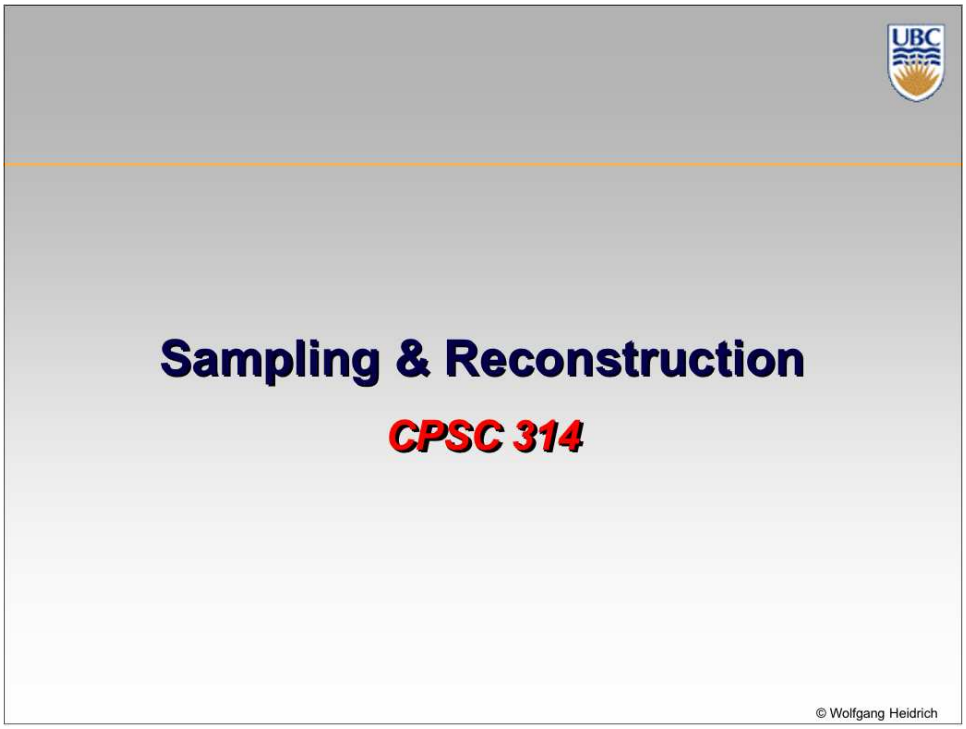
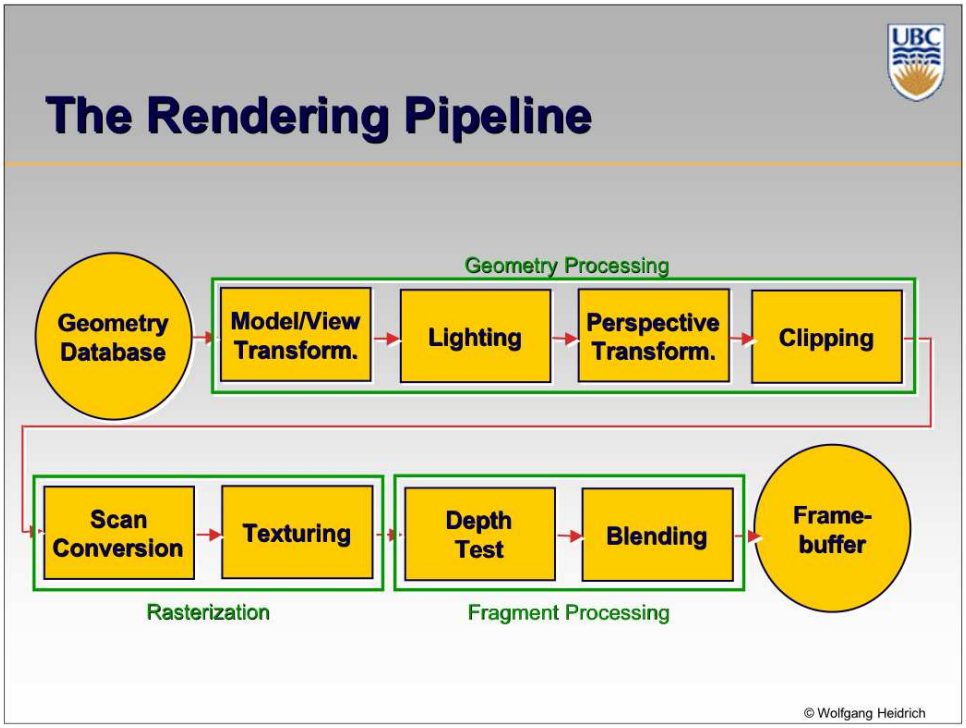

Sampling & Reconstruction
CPSC 314
© Wolfgang Heidrich


The Rendering Pipeline
Geometry Processing
Geometry Database
Model/View Transform.
Lighting
Perspective Transform.
Clipping
Scan Conversion
Texturing
Depth Test
Blending
Frame-buffer
Rasterization
Fragment Processing
© Wolfgang Heidrich

