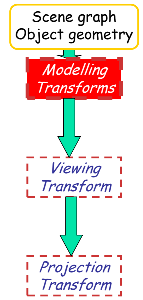
Chapter 5

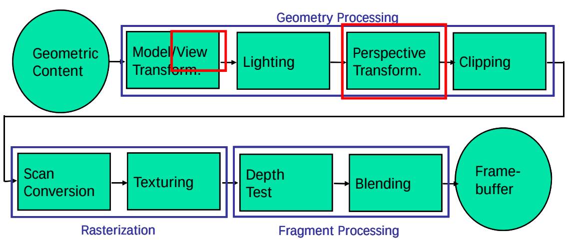
Viewing/Perspective Transformations

Rendering Pipeline



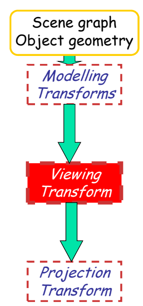
- result
  - all vertices of scene in shared 3D world coordinate system

Rendering Pipeline



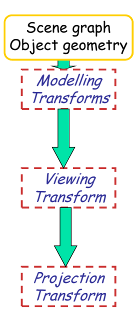
- Specify view point (change of coordinate system)
- Project from 3D to 2D (introduce perspective)

Rendering Pipeline

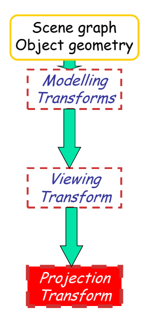


- result
  - scene vertices in 3D view (camera) coordinate system

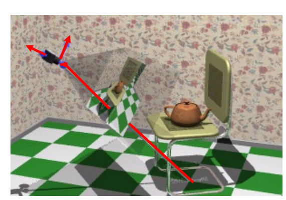
Rendering Pipeline



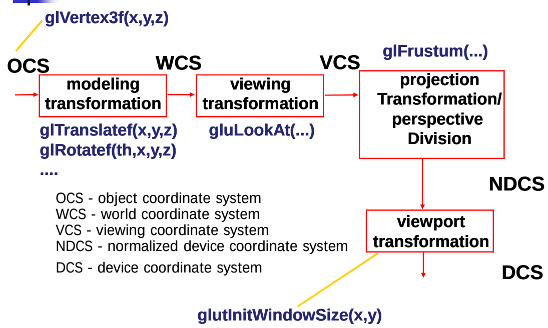
Rendering Pipeline



- result
  - 2D screen coordinates of clipped vertices

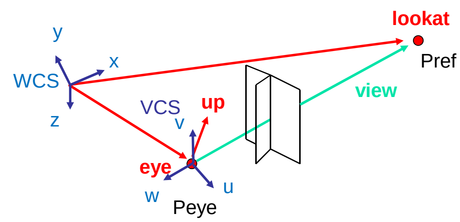


Projective Rendering Pipeline



From World to View Coordinates: W2V

- translate **eye** to origin
- rotate **view** vector (**lookat** – **eye**) to **w** axis
- rotate around **w** to bring **up** into **vw**-plane



Basic Viewing

- Starting spot - OpenGL
  - camera at world origin
    - probably inside an object
  - y axis is up
  - looking down negative z axis
    - why? RHS with x horizontal, y vertical, z out of screen
- To position – coordinate frame change
- Intuitive description
  - eye point, gaze/lookat direction, up vector

Deriving W2V Transformation

- M=RT

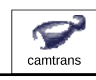
$$\mathbf{R} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{u} = \frac{\mathbf{t}\times\mathbf{w}}{\|\mathbf{t}\times\mathbf{w}\|}$$

$$\mathbf{v} = \mathbf{w}\times\mathbf{u}$$

$$\mathbf{w} = -\hat{\mathbf{g}} = -\frac{\mathbf{g}}{\|\mathbf{g}\|}$$

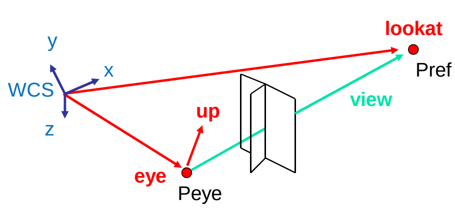
$$\mathbf{M}_{world->view} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} u_x & u_y & u_z & -\mathbf{u}\cdot\mathbf{e} \\ v_x & v_y & v_z & -\mathbf{v}\cdot\mathbf{e} \\ w_x & w_y & w_z & -\mathbf{w}\cdot\mathbf{e} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Notations/derivation from the board in class

