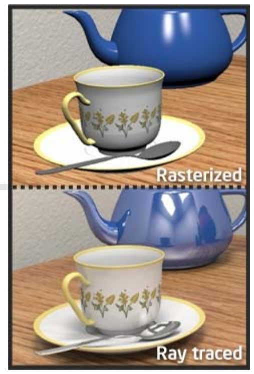
Chapter 11

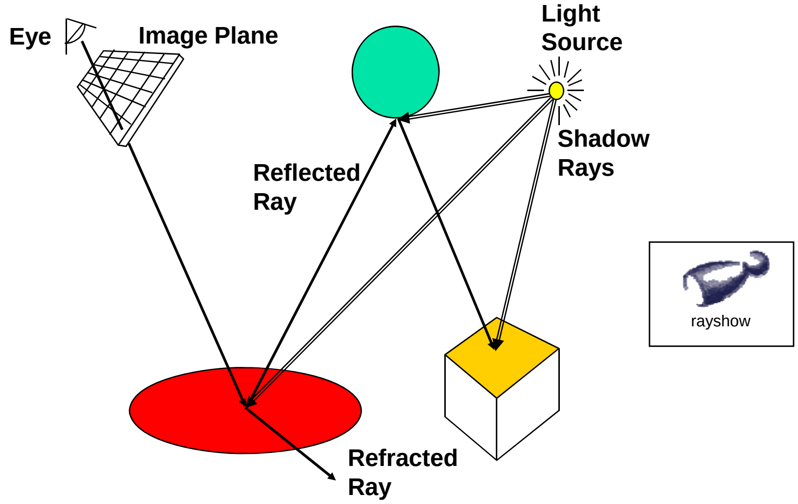
Ray-Tracing



Global Illumination Models

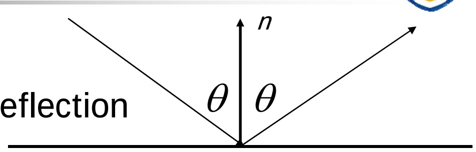
- Basic shading (rendering pipeline) = local illumination model
  - No object interaction
- Global illumination models require more sophisticated, computation-intensive algorithms
  - Ray Tracing
  - Global Illumination/Radiosity
- Ray-tracing
  - Usually offline (e.g. movies etc.)
    - research on making real-time
  - Flexible – can incorporate lots of phenomena

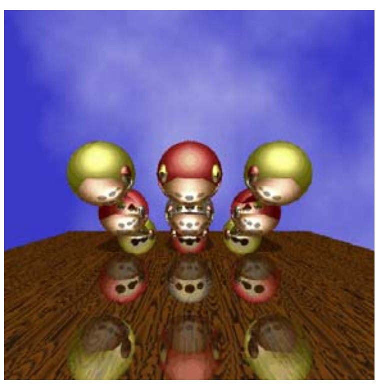
Ray-Tracing Algorithm



Reflection

- Mirror effects
  - Perfect specular reflection



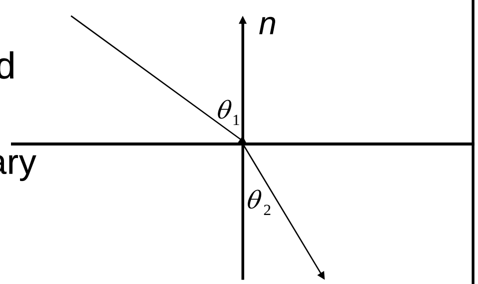


Refraction

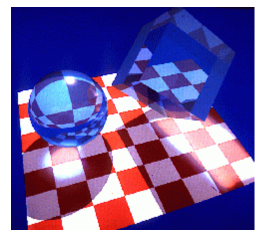
- Interface between transparent object and surrounding medium
  - E.g. glass/air boundary

$$c_2 \sin\theta_1 = c_1 \sin\theta_2$$

Snell's Law



- Light ray breaks (changes direction) based on refractive indices c_1, c_2



Basic Ray-Tracing Algorithm

```
RayTrace(r,scene)
obj := FirstIntersection(r,scene)
if (no obj)  return BackgroundColor;
else begin
   if ( Reflect(obj) ) then
      reflect_color := RayTrace(ReflectRay(r,obj));
   else
      reflect_color := Black;
  if ( Transparent(obj) ) then
      refract_color := RayTrace(RefractRay(r,obj));
   else
      refract_color := Black;
  return Shade(reflect_color,refract_color,obj);
end;
```

Sub-Routines

- ReflectRay(r,obj) – computes reflected ray (use obj normal at intersection)
- RefractRay(r,obj) - computes refracted ray
  - Note: ray is inside obj
- Shade(reflect_color,refract_color,obj) – compute illumination given three components

More About Ray-Tracing

- Algorithm above has a BUG....
- Does not terminate
- Termination Criteria
  - No intersection
  - Contribution of secondary ray attenuated below threshold – each reflection/refraction attenuates ray
  - Maximal depth is reached

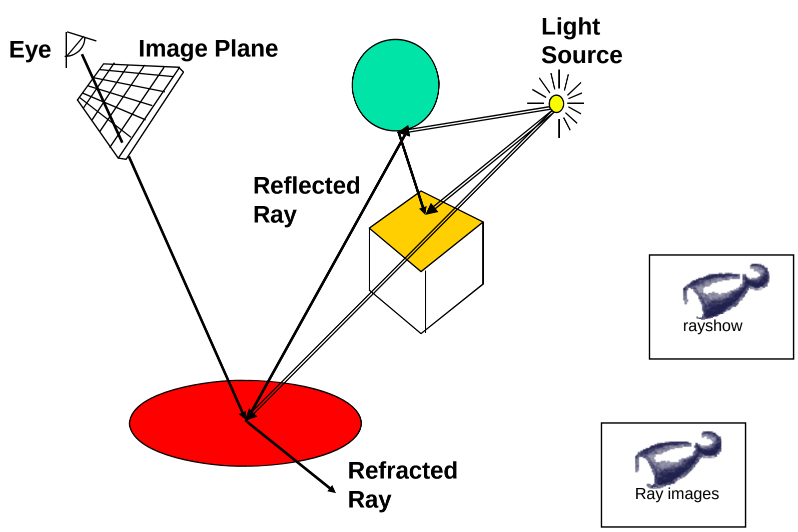
Simulating Shadows

- Trace ray from each ray-object intersection point to light sources
  - If the ray intersects an object in between ⇒ point is shadowed from the light source

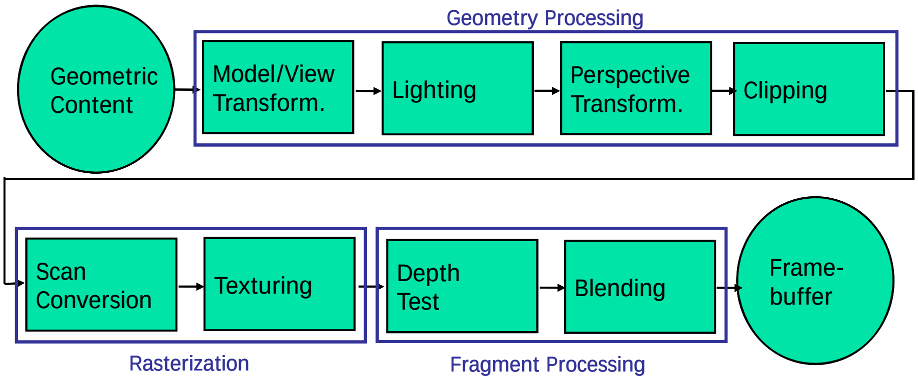
```
shadow = RayTrace(LightRay(obj,r,light));


return Shade(shadow,reflect_color,refract_color,obj);
```

Ray-Tracing With Shadows



Replaces Rendering Pipeline!!!