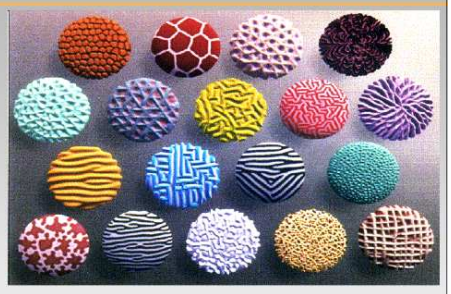
Scan Conversion of Lines - Digital Differential Analyzer

First Attempt:

```
dda( float xs, ys, xe, ye ) {
    // assume xs < xe, and slope m between 0 and 1
    float m= (ye-ys)/(xe-xs);
    float y= round( ys );
    for( int x= round( xs ) ; x<= xe ; x++ ) {
        drawPixel( x, round( y ) );
        y= y+m;
    }
}
```


Texture Mapping

- Real life objects have nonuniform colors, normals
- To generate realistic objects, reproduce coloring & normal variations = **texture**
- Can often replace complex geometric details

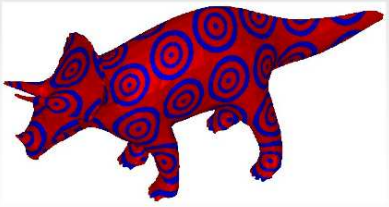


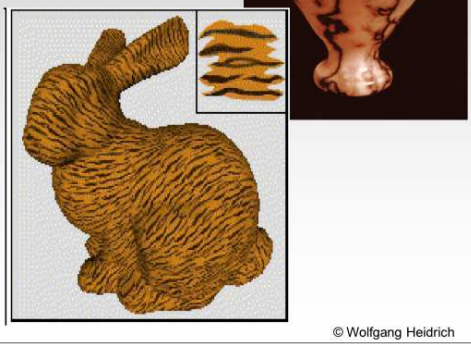
Color Texture Mapping

Define color (RGB) for each point on object surface

Two approaches

- Surface texture map
- Volumetric texture





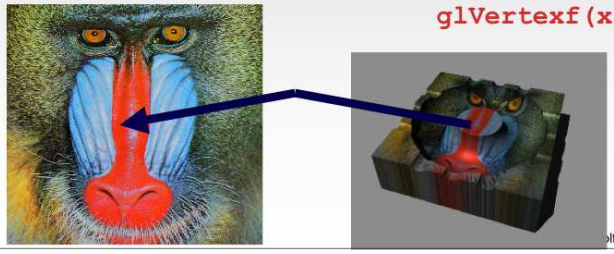
Texture Coordinates

Texture image: 2D array of color values (texels)

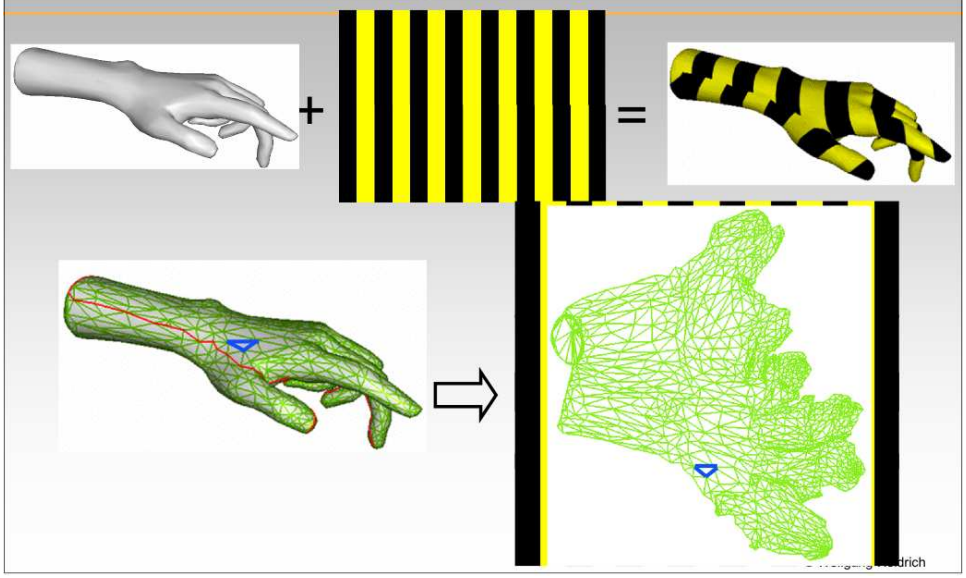
Assigning texture coordinates (s,t) at vertex with object coordinates (x,y,z,w)

- Use interpolated (s,t) for texel lookup at each pixel
- Use value to modify a polygon's color
  - *Or other surface property*
- Specified by programmer or artist

```
glTexCoord2f(s,t)
glVertexf(x,y,z,w)
```

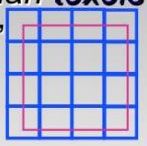


Texture Mapping

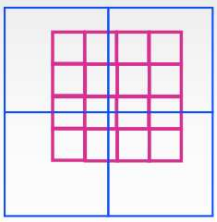


Reconstruction

- How to deal with:
  - *Pixels* that are much larger than ***texels***?
    - Apply filtering, "averaging"

