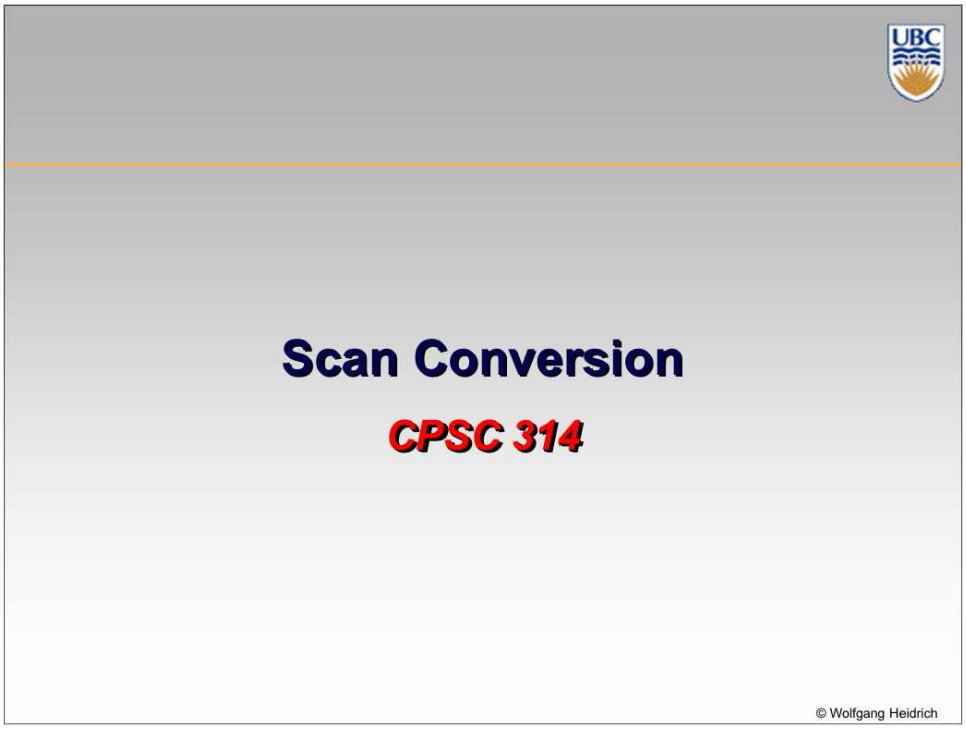
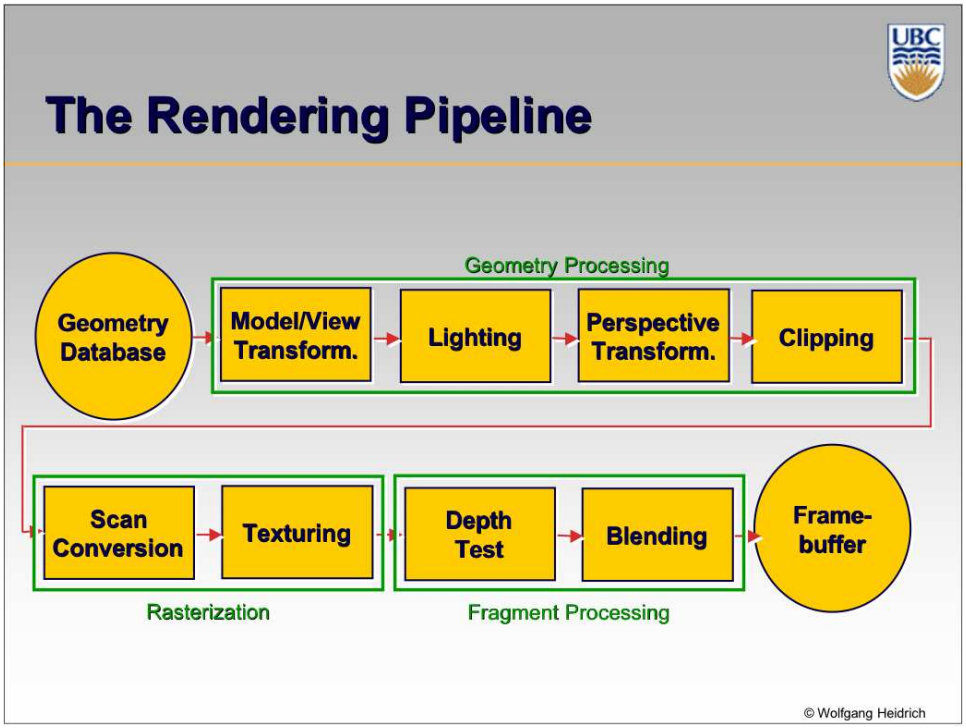
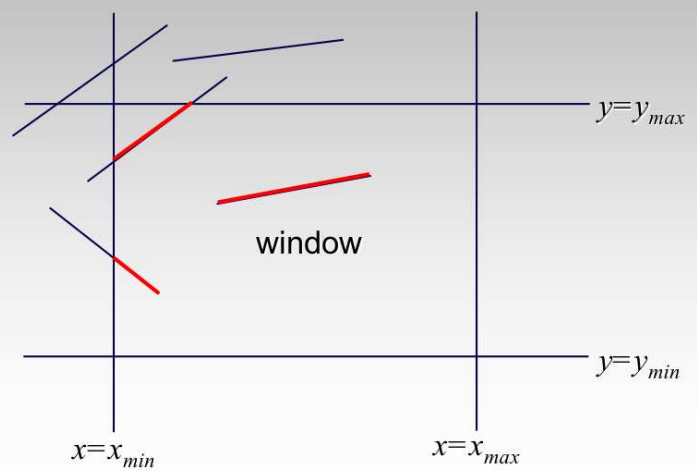