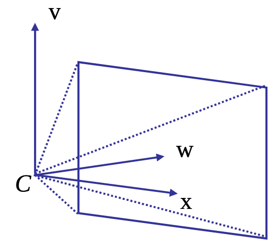


Ray-Tracing: Practicalities

- Generation of rays
- Intersection of rays with geometric primitives
- Geometric transformations
- Lighting and shading
- Speed: Reducing number of intersection tests
  - E.g. use BSP trees or other types of space partitioning

Ray-Tracing: Generation of Rays

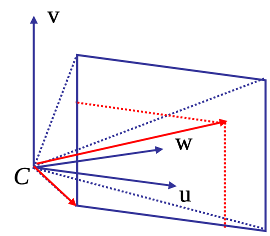
- Camera Coordinate System
  - Origin: C (camera position)
  - Viewing direction: **w**
  - Up vector: v
  - u direction: **u**= **w**×**v**



- Note:
  - Corresponds to viewing transformation in rendering pipeline!
  - See gluLookAt…

Ray-Tracing: Generation of Rays

- Other parameters:
  - Distance to image plane: d
  - Image resolution (in pixels): x, h
  - Left, right, top, bottom boundaries in image plane: l, r, t, b



- Then:
  - Lower left corner of image: $O = C + d \cdot w + l \cdot u + b \cdot v$
  - Pixel at position i, j (i=0..x-1, j=0..h-1):

$$P_{i,j} = O + i \cdot \frac{r-l}{x-1} \cdot \mathrm{u} - j \cdot \frac{t-b}{h-1} \cdot \mathrm{v}$$

$$= O + i \cdot \Delta u \cdot \mathrm{u} - j \cdot \Delta v \cdot \mathrm{v}$$

Ray-Tracing: Generation of Rays

- Ray in 3D Space:

$$R_{i,j}(t) = C + t \cdot (P_{i,j} - C) = C + t \cdot \mathbf{v}_{i,j}$$

where $t = 0 \ldots \infty$

Ray-Tracing: Practicalities

- Generation of rays
- **Intersection of rays with geometric primitives**
- Geometric transformations
- Lighting and shading
- Speed: Reducing number of intersection tests
  - E.g. use BSP trees or other types of space partitioning

Ray-Object Intersections

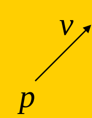
- Kernel of ray-tracing $\Rightarrow$ must be extremely efficient
- Usually involves solving a set of equations
  - Using implicit formulas for primitives

Example: Ray-Sphere intersection

ray: $x(t) = p_x + v_x t,\ \ y(t) = p_y + v_y t,\ \ z(t) = p_z + v_z t$

(unit) sphere: $x^2 + y^2 + z^2 = 1$

quadratic equation in *t* :



$$0 = (p_x + v_x t)^2 + (p_y + v_y t)^2 + (p_z + v_z t)^2 - 1$$
$$= t^2 (v_x^2 + v_y^2 + v_z^2) + 2t(p_x v_x + p_y v_y + p_z v_z)$$
$$+ (p_x^2 + p_y^2 + p_z^2) - 1$$

Ray Intersections

- Other Primitives:
  - Implicit functions:
    - Spheres at arbitrary positions
      - Same thing
    - Conic sections (hyperboloids, ellipsoids, paraboloids, cones, cylinders)
      - Same thing (all are quadratic functions!)
    - Higher order functions (e.g. tori and other quartic functions)
      - In principle the same
      - But root-finding difficult
      - Net to resolve to numerical methods

Ray Intersections

- Other Primitives (cont)
  - Polygons:
    - First intersect ray with plane
      - linear implicit function
    - Then test whether point is inside or outside of polygon (2D test)
    - For convex polygons
      - Suffices to test whether point in on the right side of every boundary edge
      - Similar to computation of outcodes in line clipping

Ray-Tracing: Practicalities

- Generation of rays
- Intersection of rays with geometric primitives
- **Geometric transformations**
- Lighting and shading
- Speed: Reducing number of intersection tests
  - E.g. use BSP trees or other types of space partitioning

Ray-Tracing: Transformations

- Note: rays replace perspective transformation
- Geometric Transformations:
  - Similar goal as in rendering pipeline:
    - Modeling scenes convenient using different coordinate systems for individual objects
  - Problem:
    - Not all object representations are easy to transform
      - This problem is fixed in rendering pipeline by restriction to polygons (affine invariance!)

Ray-Tracing: Transformations

- Ray Transformation:
  - For intersection test, it is only important that ray is in same coordinate system as object representation
  - Transform all rays into object coordinates
    - Transform camera point and ray direction by <u>inverse</u> of model/view matrix
  - Shading has to be done in world coordinates (where light sources are given)
    - Transform object space intersection point to world coordinates
    - Thus have to keep both world and object-space ray

Ray-Tracing: Practicalities

- Generation of rays
- Intersection of rays with geometric primitives
- Geometric transformations
- **Lighting and shading**
- Speed: Reducing number of intersection tests
  - E.g. use BSP trees or other types of space partitioning

Ray-Tracing: Local Lighting

- Light sources:
  - For the moment: point and directional lights
  - More complex lights are possible
    - Area lights
    - Global illumination
      - Other objects in the scene reflect light
      - Everything is a light source!
      - Talk about this on Monday

Ray-Tracing: Local Lighting

- Local surface information (normal…)
  - For implicit surfaces $F(x,y,z)=0$: normal $\mathbf{n}(x,y,z)$ can be easily computed at every intersection point using the gradient

$$\mathbf{n}(x,y,z) = \begin{pmatrix} \partial F(x,y,z)/\partial x \\ \partial F(x,y,z)/\partial y \\ \partial F(x,y,z)/\partial z \end{pmatrix}$$

  - Example:

$$F(x,y,z) = x^2 + y^2 + z^2 - r^2$$

$$\mathbf{n}(x,y,z) = \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix}$$

Needs to be normalized!

Ray-Tracing: Local Lighting

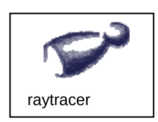
- Local surface information
  - Alternatively: can interpolate per-vertex information for triangles/meshes as in rendering pipeline
    - Phong shading!
    - Same as discussed for rendering pipeline
  - Difference to rendering pipeline:
    - Have to compute Barycentric coordinates for every intersection point (e.g plane equation for triangles)

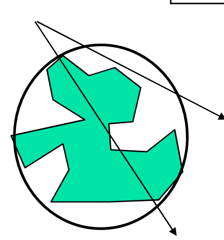
Ray-Tracing: Practicalities

- Generation of rays
- Intersection of rays with geometric primitives
- Geometric transformations
- Lighting and shading
- **Speed:** Reducing number of intersection tests

Optimized Ray-Tracing

- Basic algorithm simple but VERY expensive
- Optimize...
  - Reduce number of rays traced
  - Reduce number of ray-object intersection calculations
- Methods
  - Bounding Boxes
  - Spatial Subdivision
    - Visibility & Intersection
  - Tree Pruning