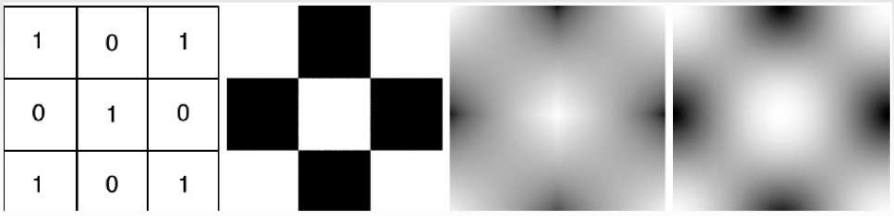


  - *Pixels* that are much smaller than ***texels*** ?
    - Interpolate



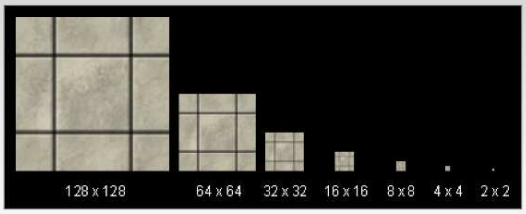
Interpolating Textures

- Nearest neighbor
- Bilinear
- Hermite

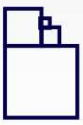


MIPmapping

use "image pyramid" to precompute averaged versions of the texture



store whole pyramid in single block of memory



Without MIP-mapping

With MIP-mapping

MIPmaps

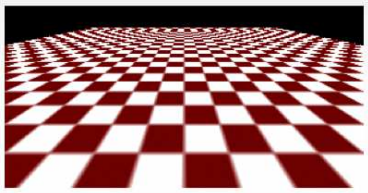
Multum in parvo -- ***many things in a small place***

- Prespecify a series of prefiltered texture maps of decreasing resolutions
- Requires more texture storage
- Avoid shimmering and flashing as objects move

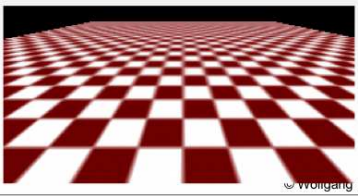
gluBuild2DMipmaps

- Automatically constructs a family of textures from original texture size down to 1x1

without

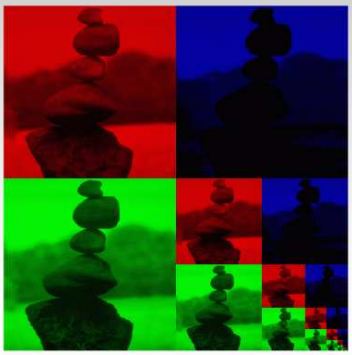


with



MIPmap storage

only 1/3 more space required



Texture Parameters

In addition to color can control other material/object properties

- Surface normal (bump mapping)
- Reflected color (environment mapping)

Sampling & Reconstruction

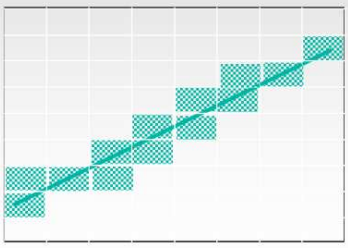
CPSC 314

Samples

- Most things in the real world are continuous
- Everything in a computer is discrete
- The process of mapping a continuous function to a discrete one is called sampling
- The process of mapping a discrete function to a continuous one is called reconstruction
- The process of mapping a continuous variable to a discrete one is called quantization
- Rendering an image requires sampling and quantization
- Displaying an image involves reconstruction

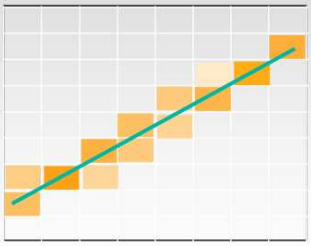
Line Segments

- We tried to sample a line segment so it would map to a 2D raster display
- We quantized the pixel values to 0 or 1
- We saw stair steps, or jaggies



Line Segments

- Instead, quantize to many shades
- But what sampling algorithm is used?



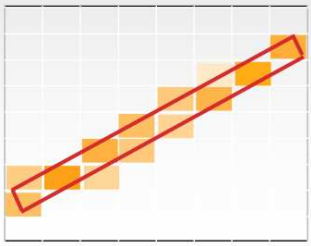
Unweighted Area Sampling

Shade pixels wrt area covered by thickened line

Equal areas cause equal intensity, regardless of distance from pixel center to area

- Rough approximation formulated by dividing each pixel into a finer grid of pixels

Primitive cannot affect intensity of pixel if it does not intersect the pixel

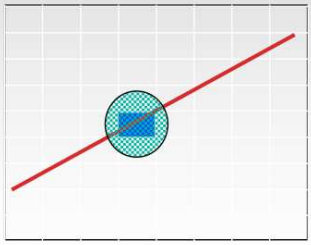


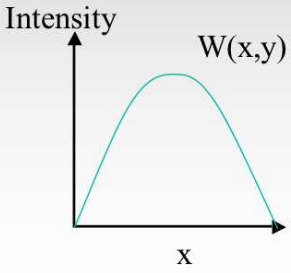
Weighted Area Sampling

Intuitively, pixel cut through the center should be more heavily weighted than one cut along corner