Scan Conversion
CPSC 314
© Wolfgang Heidrich


The Rendering Pipeline
Geometry Processing
Geometry Database
Model/View Transform.
Lighting
Perspective Transform.
Clipping
Scan Conversion
Texturing
Depth Test
Blending
Frame-buffer
Rasterization
Fragment Processing
© Wolfgang Heidrich

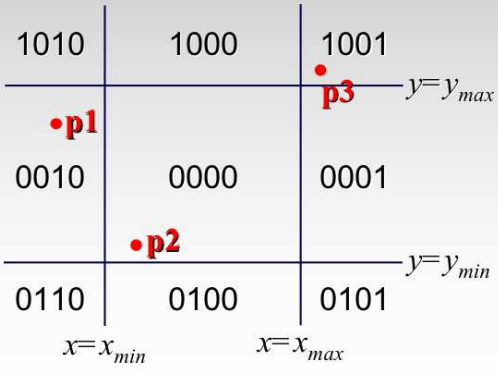
Line Clipping



Line Clipping

Outcodes (Cohen, Sutherland '74)

- 4 flags encoding position of a point relative to top, bottom, left, and right boundary
- E.g.:
  - OC(**p1**)=0010
  - OC(**p2**)=0000
  - OC(**p3**)=1001



Line Clipping

Line segment:

- $(\mathbf{p1},\mathbf{p2})$

Trivial cases:

- OC($\mathbf{p1}$)== 0 && OC($\mathbf{p2}$)==0
  - *Both points inside window, thus line segment completely visible (trivial accept)*
- (OC($\mathbf{p1}$) & OC($\mathbf{p2}$))!= 0
  - *There is (at least) one boundary for which both points are outside (same flag set in both outcodes)*
  - *Thus line segment completely outside window (trivial reject)*

Line Clipping

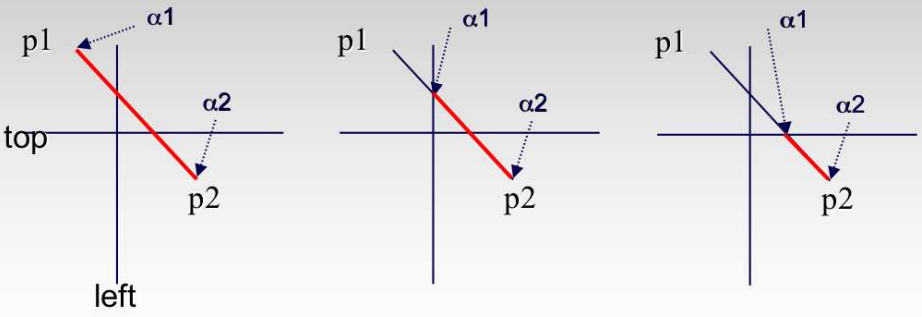
α-Clipping

- Handling of all the non-trivial cases
- Improvement of earlier algorithms (Cohen/Sutherland, Cyrus/Beck, Liang/Barsky)
- Define *window-edge-coordinates* of a point $\mathbf{p}=(x,y)^T$
  - $WEC_L(\mathbf{p}) = x - x_{min}$
  - $WEC_R(\mathbf{p}) = x_{max} - x$
  - $WEC_B(\mathbf{p}) = y - y_{min}$
  - $WEC_T(\mathbf{p}) = y_{min} - y$

Negative if outside!