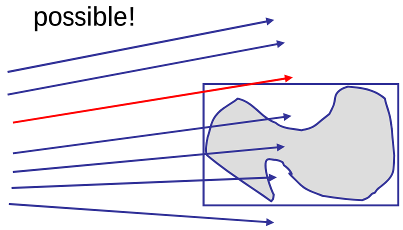


Ray Tracing

- Data Structures
  - Goal: reduce number of intersection tests per ray
  - Lots of different approaches:
    - (Hierarchical) bounding volumes
    - Hierarchical space subdivision
      - Octree, k-D tree, BSP tree

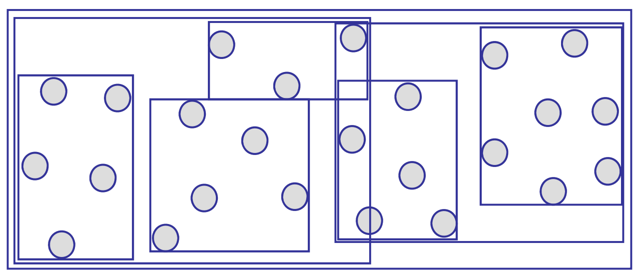
Bounding Volumes

- Idea:
  - Rather than test every ray against a potentially very complex object (e.g. triangle mesh), do a quick *conservative* test first which eliminates most rays
    - Surround complex object by simple, easy to test geometry (typically sphere or axis-aligned box)
      - Reduce false positives: make bounding volume as tight as possible!



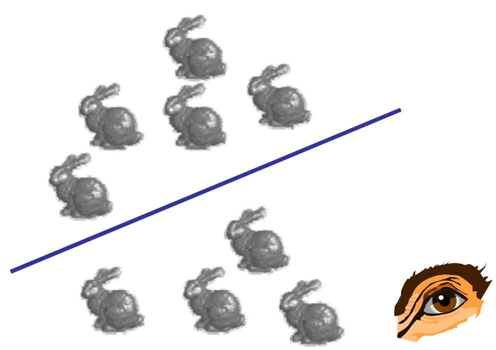
Hierarchical Bounding Volumes

- Extension of previous idea:
  - Use bounding volumes for groups of objects



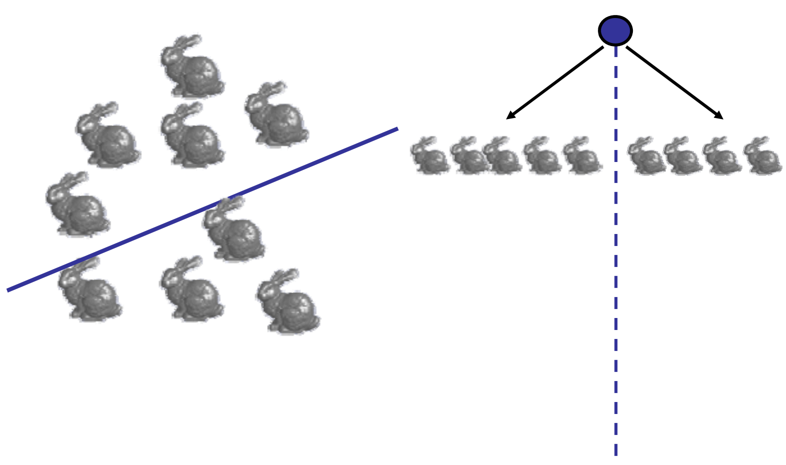
BSP Trees: Idea

- For any plane (3D) objects on the *same* side of plane as viewer CANNOT be occluded by objects on other side
- Idea:
  - Recursively split space by planes
  - Traverse resulting tree to establish rendering order
    - Test eye location w.r.t. each plane