Weighting function, W(x,y)

- Specifies the contribution of primitive passing through the point (x, y) from pixel center





Images

An image is a 2D function I(x, y)

- Specifies intensity for each point (x, y)
- (we consider each color channel independently)

Image Sampling and Reconstruction

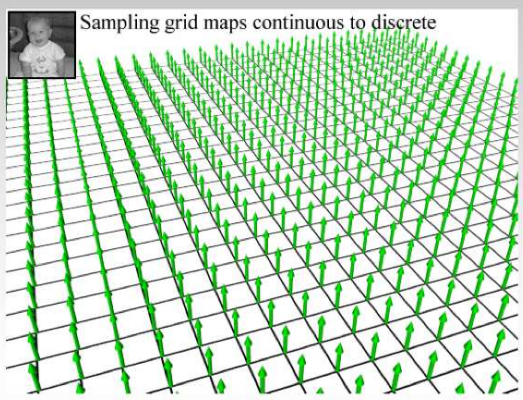
- Convert continuous image to discrete set of samples
- Display hardware reconstructs samples into continuous image
  - *Finite sized source of light for each pixel*

discrete input values

continuous light output

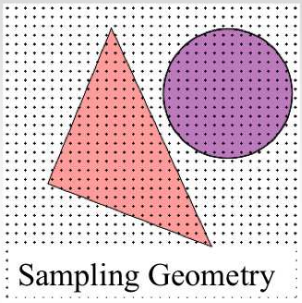
Point Sampling an Image

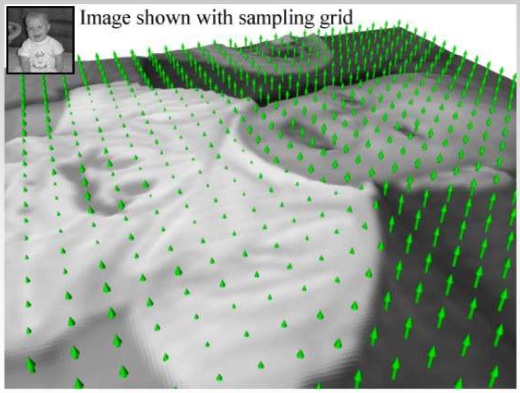
- Simplest sampling is on a grid
- Sample depends solely on value at grid points



Point Sampling

Multiply sample grid by image intensity to obtain a discrete set of points, or samples.





Sampling Errors

Some objects missed entirely, others poorly sampled

- Could try unweighted or weighted area sampling
- But how can we be sure we show everything?

Need to think about entire class of solutions!

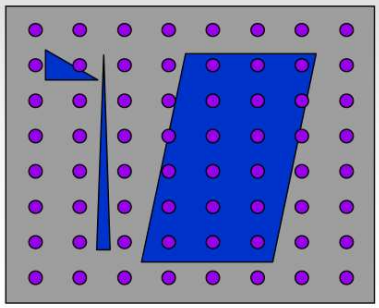


Image As Signal

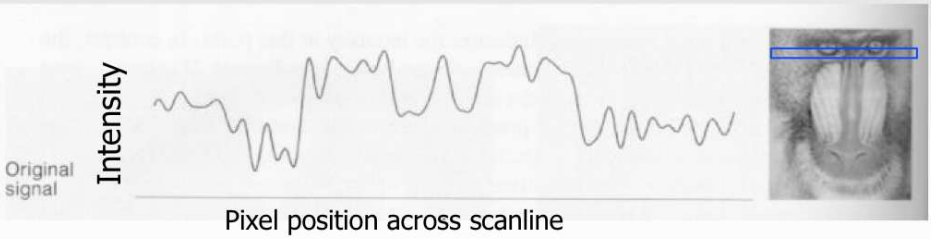
Image as spatial signal

2D raster image

- Discrete sampling of 2D spatial signal

1D slice of raster image

- Discrete sampling of 1D spatial signal



Examples from Foley, van Dam, Feiner, and Hughes

Sampling Theory

How would we generate a signal like this out of simple building blocks?

Theorem

- Any signal can be represented as an (infinite) sum of sine waves at different frequencies

