



Painter's Algorithm: Problems



- Intersecting polygons present a problem
- Even non-intersecting polygons can form a cycle with no valid visibility order:





Hidden Surface Removal



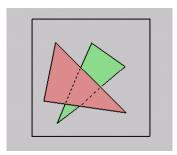
- Object Space Methods:
 - Work in 3D before scan conversion
 - E.g. Painter's algorithm
 - Usually independent of resolution
 - Important to maintain independence of output device (screen/printer etc.)
- Image Space Methods:
 - Work on per-pixel/per fragment basis after scan conversion
 - Z-Buffer/Depth Buffer
 - Much faster, but resolution dependent

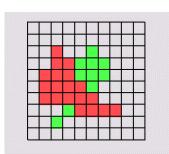


The Z-Buffer Algorithm



- What happens if multiple primitives occupy the same pixel on the screen?
- Which is allowed to paint the pixel?







The Z-Buffer Algorithm



- Idea: retain depth after projection transform
 - Each vertex maintains z coordinate
 - Relative to eye point
 - Can do this with canonical viewing volumes



The Z-Buffer Algorithm



- Augment color framebuffer with Z-buffer
 - Also called depth buffer
 - Stores z value at each pixel
 - At frame beginning, initialize all pixel depths to
- When scan converting: interpolate depth (z) across polygon
- Check z-buffer before storing pixel color in framebuffer and storing depth in z-buffer
 - don't write pixel if its z value is more distant than the z value already stored there



Z-Buffer



- Store (r,g,b,z) for each pixel
 - typically 8+8+8+24 bits, can be more

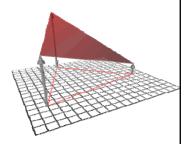
```
for all i,j {
Depth[i,j] = MAX_DEPTH
 Image[i,j] = BACKGROUND COLOUR
for all polygons P {
  for all pixels in P {
    if (Z_pixel < Depth[i,j]) {</pre>
      Image[i,j] = C_pixel
      Depth[i,j] = Z_pixel
  }
}
```



Interpolating Z



- Use barycentric coordinates
 - Interpolate z like other parameters
 - E.g. color
 - Use on of three formulas shown
 - Plane/edge walk/barycentric





The Z-Buffer Algorithm (mid-70's)



- History:
 - Object space algorithms were proposed when memory was expensive
 - First 512x512 framebuffer was >\$50,000!
- Radical new approach at the time
 - The big idea:
 - Resolve visibility independently at each pixel



Depth Test Precision



- Reminder: projective transformation maps eye-space z to generic z-range (NDC)
- Simple example:

$$T \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Thus:

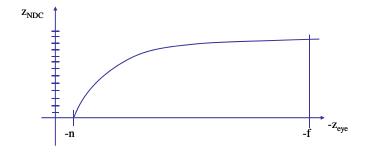
$$z_{NDC} = \frac{a \cdot z_{eye} + b}{z_{eye}} = a + \frac{b}{z_{eye}}$$



Depth Test Precision



- Therefore, depth-buffer essentially stores 1/z, rather than z!
- Issue with integer depth buffers
 - High precision for near objects
 - Low precision for far objects





Depth Test Precision



- Low precision can lead to depth fighting for far objects
 - Two different depths in eye space get mapped to same depth in framebuffer
 - Which object "wins" depends on drawing order and scan-conversion
- Gets worse for larger ratios f:n
 - Rule of thumb: f:n < 1000 for 24 bit depth buffer
- With 16 bits cannot discern cm differences in objects at 1 km distance



Z-Buffer Algorithm Questions



- How much memory does the Z-buffer use?
- Does the image rendered depend on the drawing order?
- Does the time to render the image depend on the drawing order?
- How does Z-buffer load scale with visible polygons? with framebuffer resolution?