Line Clipping

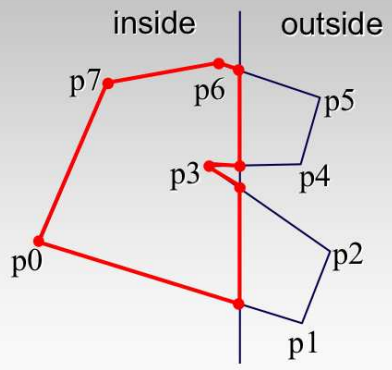
α-Clipping: example for clipping p1



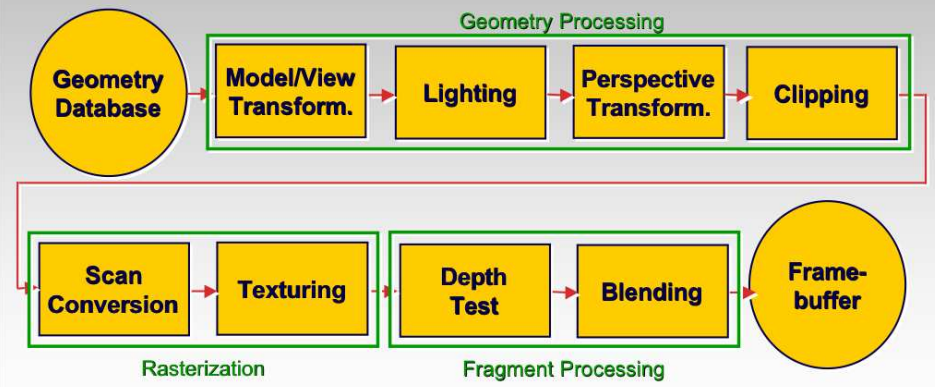
Start configuration | After clipping to left | After clipping to top

Polygon Clipping

Example



The Rendering Pipeline

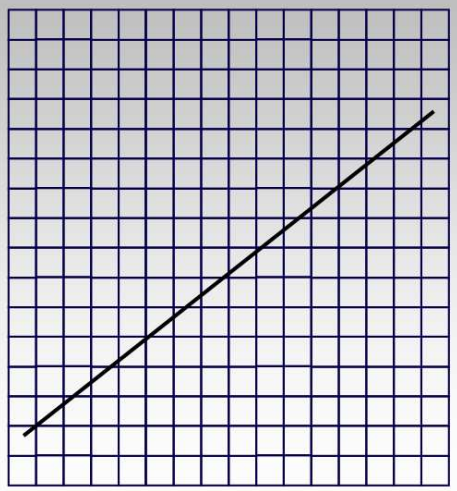


Scan Conversion - Rasterization

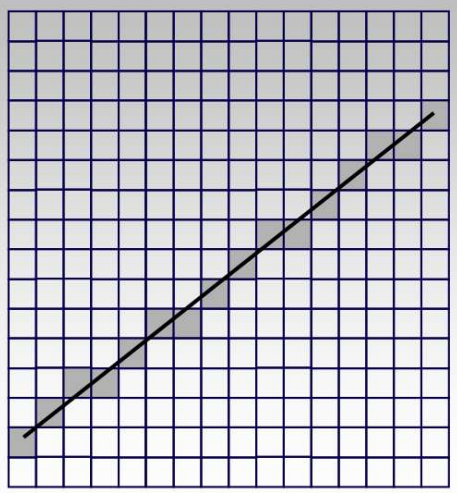
Convert continuous rendering primitives into discrete fragments/pixels

- Lines
  - *Midpoint/Bresenham*
- Triangles
  - *Flood fill*
  - *Scanline*
  - *Implicit formulation*
- Interpolation

Scan Conversion - Lines



Scan Conversion - Lines



Scan Conversion - Lines

First Attempt:

- Line (s,e) given in device coordinates
- Create the thinnest line that connects start point and end point without gap

Assumptions for now:

- Start point to the left of end point: xs< xe
- Slope of the line between 0 and 1 (I.e. elevation between 0 and 45 degrees:

$$0 \le \frac{ye - ys}{xe - xs} \le 1$$

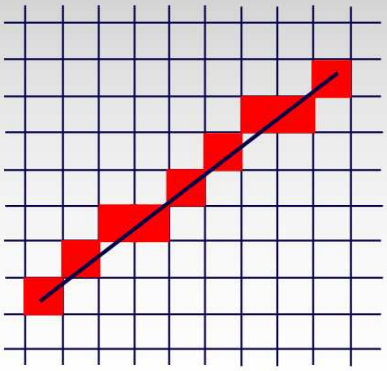
Scan Conversion of Lines - Digital Differential Analyzer

First Attempt:

```
dda( float xs, ys, xe, ye ) {
    // assume xs < xe, and slope m between 0 and 1
    float m= (ye-ys)/(xe-xs);
    float y= round( ys );
    for( int x= round( xs ) ; x<= xe ; x++ ) {
        drawPixel( x, round( y ) );
        y= y+m;
    }
}
```

Scan Conversion of Lines

DDA:



Scan Conversion of Lines
Midpoint Algorithm

Moving horizontally along x direction

- Draw at current y value, or move up vertically to y+1?
  - *Check if midpoint between two possible pixel centers above or below line*

Candidates

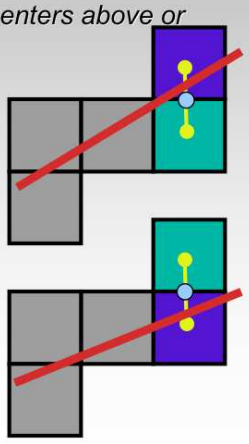
- Top pixel: (x+1,y+1)
- Bottom pixel: (x+1, y)

Midpoint: (x+1, y+.5)

Check if midpoint above or below line

- Below: top pixel
- Above: bottom pixel

Key idea behind Bresenham Alg.