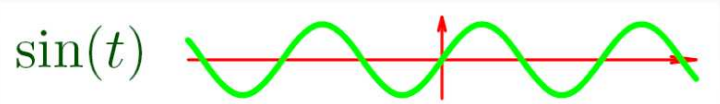
Sampling Theory in a Nutshell

Terminology

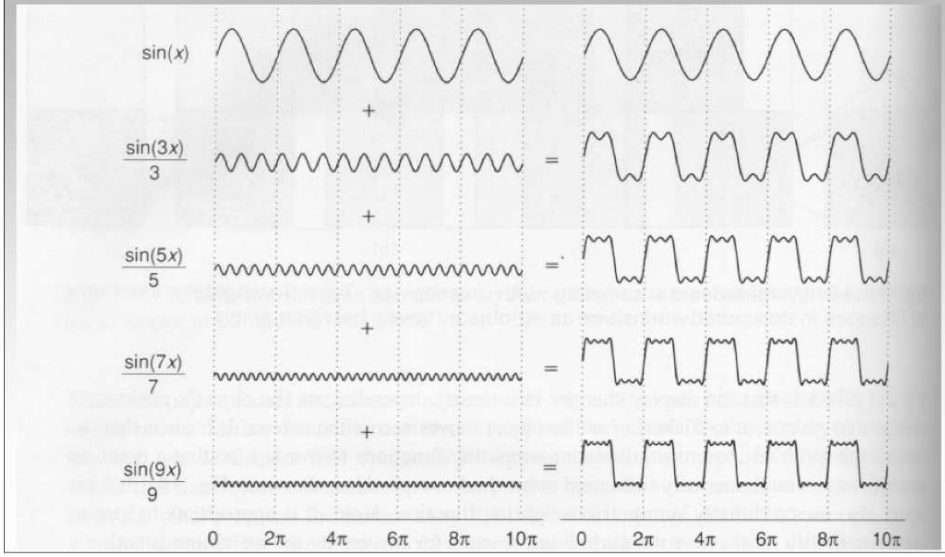
- Wavelength – length of repeated sequence on infinite signal
- Frequency – 1/wavelength (number of repeated sequences in unit length)

Example – sine wave

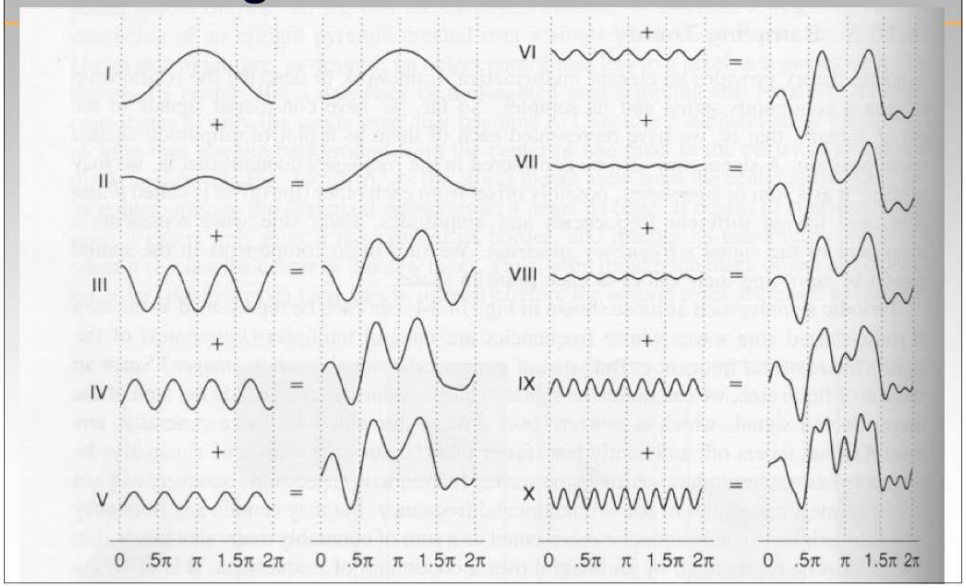
- Wavelength = 2π
- Frequency = $1/2\pi$



