta_{ne}$ ;
$x \Leftarrow x_1$ ;        $y \Leftarrow y_1$ ;
$dx \Leftarrow x_2 - x_1$ ;      $dy \Leftarrow y_2 - y_1$ ;
$d \Leftarrow 2 * dy - dx$ ;
$\Delta_e \Leftarrow 2 * dy$ ;      $\Delta_{ne} \Leftarrow 2 * (dy - dx)$ ;
PlotPixel ( $x, y$ ) ;
while ( $x < x_2$ ) do
      if ( $d < 0$ ) then
      begin
            $d \Leftarrow d + \Delta_e$ ;
      end ;
      else begin
            $d \Leftarrow d + \Delta_{ne}$ ;
            $y \Leftarrow y + 1$ ;
      end ;
      $x \Leftarrow x + 1$ ;
      PlotPixel ( $x, y$ ) ;
end ;
end ;
```



Implicit test

- Instead of clipping line in continuous space
  - For each integer value of (x,y) test if inside window just before drawing
  - Inefficient on CPU
  - On a parallel (GPU) processor can be surprisingly fast

```
Line ($x_1, y_1, x_2, y_2$)
begin
float $dx, dy, x, y, slope$ ;
$dx \Leftarrow x_2 - x_1$;
$dy \Leftarrow y_2 - y_1$;
$slope \Leftarrow dy/dx$ ;
$y \Leftarrow y_1$
for $x$ from $x_1$ to $x_2$ do
begin
      $y\_int = $ Round ($y$);
      if inside ($x, y\_int$)   PlotPixel ( $x$, y_int ) ;
      $y \Leftarrow y + slope$ ;
end ;
end ;
```

Scan Conversion of Lines

Discussion

- Integer: Bresenham
  - Good for hardware implementations (integer!)
- Floating Point
  - May be faster for software (depends on system)!
  - Easier to parallelize