Z-Buffer Pros



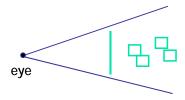
- Simple!!!
- Easy to implement in hardware
 - Hardware support in all graphics cards today
- Polygons can be processed in arbitrary order
- Easily handles polygon interpenetration



Z-Buffer Cons



- Poor for scenes with high depth complexity
 - Need to render all polygons, even if most are invisible



- Shared edges/overlaps handled inconsistently
 - Ordering dependent



Z-Buffer Cons



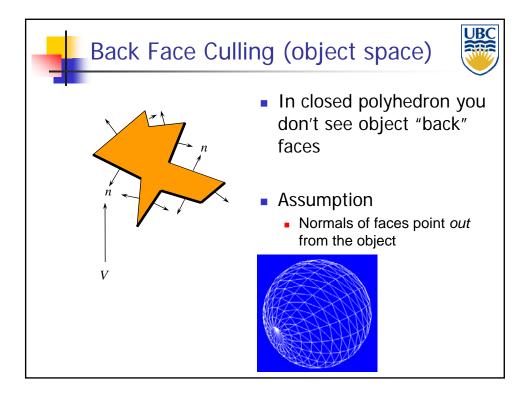
- Requires "lots" of memory
 - (e.g. 1280x1024x32 bits)
- Requires fast memory
 - Read-Modify-Write in inner loop
- Hard to simulate transparent polygons
 - We throw away color of polygons behind closest one
 - Works if polygons ordered back-to-front
 - Extra work throws away much of the speed advantage



Object Space Algorithms



- Determine visibility on object or polygon level
 - Using camera coordinates
- Resolution independent
 - Explicitly compute visible portions of polygons
- Early in pipeline
 - After clipping
- Requires depth-sorting
 - Painter's algorithm
 - BSP trees





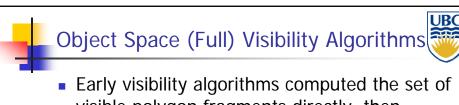
Back Face Culling



Determine back & front faces using sign of inner product nv

$$n \cdot v = n_x v_x + n_y v_y + n_z v_z = ||n|| \cdot ||v|| \cos \theta$$

- In a convex object :
 - Invisible back faces
 - All front faces entirely visible ⇒ solves hidden surfaces problem
- In non-convex object:
 - Invisible back faces
 - Front faces can be visible, invisible, or partially visible



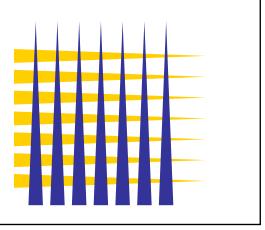
visible polygon fragments directly, then rendered the fragments to a display:



Object Space Visibility Algorithms



- What is the worst-case cost of computing the fragments for a scene composed of n polygons?
- Answer: O(n²)





Object Space Visibility Algorithms



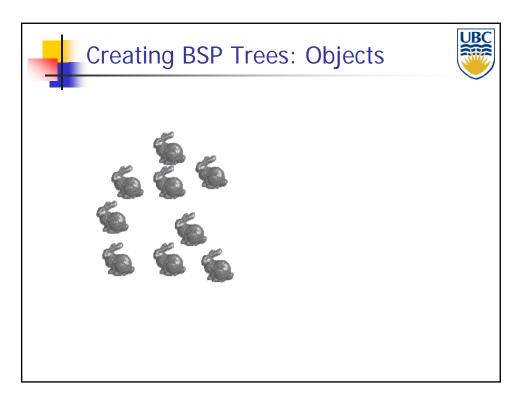
- Not optimal for our "cheap" memory rendering pipeline setup
 - But very useful for other tasks (e.g. RayTracing) or to speed pipeline rendering for static scenes
- Example:
 - Binary Space Partition (BSP) Trees