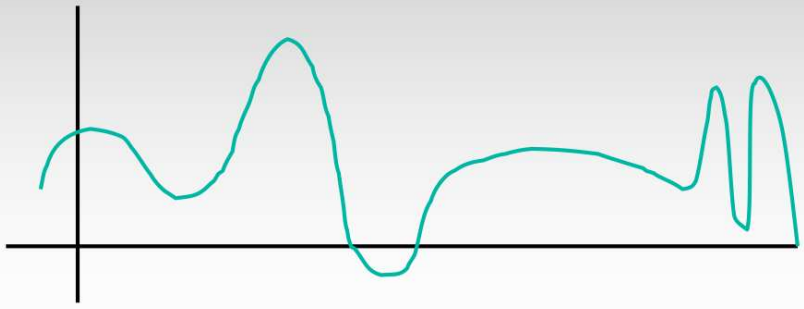
Summing Waves I



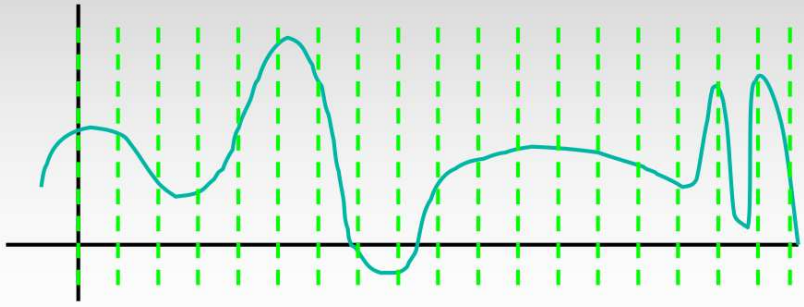
Summing Waves II



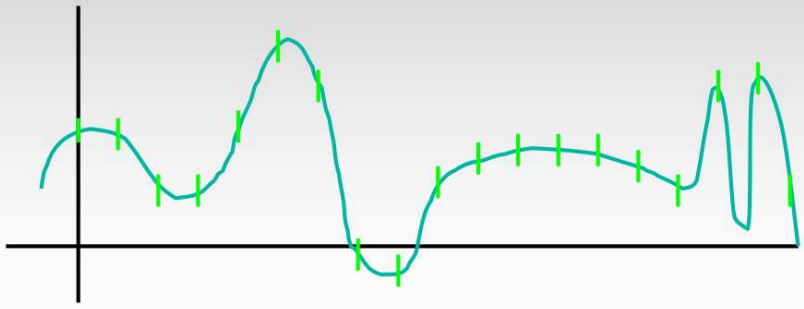
1D Sampling and Reconstruction



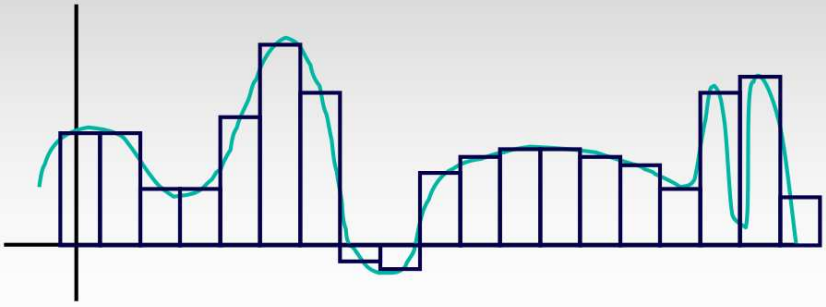
1D Sampling and Reconstruction



1D Sampling and Reconstruction



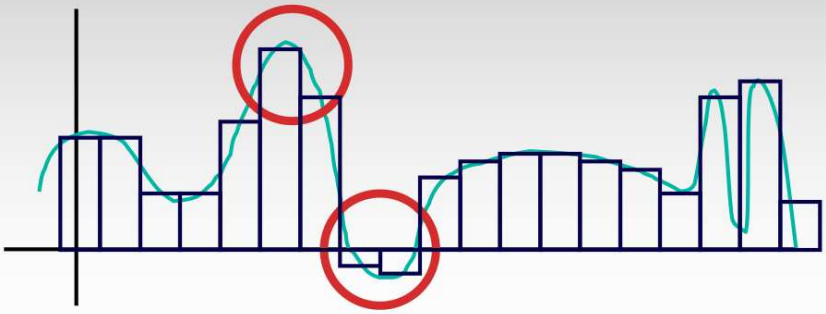
1D Sampling and Reconstruction



1D Sampling and Reconstruction

Problems

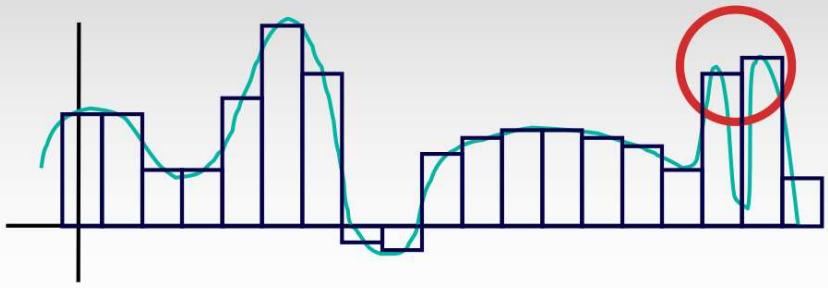
- Jaggies – abrupt changes



1D Sampling and Reconstruction

Problems

- Jaggies – abrupt changes
- Lose data



Sampling Theorem

- Continuous signal can be completely recovered from its samples

Iff

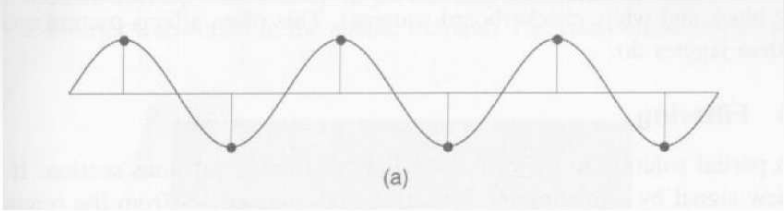
- Sampling rate greater than twice highest frequency present in signal

- Claude Shannon

Nyquist Rate

Lower bound on sampling rate

- Twice the highest frequency component in the image's spectrum



Falling Below Nyquist Rate

When sampling below Nyquist Rate, resulting signal looks like a lower-frequency one

- This is aliasing!