Scan Conversion of Lines

Idea: decision variable

```
dda( float xs, ys, xe, ye ) {
    float d= 0.0;
    float m= (ye-ys)/(xe-xs);
    int y= round( ys );
    for( int x= round( xs ) ; x<= xe ; x++ ) {
        drawPixel( x, y );
        d= d+m;
        if( d>= 0.5 ) { d= d-1.0; y++; }
    }
}
```


Scan Conversion of Lines
Bresenham Algorithm ('63)

- Use decision variable to generate purely integer algorithm
- Explicit line equation:

$$y = \frac{(y_e - y_s)}{(x_e - x_s)}(x - x_s) + y_s$$

- Implicit version:

$$L(x,y) = \frac{(y_e - y_s)}{(x_e - x_s)}(x - x_s) - (y - y_s) = 0$$

- In particular for specific x, y, we have
  - *L(x,y)>0 if (x,y) below the line, and*
  - *L(x,y)<0 if (x,y) above the line*

Scan Conversion of Lines
Bresenham Algorithm

- Decision variable: after drawing point (x,y) decide whether to draw
  - $(x+1,y)$: *case E (for "east")*
  - $(x+1,y+1)$: *case NE (for "north-east")*
- Check whether $(x+1,y+1/2)$ is above or below line

$$d = L(x+1, y+\frac{1}{2})$$

- Point above line if and only if $d<0$

Scan Conversion of Lines

Bresenham Algorithm

- Problem: how to update d?
- Case E (point above line, $d<=0$)
  - *x= x+1;*
  - $d= L(x+2, y+1/2)= d+ (y_e-y_s)/(x_e-x_s)$
- Case NE (point below line, $d>0$)
  - *x= x+1; y= y+1;*
  - $d= L(x+2, y+3/2)= d+ (y_e-y_s)/(x_e-x_s) -1$
- Initialization:
  - $d= L(x_s+1, y_s+1/2)= (y_e-y_s)/(x_e-x_s) -1/2$

Scan Conversion of Lines

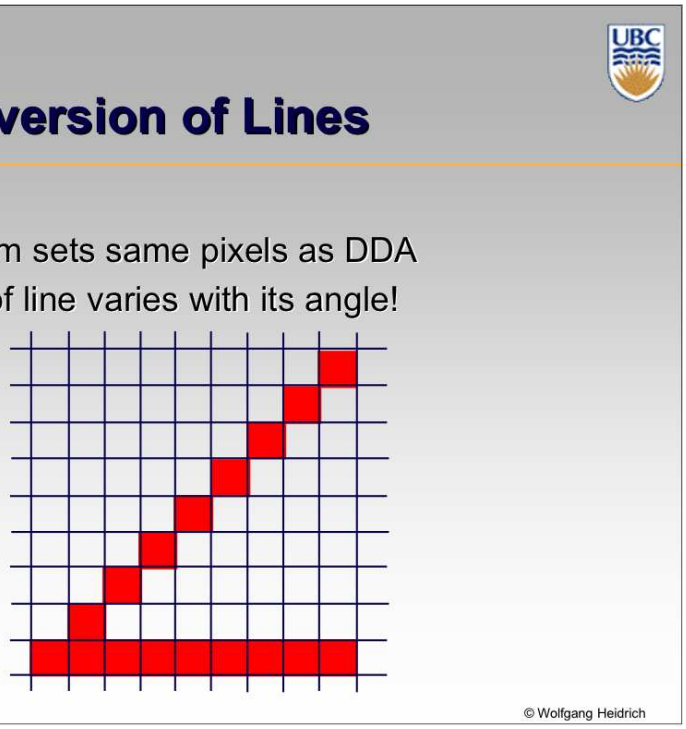
Bresenham Algorithm

- This is still floating point
- But: only sign of d matters
- Thus: can multiply everything by $2(x_e-x_s)$