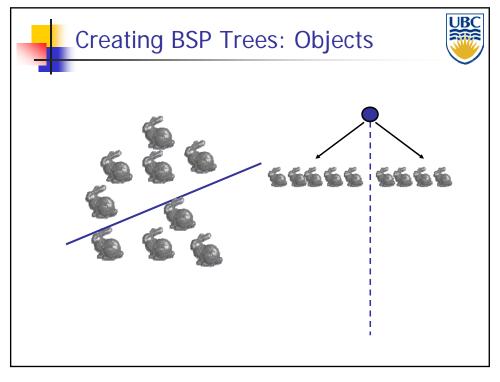


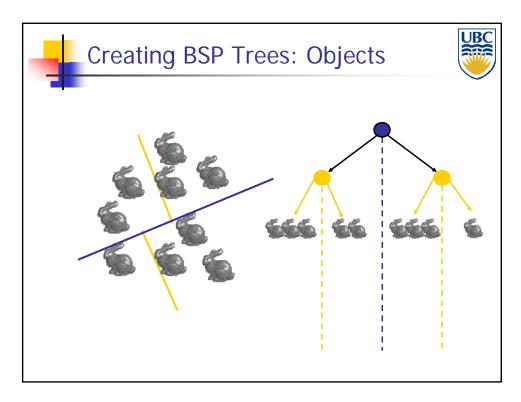
Binary Space Partition Trees

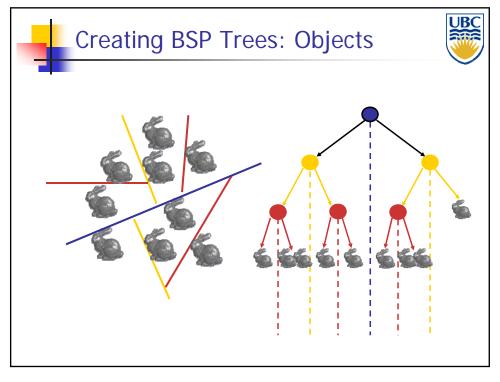


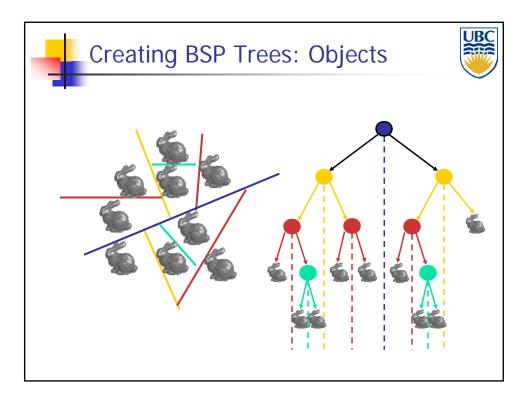
- BSP Tree: partition space with binary tree of planes
- Idea: divide space recursively into half-spaces by choosing splitting planes that separate objects in scene
 - Now we can define partial view order between halves
- Preprocessing: create binary tree of planes
- Runtime: correctly traversing this tree enumerates objects from back to front













Splitting Objects



- No bunnies were harmed in previous example
- But what if a splitting plane passes through an object?
 - Split the object; give half to each node





Traversing BSP Trees



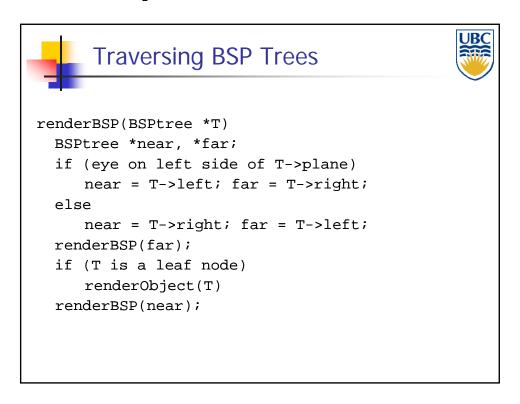
- Tree creation independent of viewpoint
 - Preprocessing step
- Tree traversal uses viewpoint
 - Runtime, happens for many different viewpoints