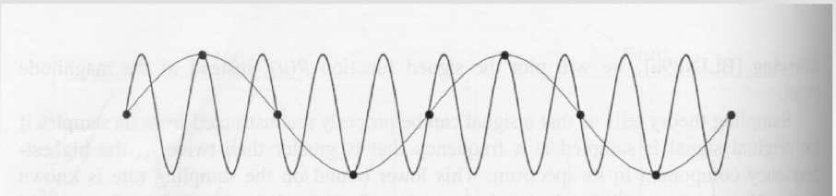
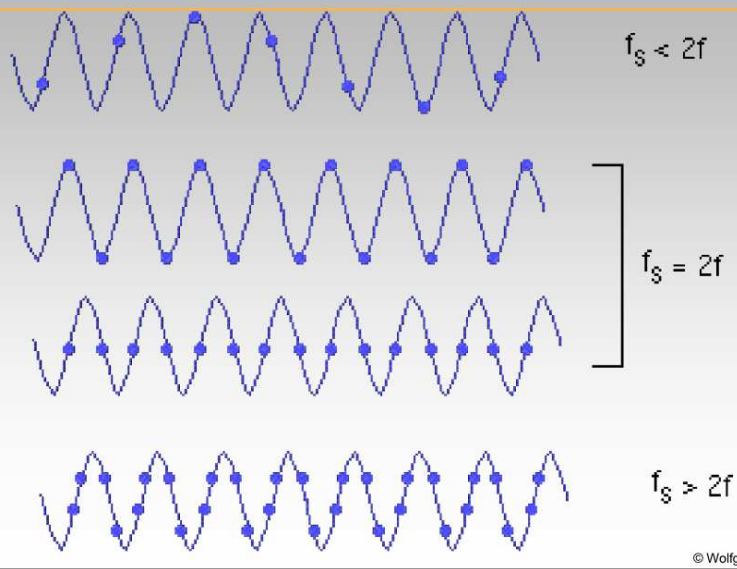


Fig. 14.17 Sampling below the Nyquist rate. (Courtesy of George Wolberg, Columbia University.)

Nyquist Rate



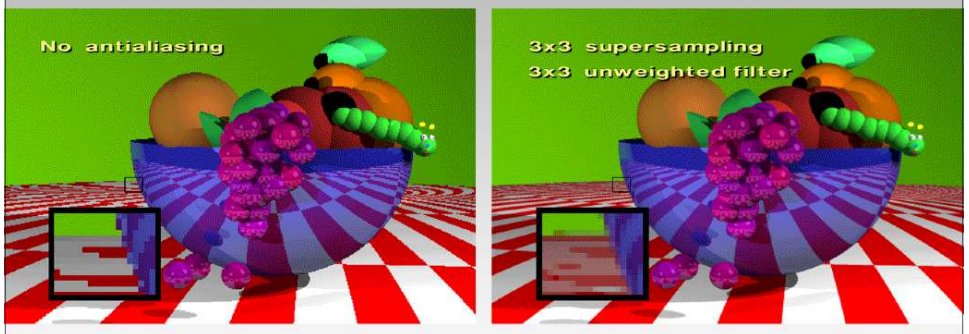
Aliasing

Incorrect appearance of high frequencies as low frequencies

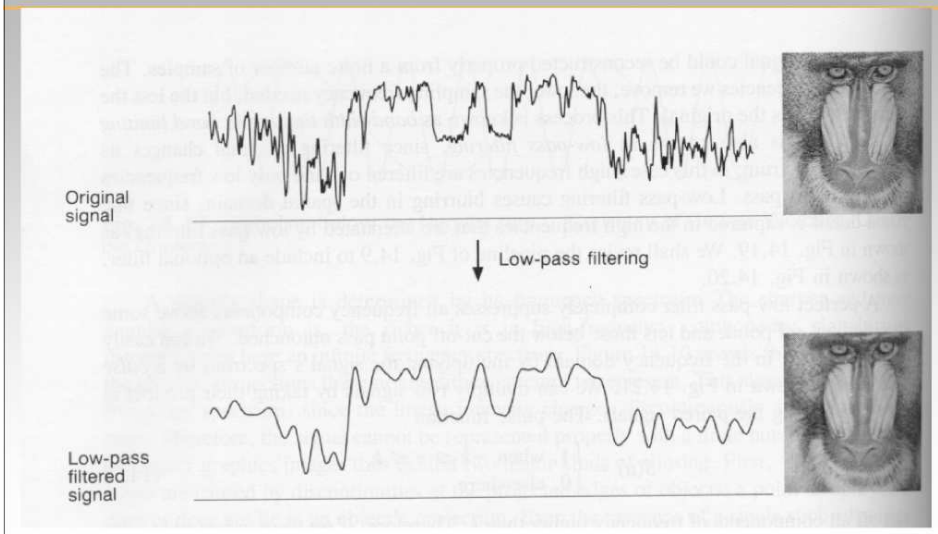
To avoid: anti-aliasing

- Supersample
  - *Sample at higher frequency*
- Low pass filtering
  - *Remove high frequency function parts*
  - *Aka prefiltering, band-limiting*