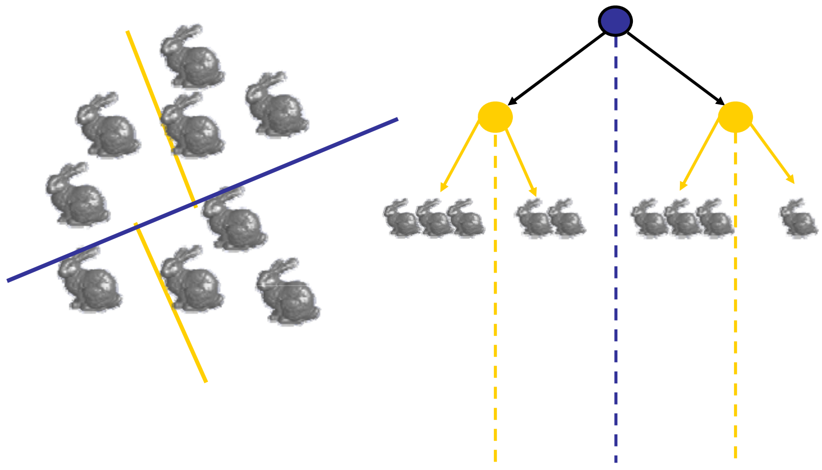


Creating BSP Trees: Objects

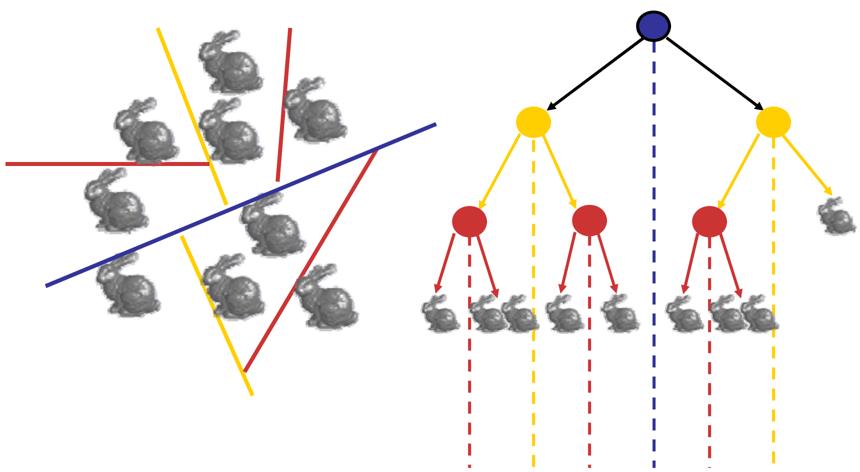
Creating BSP Trees: Objects



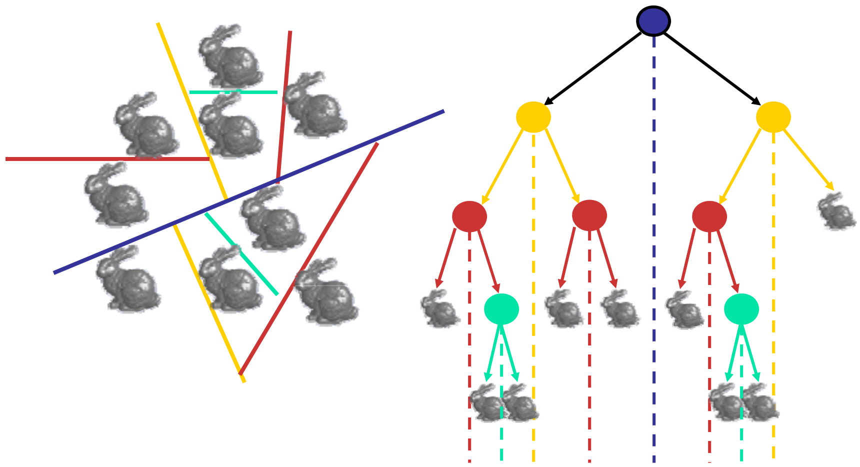
Creating BSP Trees: Objects



Creating BSP Trees: Objects

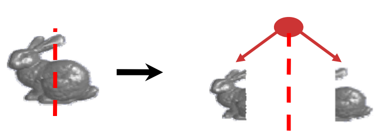


Creating BSP Trees: Objects



Splitting Objects

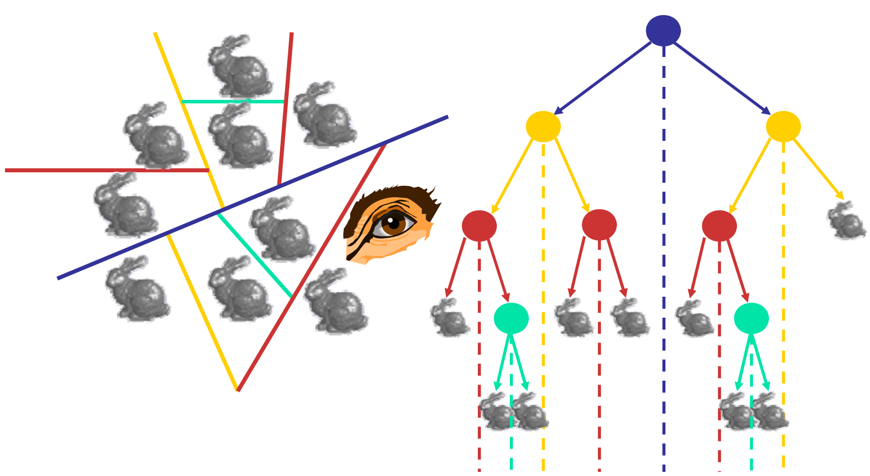
- No bunnies were harmed in previous example
- But what if a splitting plane passes through an object?
  - Split the object; give half to each node



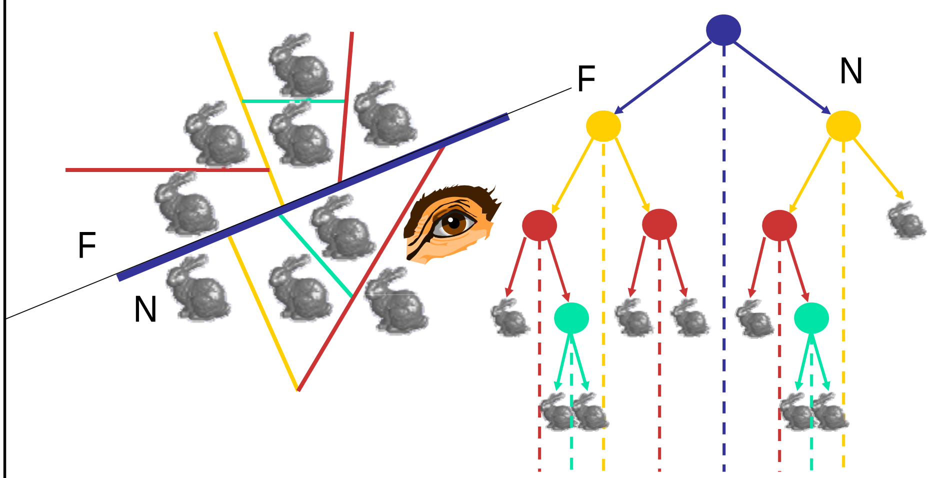
Traversing BSP Trees

- Tree creation independent of viewpoint
  - Preprocessing step
- Tree traversal uses viewpoint
  - Runtime, happens for many different viewpoints

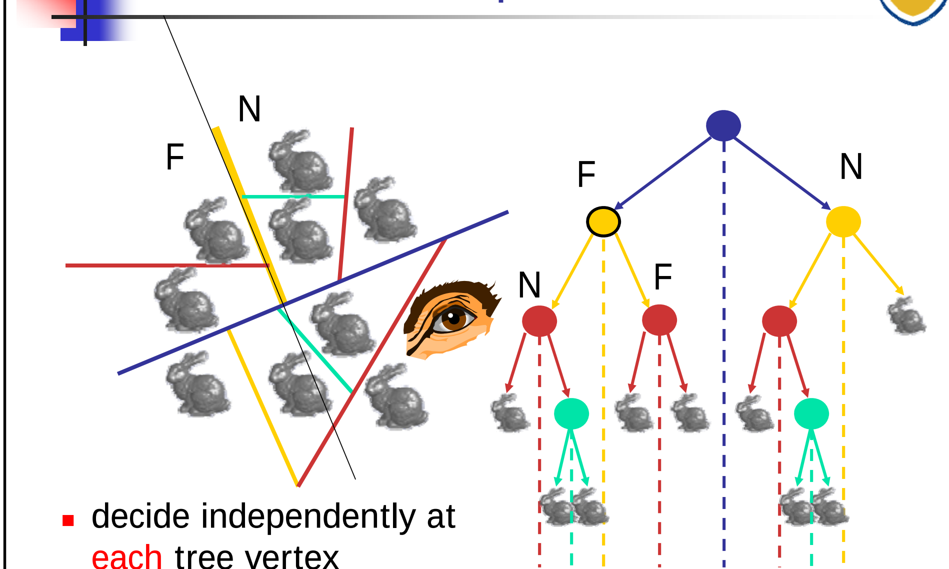
BSP Trees : Viewpoint A



BSP Trees : Viewpoint A

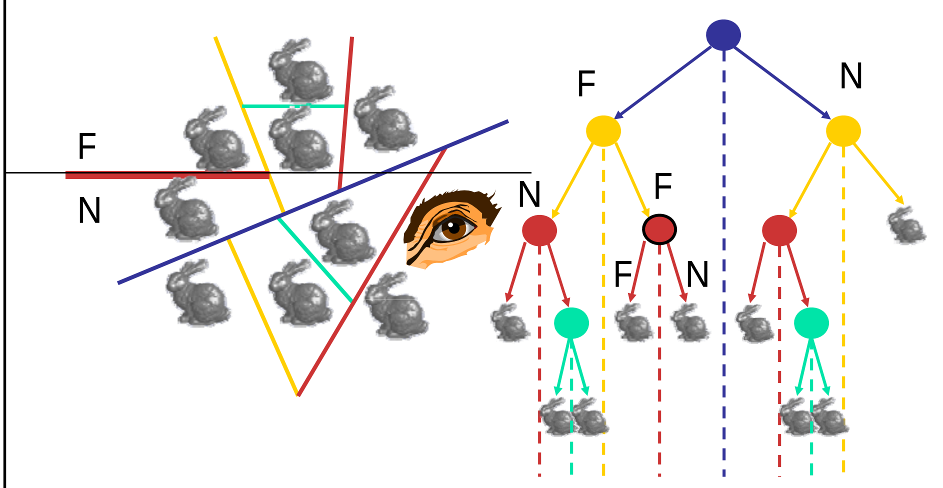


BSP Trees : Viewpoint A

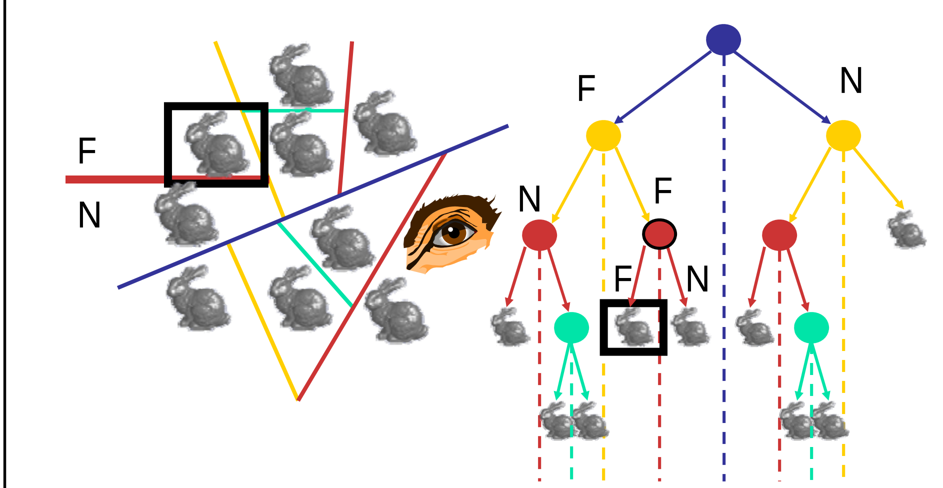


- decide independently at each tree vertex
- not just left or right child!