Scan Conversion of Lines

Bresenham Algorithm

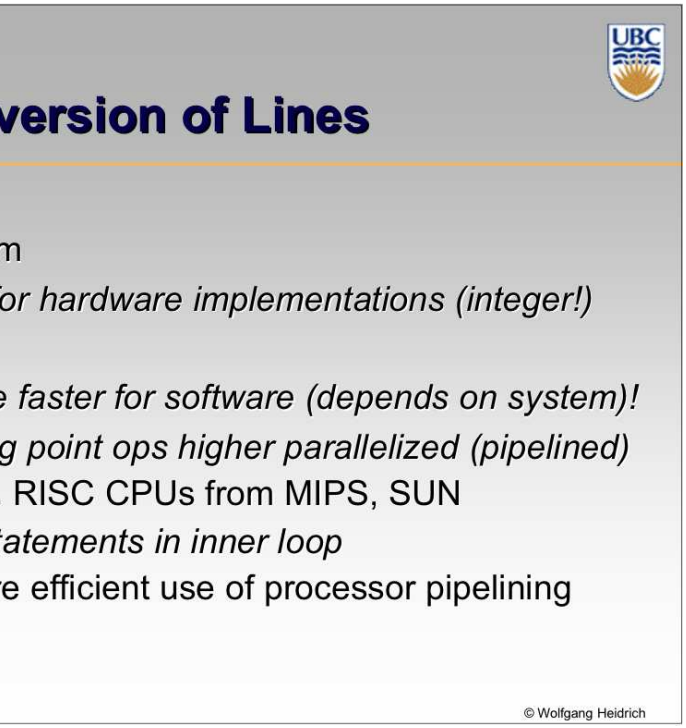
```
Bresenham( int xs, ys, xe, ye ) {
    int y= ys;
    incrE= 2(ye - ys);
    incrNE= 2((ye - ys) – (xe-xs));
    for( int x= xs ; x<= xe ; x++ ) {
        drawPixel( x, y );
        if( d<= 0 ) d+= incrE;
        else { d+= incrNE; y++; }
    }
}
```



Scan Conversion of Lines

Discussion

- Bresenham sets same pixels as DDA
- Intensity of line varies with its angle!

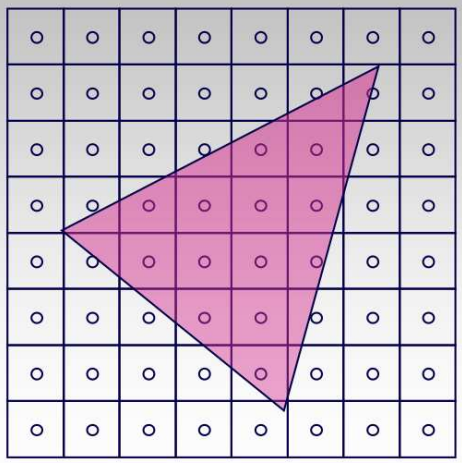


Scan Conversion of Lines

Discussion

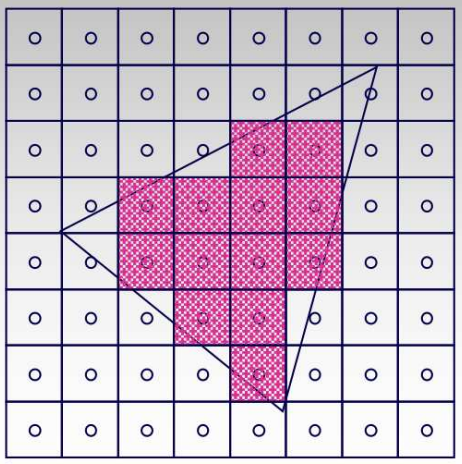
- Bresenham
  - *Good for hardware implementations (integer!)*
- DDA
  - *May be faster for software (depends on system)!*
  - *Floating point ops higher parallelized (pipelined)*
    - E.g. RISC CPUs from MIPS, SUN
  - *No if statements in inner loop*
    - More efficient use of processor pipelining

Scan Conversion of Polygons



Scan Conversion of Polygons

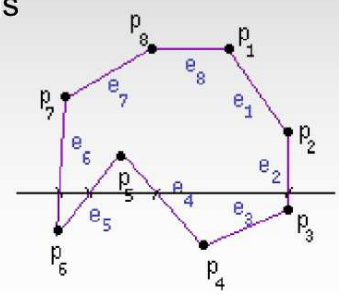
One possible scan conversion



Scan Conversion of Polygons

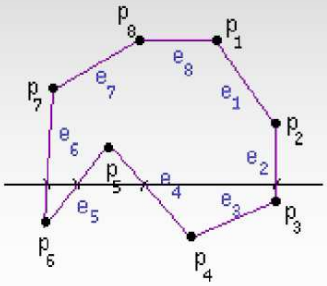
A General Algorithm

- Intersect each scanline with all edges
- Sort intersections in x
- Calculate parity to determine in/out
- Fill the 'in' pixels



Scan Conversion of Polygons

- Works for arbitrary polygons
- Efficiency improvement:
  - *Exploit row-to-row coherence using "edge table"*