Supersampling



Low-Pass Filtering



Low-Pass Filtering

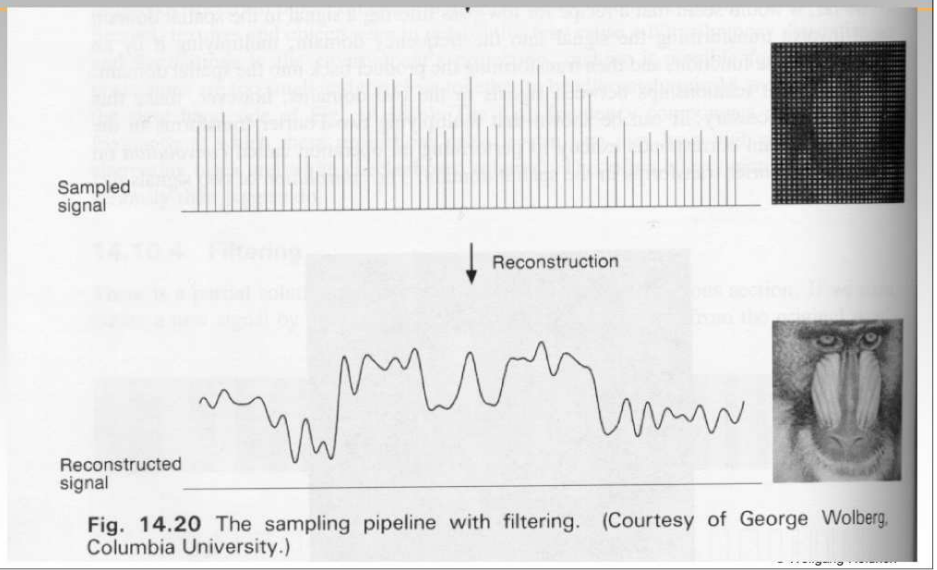
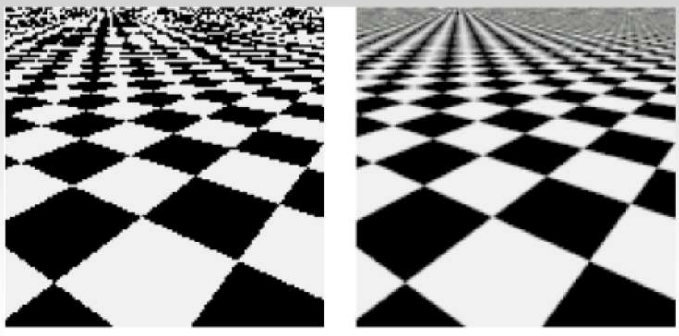


Fig. 14.20 The sampling pipeline with filtering. (Courtesy of George Wolberg, Columbia University.)

Previous Antialiasing Example

Texture mipmapping: low pass filter



(a) (b)

Discussion

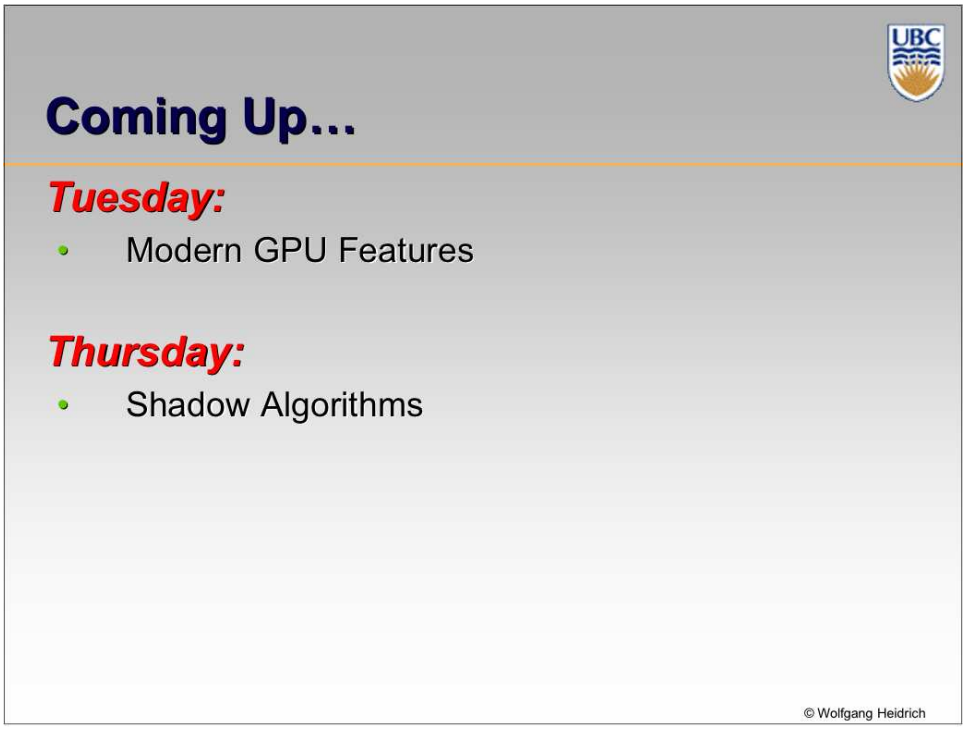
Sampling & Reconstruction

- Fundamental issue in graphics, vision, and many other areas of computer science
  - *Whenever continuous signals need to be represented in a computer*
- Aliasing refers to the problem of reconstruction errors due to frequencies above the Nyquist limit
  - *These frequencies show up as erroneous low frequency content*

Discussion

Anti-Aliasing Approaches

- Low-pass filtering (before sampling!)
  - *Avoids aliasing*
  - *May not be practical in all settings*
  - *For images: artifacts around edges?!*
- Supersampling
  - *General algorithmic approach*
  - *Hoever: even the higher resolution image has a Nyquist limit!*
  - *Slow*


Coming Up...
Tuesday:
Modern GPU Features
Thursday:
Shadow Algorithms
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