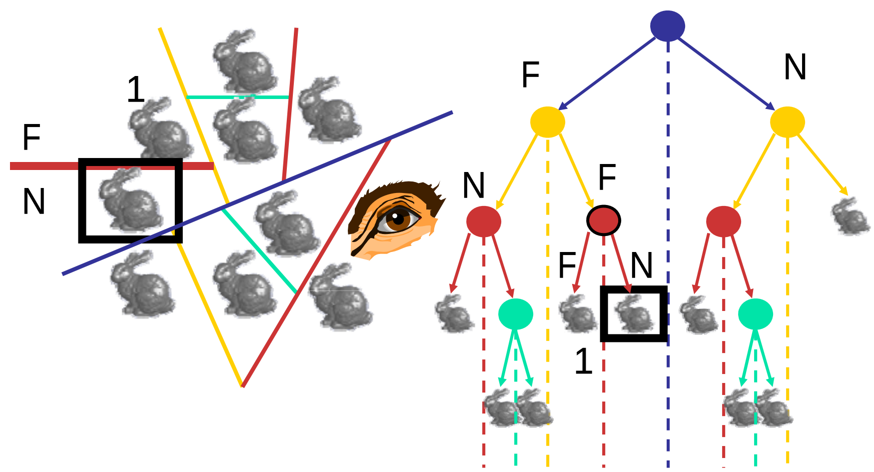
BSP Trees : Viewpoint A



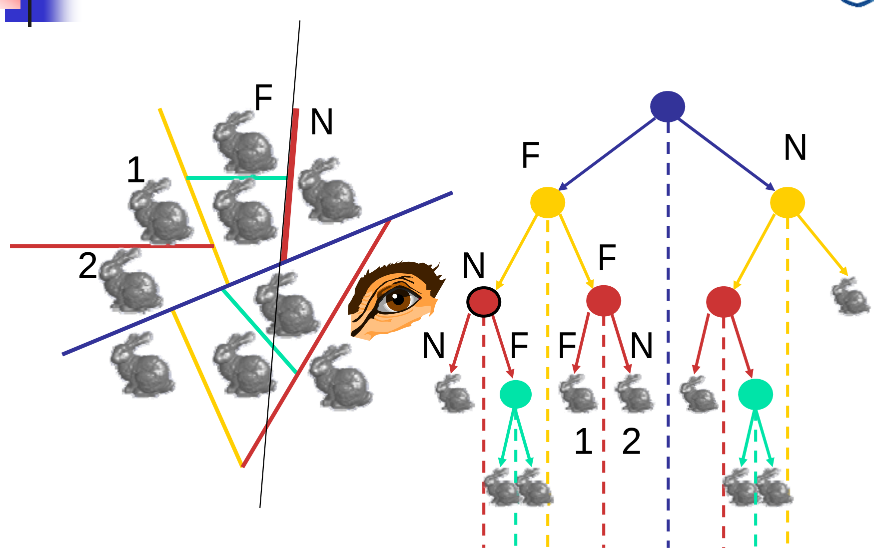
BSP Trees : Viewpoint A



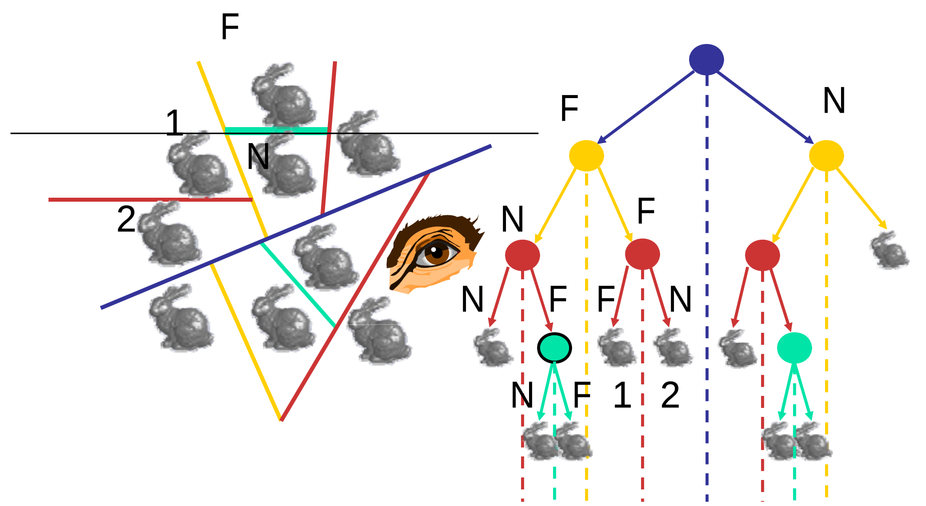
BSP Trees : Viewpoint A



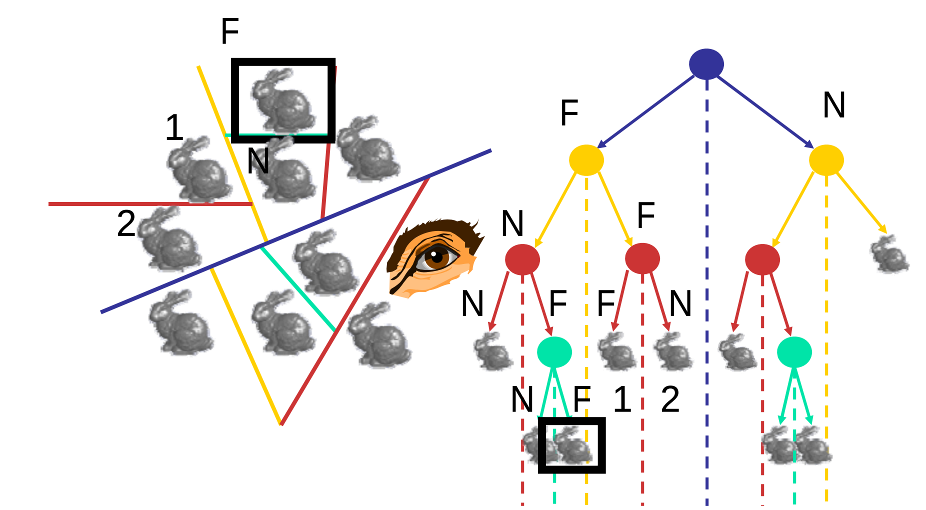
BSP Trees : Viewpoint A

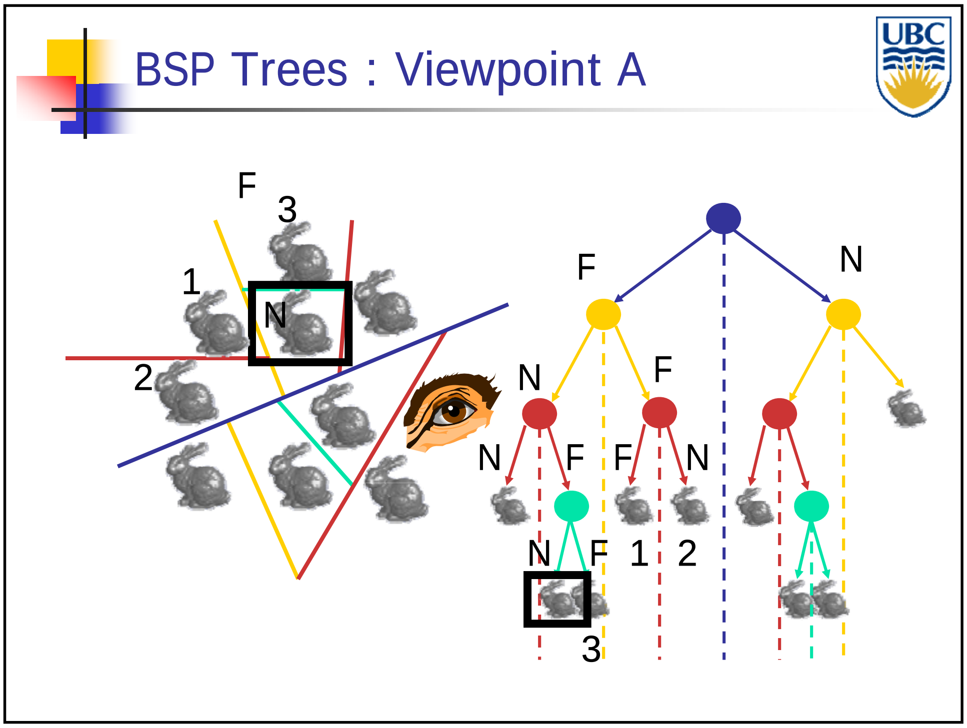


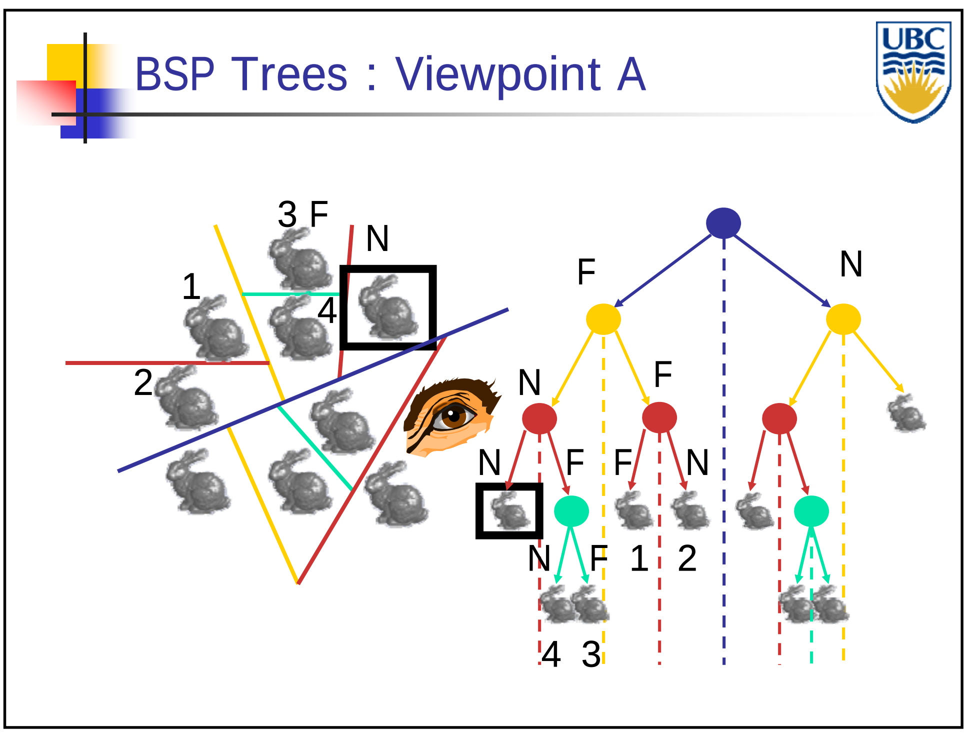
BSP Trees : Viewpoint A



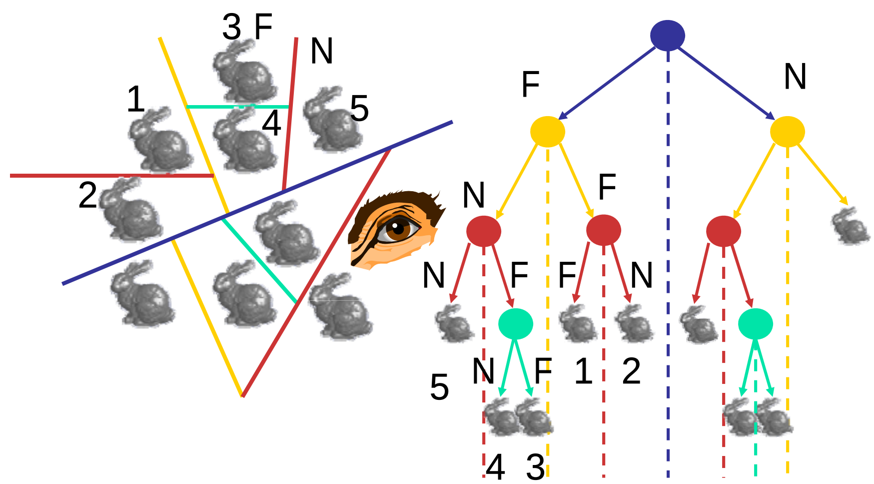
BSP Trees : Viewpoint A



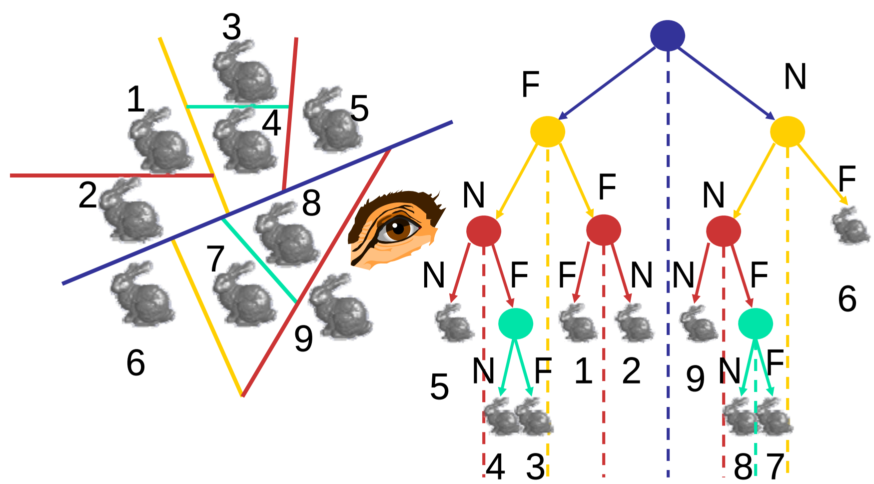

BSP Trees : Viewpoint A
UBC
F
3
1
N
2
F
N
N
F
N
F
F
N
N
F
1
2
3


BSP Trees : Viewpoint A
UBC
3 F
N
1
4
2
F
N
N
F
N
F
F
N
N
F
1
2
4
3

BSP Trees : Viewpoint A



BSP Trees : Viewpoint A



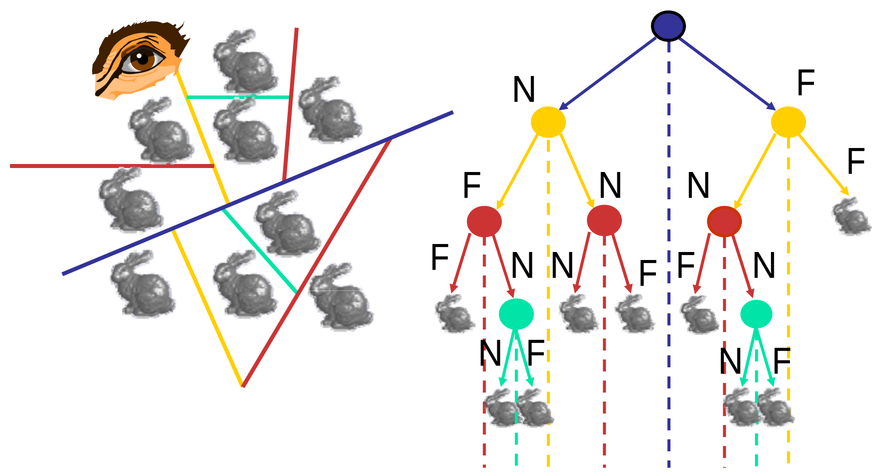
Traversing BSP Trees

- Each plane divides world into near and far
  - For given viewpoint, decide which side is near and which is far
    - Check which side of plane viewpoint is on independently for each tree vertex
    - Tree traversal differs depending on viewpoint!
  - Recursive algorithm
    - Recurse on far side
    - Draw object