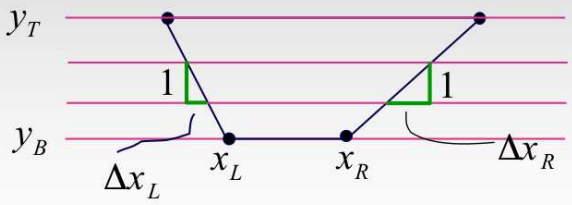


Edge Walking

Past graphics hardware

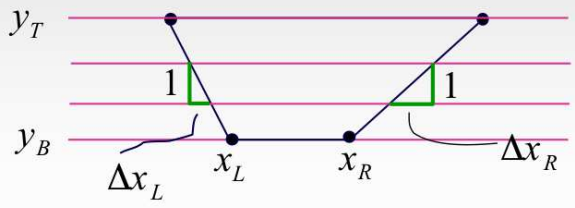
- Exploit continuous L and R edges on trapezoid

scanTrapezoid($x_L, x_R, y_B, y_T, \Delta x_L, \Delta x_R$)



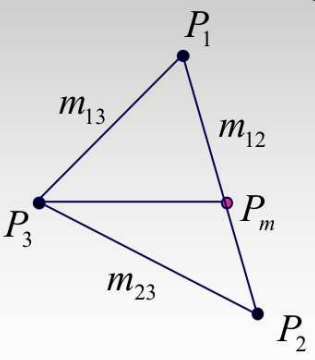
Edge Walking

```
for (y=yB; y<=yT; y++) {
    for (x=xL; x<=xR; x++)
     setPixel(x,y);
    xL += DxL;
    xR += DxR;
}
```



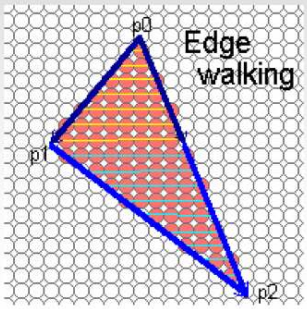
Edge Walking Triangles

- Split triangles into two regions with continuous left and right edges



scanTrapezoid($x_3, x_m, y_3, y_1, \frac{1}{m_{13}}, \frac{1}{m_{12}}$ **)**

scanTrapezoid($x_2, x_2, y_2, y_3, \frac{1}{m_{23}}, \frac{1}{m_{12}}$ **)**

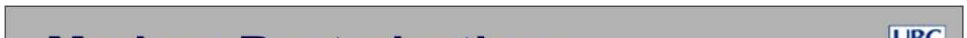


Edge Walking Triangles

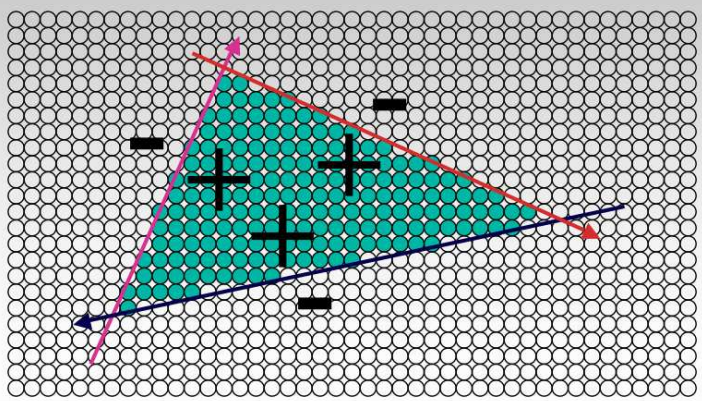
Issues

- Many applications have small triangles
  - *Setup cost is non-trivial*
- Clipping triangles produces non-triangles
  - *This can be avoided through re-triangulation, as discussed*

Modern Rasterization: Edge Equations



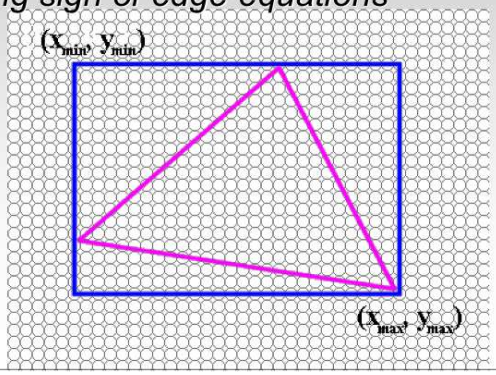
Define a triangle as follows:



Using Edge Equations

Usage:

- Go over each pixel in bounding rectangle
- Check if pixel is inside/outside of triangle
  - *Using sign of edge equations*



Computing Edge Equations

Implicit equation of a triangle edge:

$$L(x,y) = \frac{(y_e - y_s)}{(x_e - x_s)}(x - x_s) - (y - y_s) = 0$$

(see Bresenham algorithm)

- L(x,y) positive on one side of edge, negative on the other

Question:

- How do we know which side is in, and which side out?
  - *And how do we make the L(x,y) positive for points inside?*

