Name: _____ **Student ID:** _____

Write your answers in the spaces provided.

No aids (books, notes, calculators, mobile phones, PDA's, music players, killer robots, other electronic devices) are permitted.

1) Given a point with pixel coordinates (p, q) in an $m \times n$ image, construct the ray in camera space through the pixel for an *orthographic* projection specified in the usual way (left, right, bottom, top, near, far).

The origin of the ray is at

$$\left(\frac{p}{m}(\text{right} - \text{left}) + \text{left}, \quad \frac{q}{n}(\text{top} - \text{bottom}) + \text{bottom}, \quad -\text{near}\right)$$

Since the projection is orthographic, the direction is just $(0, 0, -1)$.

2) What is the "teapot in the stadium" problem for acceleration grids?

The acceleration grid must cover the entire stadium. However, if the cell size is chosen for the stadium's length scale, all of the teapot's polygons will probably land in just a few cells, so there will be almost no acceleration for the teapot render. If the cell size is chosen for the teapot, the grid will be excessively large, and offer no acceleration for rendering the stadium.

3) Give pseudo-code for checking if two axis-aligned bounding boxes intersect.

```
For a=0, 1, 2:
  if xmin1[a]>xmax2[a] or xmax1[a]<xmin2[a]:
    return false
return true
```

4) Give pseudo-code for checking if a ray with origin $\vec{x}_0$ and direction $\vec{d}$ intersects a sphere with centre $\vec{c}$ and radius r.

Plug the parametric equation of the ray, $\vec{x}(s) = \vec{x}_0 + s\vec{d}$, into the implicit equation of the sphere $\|\vec{x} - \vec{c}\|^2 - r^2 = 0$. This gives a quadratic equation:

$$\begin{aligned} \|\vec{d}\|^2 s^2 + 2\vec{d} \cdot (\vec{x_0} - \vec{c})s + \|\vec{x}_0 - \vec{x}\|^2 - r^2 &= 0 \\ \Leftrightarrow \quad as^2 + bs + c &= 0 \end{aligned}$$

If the discriminant $D = b^2 - 4ac$ is non-negative, $D \geq 0$, there are real roots:

$$s = \frac{-b \pm \sqrt{D}}{2a}$$

Check if either possible root satsifies $s \geq 0$, i.e. occurs on the ray.

5) If a ray with direction $\vec{d}$ hits a surface with unit-length normal $\hat{n}$ at point $\vec{x}$, what is the reflected ray?

The origin is $\vec{x}$ and the direction is $\vec{d} - 2(\vec{d} \cdot \hat{n})\hat{n}$.

6) Given a 4×4 transformation matrix and a 3D direction vector $\vec{d}$, how do you compute the transformed direction?

Multiply with a zero as the fourth coordinate: the transformed direction $\vec{d'}$ satisfies

$$\begin{pmatrix} \vec{d'} \\ 0 \end{pmatrix} = M \begin{pmatrix} \vec{d} \\ 0 \end{pmatrix}$$

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where M is the matrix.

7) How do we define the orientation of an ordered list of four points in 3D? (just give one definition)

$$\text{orient}(\vec{x}_0, \vec{x}_1, \vec{x}_2, \vec{x}_3) = \det \begin{pmatrix} \vec{x}_0 & \vec{x}_1 & \vec{x}_2 & \vec{x}_3 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

8) What is the "depth complexity" of a pixel in an image?

It is the number of triangles that overlap that pixel (Z-buffer perspective) or the number of geometric intersections with a ray extending through that pixel out to infinity (ray-tracing perspective).

9) Suppose a BVH of axis-aligned bounding boxes has been built on a set of n points. Give recursive pseudo-code for efficiently finding if any point lies below the plane $y = 0$.

Apply the following to the root node of the BVH:

```
check(node):
if ymin(node)>=0:
  return Empty
else if node is a leaf:
  return node's point
else for all children:
  return check(child) if not Empty:
```

10) How can you check if the line segment between points $\vec{p}$ and $\vec{q}$ intersects a plane through point $\vec{r}$ with normal $\hat{n}$?

Return true if the sign of $(\vec{p} - \vec{r}) \cdot \hat{n}$ differs from $(\vec{q} - \vec{r}) \cdot \hat{n}$.

11) Describe a set of n points where an acceleration tree structure, built by splitting the space a node occupies in half, would have depth $O(n)$.

Take $x_i = 2^i$ for $i = 1, \ldots n$.

12) Describe a physical effect in light transport that raytracing doesn't capture (without extra work).

Diffraction is the best example.