    - Recurse on near side

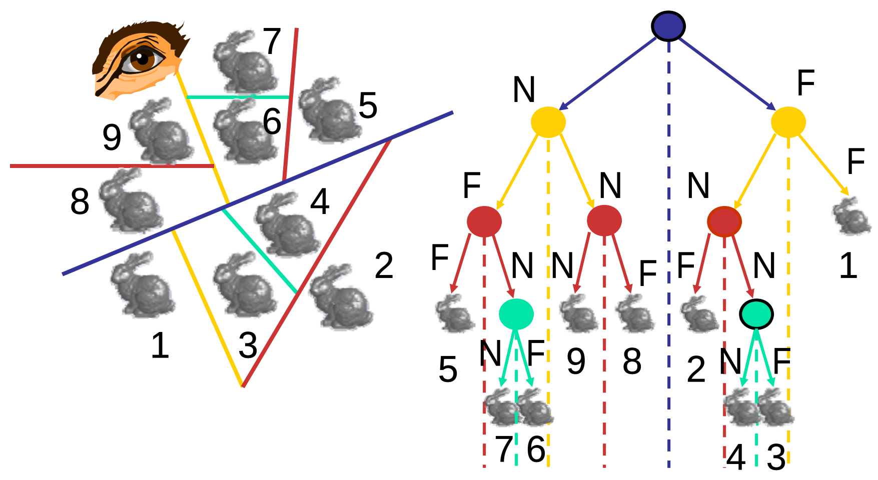
Traversing BSP Trees

```
renderBSP(BSPtree *T)
  BSPtree *near, *far;
  if (eye on left side of T->plane)
     near = T->left; far = T->right;
  else
     near = T->right; far = T->left;
  renderBSP(far);
  if (T is a leaf node)
     renderObject(T)
  renderBSP(near);
```

BSP Trees : Viewpoint B



BSP Trees : Viewpoint B

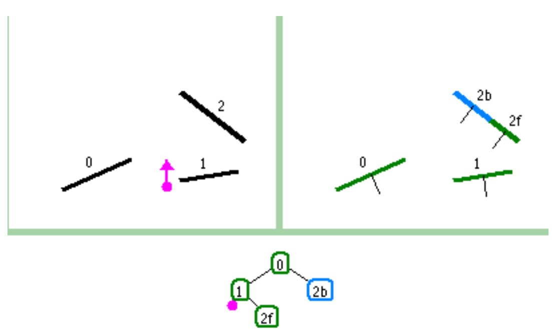


BSP Tree Traversal: Polygons

- Split along the plane defined by any polygon from scene
- Classify all polygons into positive or negative half-space of the plane
  - If a polygon intersects plane, split polygon into two and classify them both
- Recurse down the negative half-space
- Recurse down the positive half-space

BSP Demo

- Useful demo:
  - http://symbolcraft.com/graphics/bsp



Summary: BSP Trees

- Pros:
  - Simple, elegant scheme
  - Correct version of painter's algorithm back-to-front rendering approach
  - Still very popular for video games
- Cons:
  - Slow(ish) to construct tree: O(n log n) to split, sort