Computing Edge Equations

Assumption:

- Triangle vertices given in counter-clockwise order

Then:

- If $x_s < x_e$, then
  - *Use -L(x,y) as edge equation*
- Else
  - *Use +L(x,y) as edge equation*

Computing Edge Equations

Inside/Outside depends on vertex order:

- Implicit equation of a triangle interior:

$$L(x,y) \geq 0$$

with

$$L(x,y) = \begin{cases} -\dfrac{(y_e - y_s)}{(x_e - x_s)}(x - x_s) + (y - y_s), \text{ if } x_s < x_e \\ \dfrac{(y_e - y_s)}{(x_e - x_s)}(x - x_s) - (y - y_s), \text{ if } x_s > x_e \end{cases}$$

What about vertical lines?

- $x_s = x_e \Rightarrow$ division by zero

Computing Edge Equations

Solution:

- We are only interested in the sign of the equation
- Let's multiply the equation by denominator:
  - *$x_s < x_e$: $(x_e - x_s)$ is positive, so the sign is preserved*

$$L(x,y) = (x_e - x_s) \cdot \left(-\frac{(y_e - y_s)}{(x_e - x_s)}(x - x_s) + (y - y_s) \right)$$

$$= -(y_e - y_s)(x - x_s) + (y - y_s)(x_e - x_s)$$

  - *$x_s > x_e$: $(x_e - x_s)$ is negative, multiply by $-(x_e - x_s)$ to preserve sign*

$$L(x,y) = -(x_e - x_s) \cdot \left(\frac{(y_e - y_s)}{(x_e - x_s)}(x - x_s) - (y - y_s) \right)$$

$$= -(y_e - y_s)(x - x_s) + (y - y_s)(x_e - x_s)$$

Computing Edge Equations

Summary:

- Now we have only ONE equation

$$L(x,y) = -(y_e - y_s)(x - x_s) + (y - y_s)(x_e - x_s)$$

- Works for both cases
- Also works for vertical lines!

Interpolation During Scan Conversion

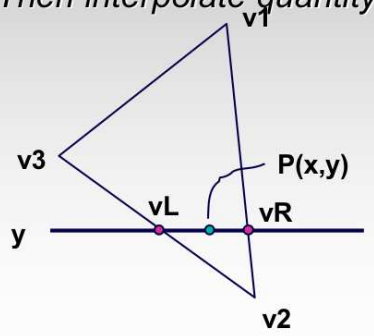
Need to propoagat vertex attributes to pixels

- Interpolate between vertices:
  - *z (depth)*
  - *r,g,b color components*
  - N_x, N_y, N_z *surface normals*
  - *u,v texture coordinates*
    - We'll discuss a better way for these next lecture
- Three equivalent ways of viewing this (for triangles)
  1. *Bilinear interpolation*
  2. *Barycentric coordinates*
  3. *Plane Equation*

1. Bilinear Interpolation

We've seen this before:

- Interpolate quantity along LH and RH edges, as a function of y
  - *Then interpolate quantity as a function of x*



2. Barycentric Coordinates

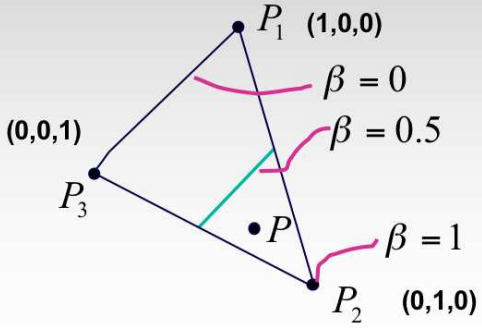
This too:

- Barycentric Coordinates: weighted combination of vertices

$$P = \alpha \cdot P_1 + \beta \cdot P_2 + \gamma \cdot P_3$$

$$\alpha + \beta + \gamma = 1$$

$$0 \le \alpha, \beta, \gamma \le 1$$



3. Plane Equation

Observatiop: Quantities vary linearly across image plane

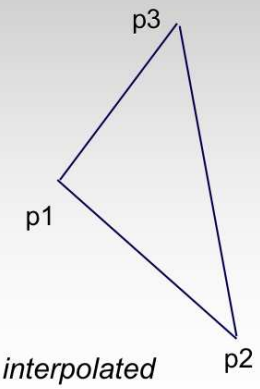
- E.g.: r = Ax + By + C
  - *r= red channel of the color*
  - *Same for g, b, Nx, Ny, Nz, z…*
- From info at vertices we know:

$$r_1 = Ax_1 + By_1 + C$$

$$r_2 = Ax_2 + By_2 + C$$

$$r_3 = Ax_3 + By_3 + C$$

  - *Solve for A, B, C*
  - *One-time set-up cost per triangle and interpolated quantity*



Discussion of Polygon Scan Conversion Algorithms

On old hardware:

- Use first scan-conversion algorithm
  - *Scan-convert edges, then fill in scanlines*
  - *Compute interpolated values by interpolating along edges, then scanlines*
- Requires clipping of polygons against viewing volume
- Faster if you have a few, large polygons
- Possibly faster in software

Discussion of Polygon Scan Conversion Algorithms

Modern GPUs:

- Use edge equations
  - *And plane equations for attribute interpolation*
  - *No clipping of primitives required*
- Faster with many small triangles

Additional advantage:

- Can control the order in which pixels are processed
- Allows for more memory-coherent traversal orders
  - *E.g. tiles or space-filling curve rather than scanlines*

Egde Equation Rasterization and Clipping

Note:

- Once we use edge equations, we no longer really have to clip the geometry against window boundary!
- Instead: clip bounding rectangle against window
  - *Only evaluate edge equations for pixels inside the window!*
- Near/far clipping: when interpolating depth values, detect whether point is closer than near or farther than far
  - *If so, don't draw it*

Triangle Rasterization Issues (Independent of Algorithm)

Exactly which pixels should be lit?

- A: Those pixels inside the triangle edgee (of course)

But what about pixels exactly on the edge?

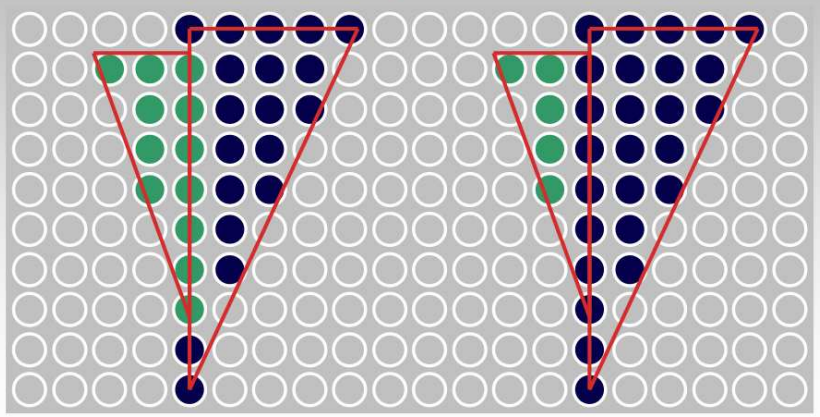
- Draw them: order of triangles matters (it shouldn't)
- Don't draw them: gaps possible between triangles

We need a consistent (if arbitrary) rule

- Example: draw pixels on left or top edge, but not on right or bottom edge

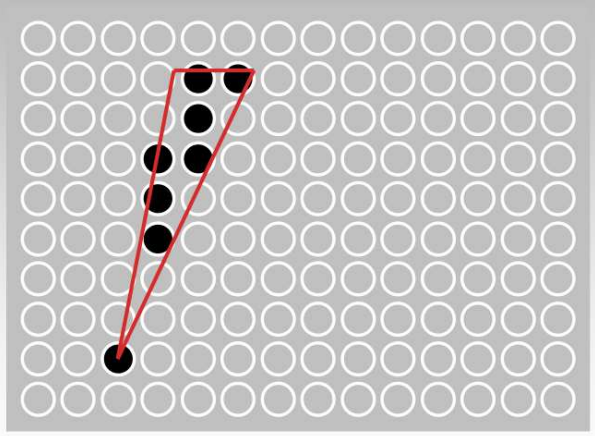
Triangle Rasterization Issues

Shared Edge Ordering



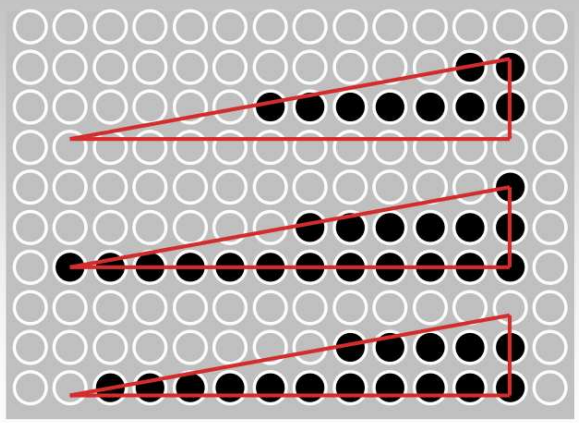
Triangle Rasterization Issues

Sliver



Triangle Rasterization Issues

Moving Slivers



Triangle Rasterization Issues

These are ALIASING Problems

- Problems associated with representing continuous functions (triangles) with finite resolution (pixels)
- More on this problem when we talk about sampling…

Coming Up…

Thursday (Oct 11):

- Visibility

Tuesday (Oct 16):

- Quiz 1 solutions (Brad)

Thursday (Oct 18):

- Double Buffering, Picking, Alpha Blending (Brad)