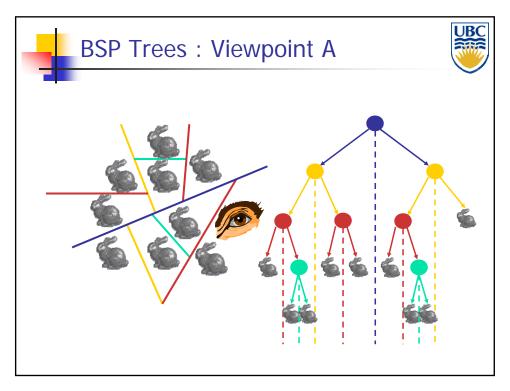


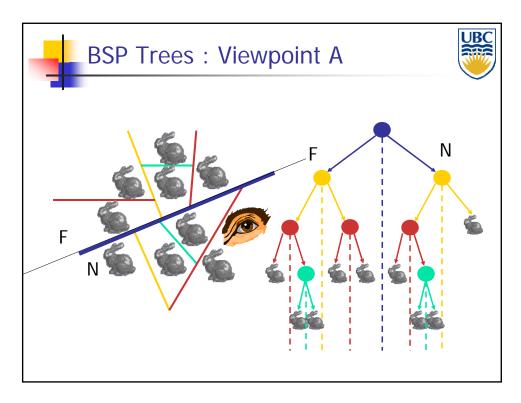
Traversing BSP Trees

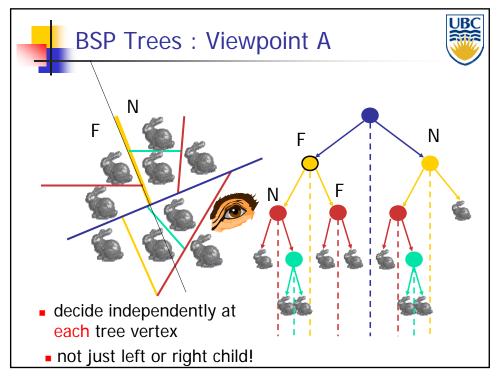


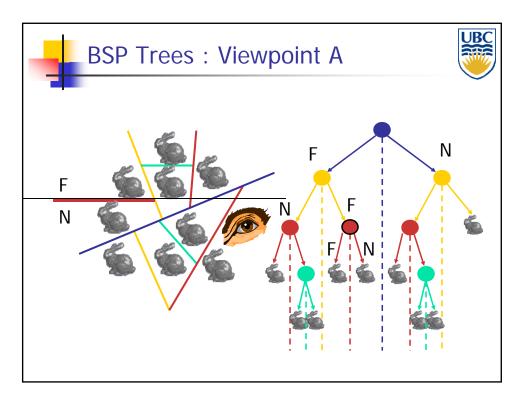
- Each plane divides world into near and far
 - For given viewpoint, decide which side is near and which is far
 - Check which side of plane viewpoint is on independently for each tree vertex
 - Tree traversal differs depending on viewpoint!
 - Recursive algorithm
 - Recurse on far side
 - Draw object
 - Recurse on near side