  - Splitting increases polygon count: $O(n^2)$ worst-case
  - Computationally intense preprocessing stage restricts algorithm to static scenes

Spatial Subdivision Data Structures

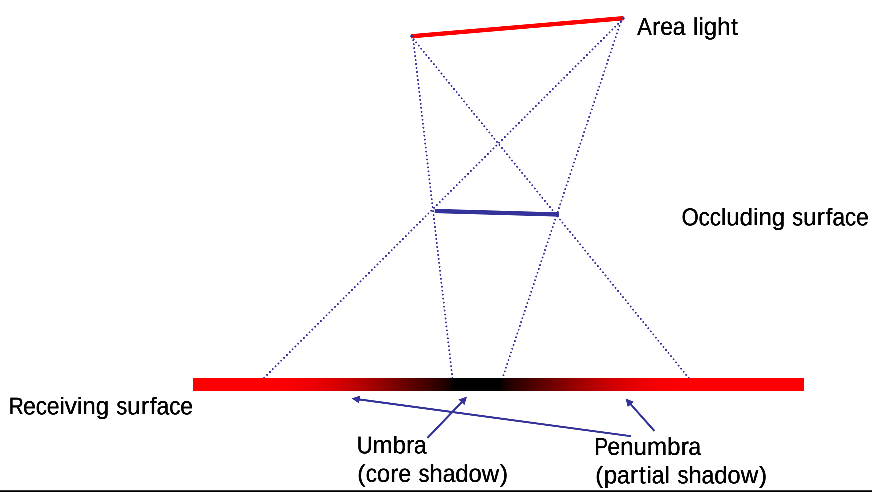
- Bounding Volumes:
  - Find simple object completely enclosing complicated objects
    - Boxes, spheres
  - Hierarchically combine into larger bounding volumes
- Spatial subdivision data structure:
  - Partition the whole space into cells
    - Grids, octrees, (BSP trees)
  - Simplifies and accelerates traversal
  - Performance less dependent on order in which objects are inserted

Soft Shadows: Area Light Sources

- So far:
  - All lights were either point-shaped or directional
    - Both for ray-tracing and the rendering pipeline
  - Thus, at every point, we only need to compute lighting formula and shadowing for ONE direction per light
- In reality:
  - All lights have a finite area
  - Instead of just dealing with one direction, we now have to integrate over all directions that go to the light source