Camera Description/Motion

- arbitrary viewing position
  - eye point, gaze/lookat direction, up vector



OpenGL Viewing Transformation

```
gluLookAt(ex,ey,ez,lx,ly,lz,ux,uy,uz)
```

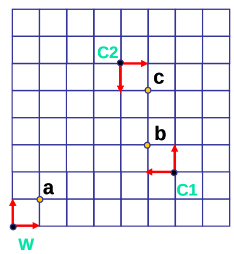
- postmultiplies current matrix, so to be safe:

```
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
gluLookAt(ex,ey,ez,lx,ly,lz,ux,uy,uz)
// now ok to do model transformations
```

Computer Graphics

Transformations: Viewing & Perspective

World vs. Camera Coordinates



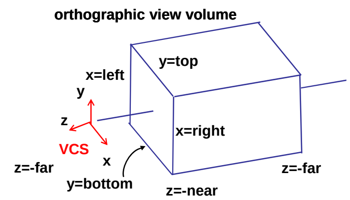
$a = (1,1)_W$

$b = (1,1)_{C1} = (5,3)_W$

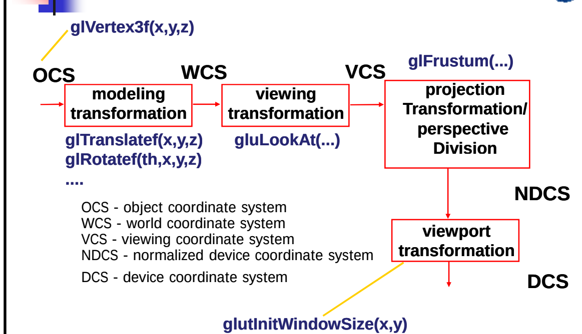
$c = (1,1)_{C2} = (1,3)_{C1} = (5,5)_W$

Clipping: View Volumes

- specifies field-of-view, used for clipping
- restricts domain of ***z*** stored for visibility test

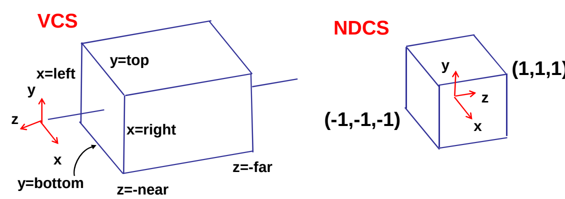


Projective Rendering Pipeline



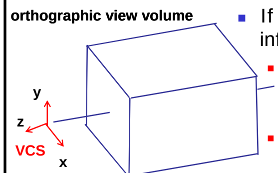
Understanding Z

- z axis flip changes coord system handedness
  - RHS before projection (eye/view coords)
  - LHS after projection (clip, norm device coords)



Projection Transformations

orthographic view volume



- Question: How to draw 3D object on 2D screen?
- If we ignore perspective (viewer at infinity)
  - Project transformed object along Z axis onto XY plane - and from there to screen (clipped)
  - Canonical ***orthographic*** projection:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- In practice "ignore" *z* axis – use *x* and *y* coordinates for screen coordinates

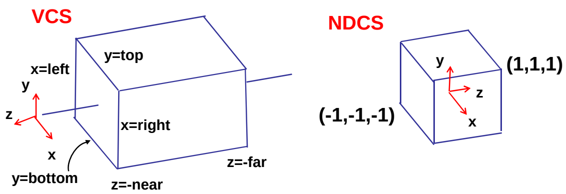
Understanding Z

- why near and far plane?
  - near plane:
    - avoid singularity for perspective projection (division by zero, or very small numbers)
  - far plane:
    - store depth in fixed-point representation (integer), thus have to have fixed range of values (0…1)
    - avoid/reduce numerical precision artifacts for distant objects