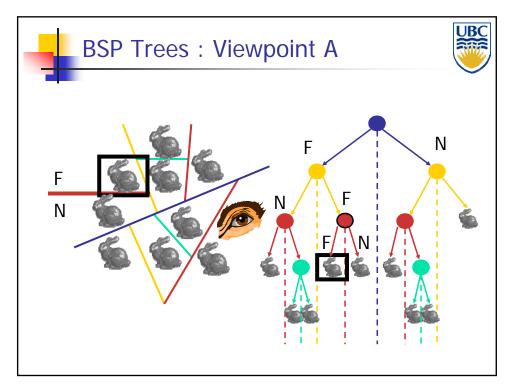


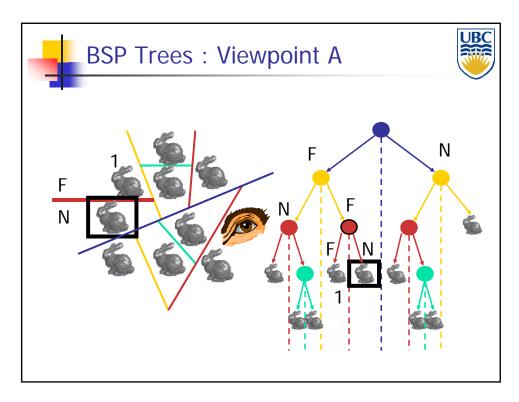


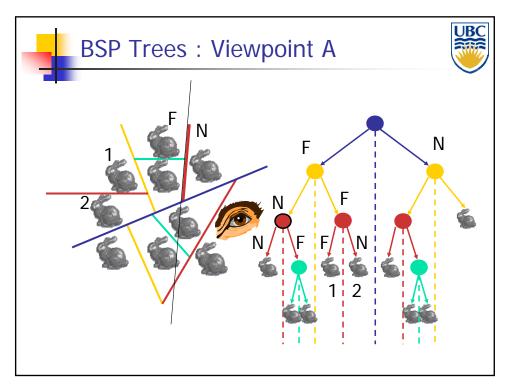


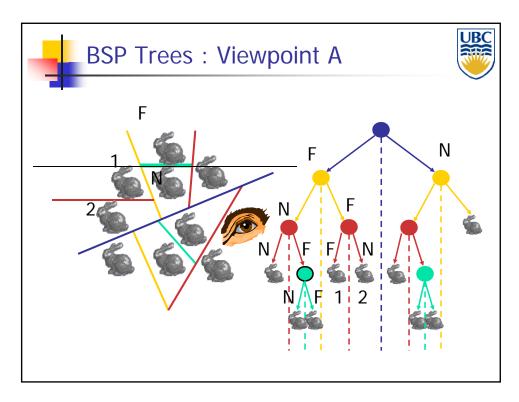


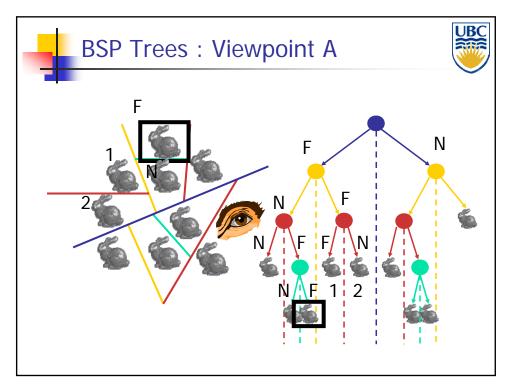


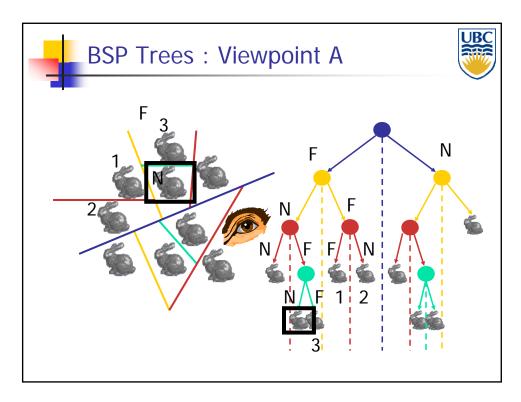


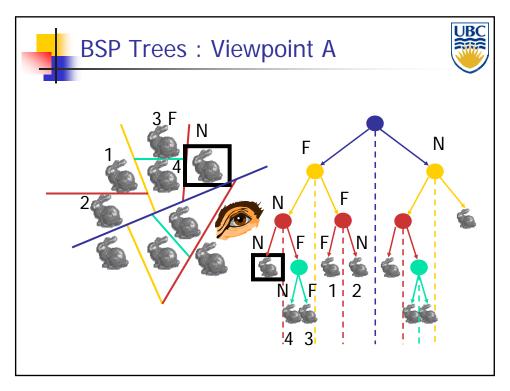


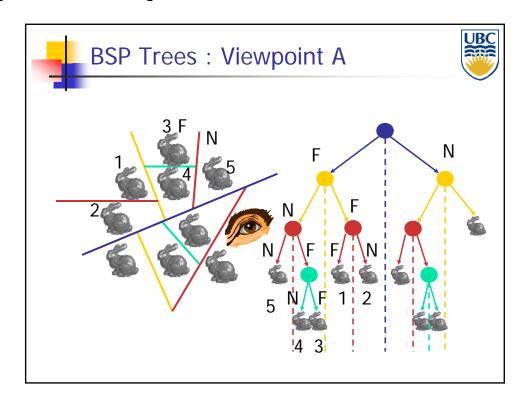


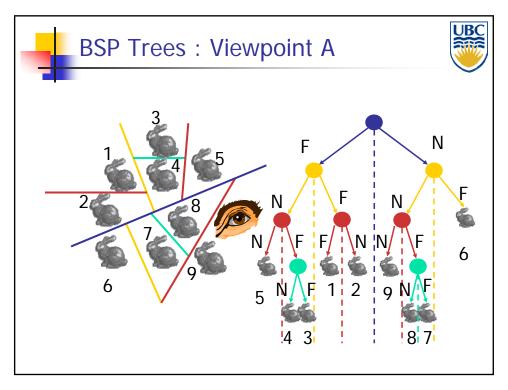


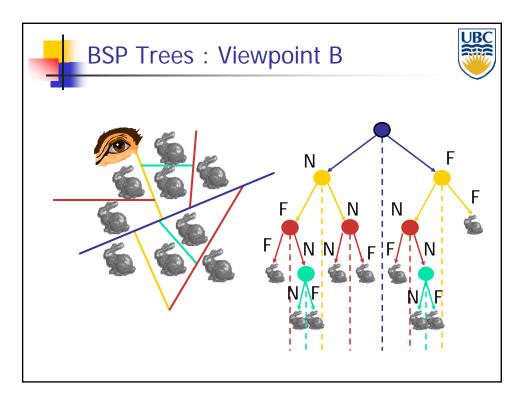


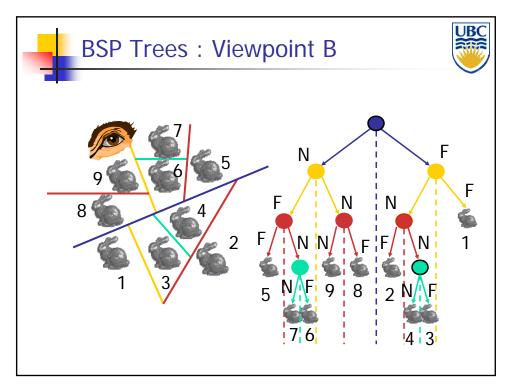










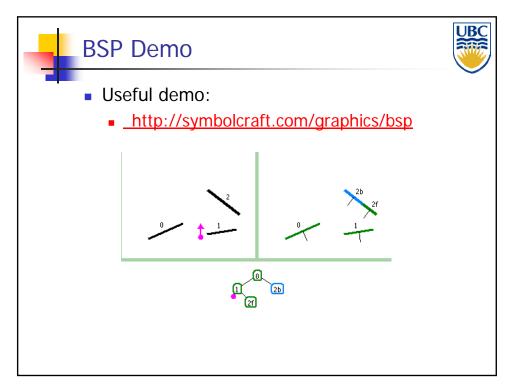




BSP Tree Traversal: Polygons



- Split along the plane defined by any polygon from scene
- Classify all polygons into positive or negative half-space of the plane
 - If a polygon intersects plane, split polygon into two and classify them both
- Recurse down the negative half-space
- Recurse down the positive half-space





Summary: BSP Trees



- Pros:
 - Simple, elegant scheme
 - Correct version of painter's algorithm back-to-front rendering approach
 - Still very popular for video games (but getting less so)
- Cons:
 - Slow(ish) to construct tree: O(n log n) to split, sort
 - Splitting increases polygon count: O(n2) worst-case
 - Computationally intense preprocessing stage restricts algorithm to static scenes