Area Light Sources

- Area lights produce soft shadows:
  - In 2D:

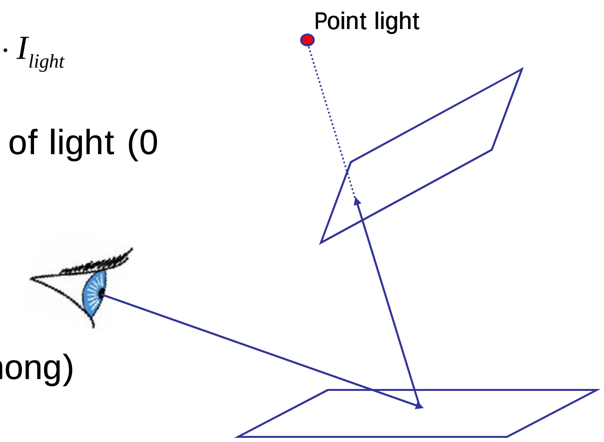


Area Light Sources

- Point lights:
  - Only one light direction:

$$I_{reflected} = \rho \cdot V \cdot I_{light}$$

  - V is visibility of light (0 or 1)
  - ρ is lighting model (e.g. diffuse or Phong)

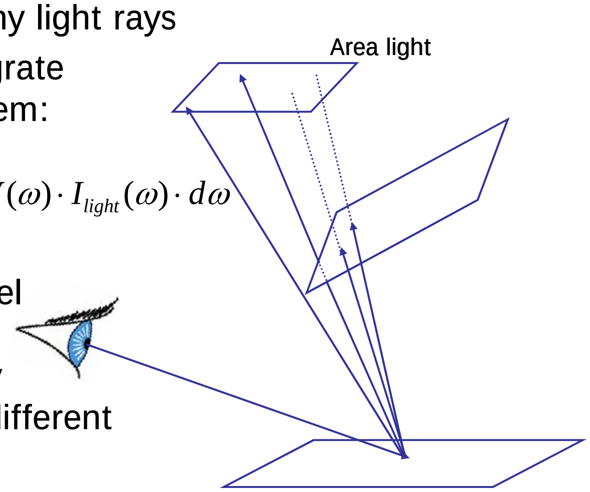


Are Light Sources

- Area Lights:
  - Infinitely many light rays
  - Need to integrate over all of them:

$$I_{reflected} = \int_{\substack{light \\ directions}} \rho(\omega) \cdot V(\omega) \cdot I_{light}(\omega) \cdot d\omega$$

  - Lighting model visibility and light intensity can now be different for every ray!



Integrating over Light Source

- Rewrite the integration
  - Instead of integrating over directions

$$I_{reflected} = \int_{\substack{light\\directions}} \rho(\omega) \cdot V(\omega) \cdot I_{light}(\omega) \cdot d\omega$$

integrate over points on the light source

$$I_{reflected}(q) = \int_{s,t} \frac{\rho(p-q) \cdot V(p-q)}{|p-q|^2} \cdot I_{light}(p) \cdot ds \cdot dt$$

where: q point on reflecting surface & p= F(s,t) point on the area light

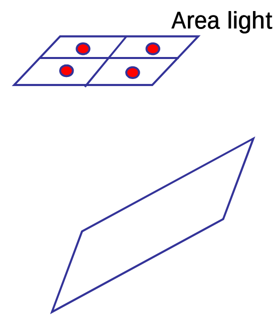
- We are integrating over p
- Denominator: quadratic falloff!

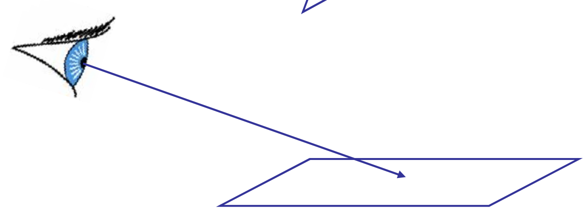
Integration

- Problem:
  - Except for the simplest of scenes, either integral is not solvable analytically!
  - This is mostly due to the visibility term, which could be arbitrarily complex depending on the scene
- So:
  - Use numerical integration
  - Effectively: approximate the light with a whole number of point lights

Numerical Integration

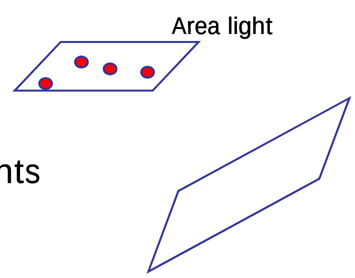
- Regular grid of point lights
  - Problem:
    will see 4 hard
    shadows rather than
    as soft shadow
  - Need LOTS of points
    to avoid this problem



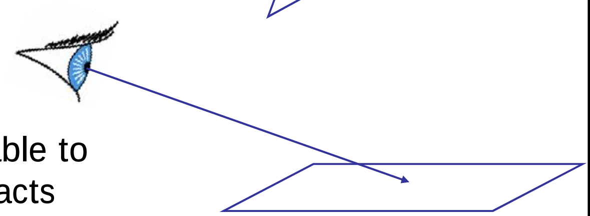


Monte Carlo Integration

- Better:
  - Randomly choose
    the points
  - Use different points on
    light for computing the
    lighting in different points
    on reflecting surface

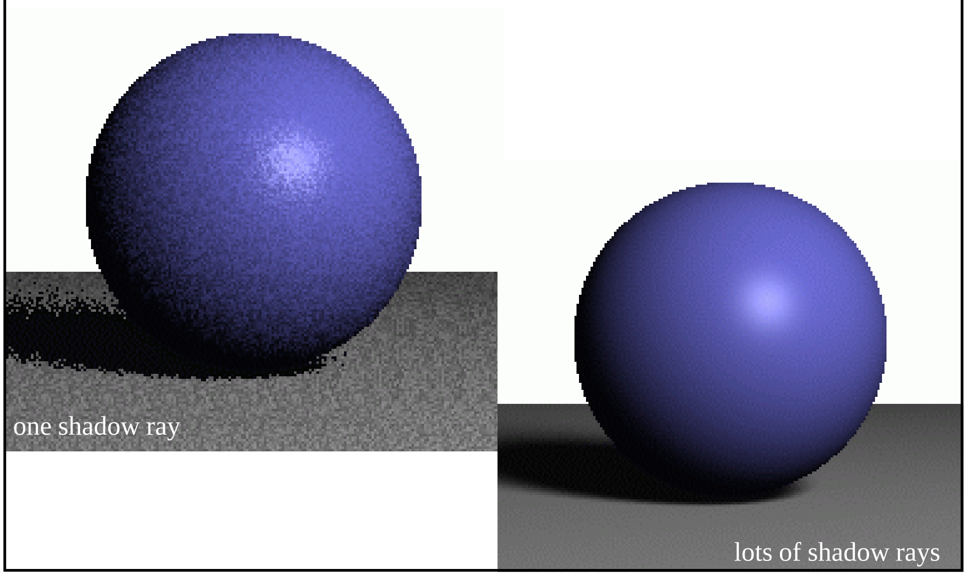


  - This produces
    random noise



  - Visually preferable to
    structured artifacts

Monte Carlo Integration



Monte Carlo Integration

- Note:
  - This approach of approximating lighting integrals with sums over randomly chosen points is much more flexible than this!
  - In particular, it can be used for global illumination
    - Light bouncing off multiple surfaces before hitting the eye