Orthographic Derivation

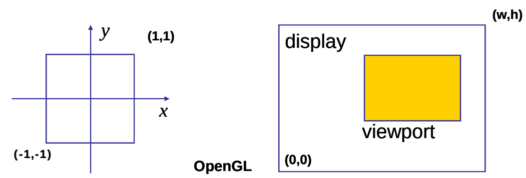
- scale, translate, reflect for new coord sys

$$y' = a \cdot y + b \qquad y = top \rightarrow y' = 1 \qquad y = bot \rightarrow y' = -1$$



NDC to Viewport Transformation

- generate pixel coordinates
  - map x, y from range –1...1 (NDC) to pixel coordinates on the display
  - involves 2D scaling and translation




```
glViewport(x,y,a,b);
```

Orthographic Derivation

- scale, translate, reflect for new coord sys

$$P' = \begin{bmatrix} \frac{2}{right-left} & 0 & 0 & -\frac{right+left}{right-left} \\ 0 & \frac{2}{top-bot} & 0 & -\frac{top+bot}{top-bot} \\ 0 & 0 & \frac{-2}{far-near} & -\frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

Origin Location

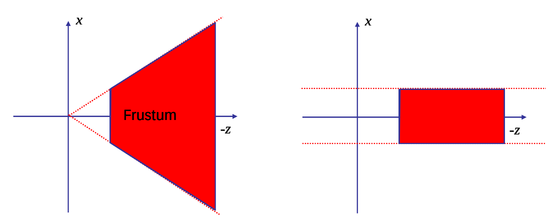
- yet more possibly confusing conventions
  - OpenGL: lower left
  - most window systems: upper left
- often have to flip your y coordinates
  - when interpreting mouse position

Orthographic OpenGL

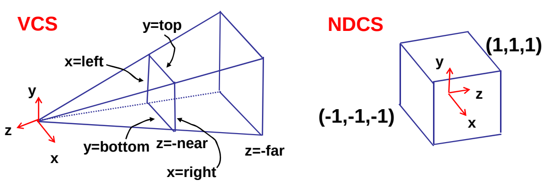
```
glMatrixMode(GL_PROJECTION);
glLoadIdentity();
glOrtho(left,right,bot,top,near,far);
```

Perspective Projection

- Viewing is from point at finite distance (origin)
  - View volume is a frustum not a box
- Conversion to device coordinates
  - Warp view frustum to box



Perspective Derivation



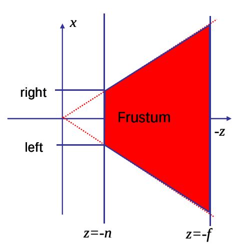
Perspective Derivation

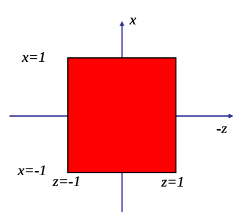
- Solve linear system to get A-F
  - 6 planes, 6 unknowns

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} E & 0 & A & 0 \\ 0 & F & B & 0 \\ 0 & 0 & C & D \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad \begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Projective Transformations

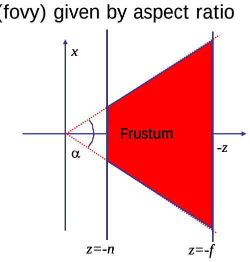
- OpenGL Convention





Projective Transformations

- Alternative specification of symmetric frusta
  - Field-of-view angles
    - In x-direction (fov) α
    - In y-direction (fovy) given by aspect ratio



Perspective Derivation

Basic (derived in class)

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}, (d = -1)$$

complete: shear, scale, projection-normalization

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} E & 0 & A & 0 \\ 0 & F & B & 0 \\ 0 & 0 & C & D \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Perspective OpenGL

```
glMatrixMode(GL_PROJECTION);
glLoadIdentity();

glFrustum(left,right,bot,top,near,far);
```

or

```
glPerspective(fovy,aspect,near,far);
```

- symmetric version

Another Transformations Quiz

- What does each transformation preserve?

	lines	parallel lines	distance	angles	normals	convexity
scaling						
rotation						
translation						
shear						
perspective						