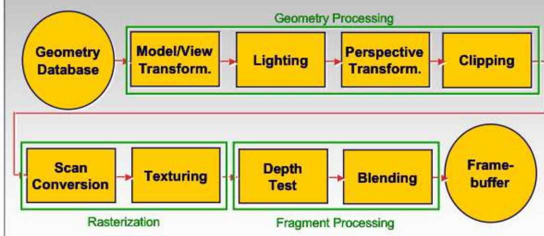
# OpenGL Transformations, Hierarchical Transformations, Accelerations

***CPSC 314***



## The Rendering Pipeline






## Homogeneous Coordinates

***Homogeneous representation of points:***

- Add an additional component $w=1$ to all *points*
- All multiples of this vector are considered to represent the same 3D point
- All points are represented as *column vectors*

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \equiv \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \equiv \begin{bmatrix} x \cdot w \\ y \cdot w \\ z \cdot w \\ w \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix}, \forall w \neq 0$$



## Homogeneous Matrices

***Affine Transformations***

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} & 0 \\ m_{2,1} & m_{2,2} & m_{2,3} & 0 \\ m_{3,1} & m_{3,2} & m_{3,3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} & 0 \\ m_{2,1} & m_{2,2} & m_{2,3} & 0 \\ m_{3,1} & m_{3,2} & m_{3,3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & t_x \\ 0 & 0 & 0 & t_y \\ 0 & 0 & 0 & t_z \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



## Homogeneous Vectors

***Representing vectors in homogeneous coordinates***

- Column vectors with $w=0$

$$T\left(\begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix}\right) = \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} & t_x \\ m_{2,1} & m_{2,2} & m_{2,3} & t_y \\ m_{3,1} & m_{3,2} & m_{3,3} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \\ 0 \end{bmatrix}$$



## Modeling Transformation

***Purpose:***

- Map geometry from local *object coordinate system* into a global *world coordinate system*
- *Same as placing objects*

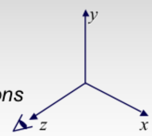
**Transformations:**

- Arbitrary affine transformations are possible
  - *Even more complex transformations may be desirable, but are not available in hardware*
    - Freeform deformations

## Viewing Transformation

### *Purpose:*

- Map geometry from *world coordinate system* into *camera coordinate system*
- Camera coordinate system is *right-handed*, viewing direction is *negative* z-axis
- Same a placing camera




### *Transformations:*

- Usually only *rigid body transformations*
  - *Rotations and translations*
- Objects have same size and shape in camera and world coordinates



## Model/View Transformation

### *Combine modeling and viewing transform.*

- Combine both into a single matrix
- Saves computation time if many points are to be transformed
- Possible because the viewing transformation directly follows the modeling transformation without intermediate operations



## Homogeneous Planes And Normals

### *Planes in Cartesian Coordinates:*

$$\{(x,y,z)^T \mid n_x x + n_y y + n_z z + d = 0\}$$

- $n_x$, $n_y$, $n_z$, and $d$ are the parameters of the plane (normal and distance from origin)

### *Planes in Homogeneous Coordinates:*

$$\{[x,y,z,w]^T \mid n_x x + n_y y + n_z z + dw = 0\}$$



## Homogeneous Planes And Normals

### *Planes in homogeneous coordinates are represented as row vectors*

- E=$[n_x, n_y, n_z, d]$
- Condition that a point $[x,y,z,w]^T$ is located in E

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \in E = [n_x, n_y, n_z, d] \Leftrightarrow [n_x, n_y, n_z, d] \cdot \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = 0$$



## Homogeneous Planes and Normals

### *Example in 2D (lines instead of planes):*

- Line $L$: $y=1-x$
- Implicit definition: $-x-y+1=0$
- Unit-length normal of that line:

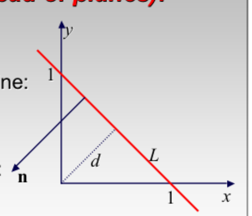
$$\mathbf{n} = \left[\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}, 0\right]^T$$

- Distance of line from origin:

$$d = \sqrt{2}/2$$

- Thus:

$$L = \left[\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right] \equiv [-1,-1,1]$$



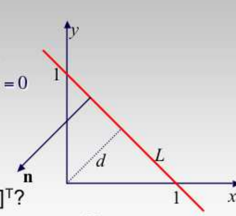



## Homogeneous Planes and Normals

### *Example in 2D (cont.):*

- Is $[1,0,1]^T$ on the line?

$$\left[\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right] \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \frac{-\sqrt{2}}{2} + 0 + \frac{\sqrt{2}}{2} = 0$$




- What about $[0,0,1]^T$, $[1,1,1]^T$?

$$\left[\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right] \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \frac{\sqrt{2}}{2}; \qquad \left[\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right] \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \frac{-\sqrt{2}}{2}$$

## Homogeneous Planes And Normals

### *Transformations of planes*

$$[n_x, n_y, n_z, d] \cdot \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = 0 \Leftrightarrow T([n_x, n_y, n_z, d]) \cdot (\mathbf{A} \cdot \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}) = 0$$



## Homogeneous Planes And Normals

### *Transformations of planes*

$$[n_x, n_y, n_z, d] \cdot \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = 0 \Leftrightarrow ([n_x, n_y, n_z, d] \cdot \mathbf{A}^{-1}) \cdot (\mathbf{A} \cdot \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}) = 0$$

- Works for $T([n_x, n_y, n_z, d]) = [n_x, n_y, n_z, d]\mathbf{A}^{-1}$
- Thus: planes have to be transformed by the *inverse* of the affine transformation (multiplied from left as a row vector)!



## Homogeneous Planes And Normals

### *Homogeneous Normals*

- The plane definition also contains its normal
- Normal written as a vector $[n_x, n_y, n_z, 0]^T$

$$(\begin{bmatrix} n_x \\ n_y \\ n_z \\ 0 \end{bmatrix} \cdot \begin{bmatrix} v_x \\ v_y \\ v_z \\ 0 \end{bmatrix}) = 0 \Leftrightarrow ((\mathbf{A}^{-T} \cdot \begin{bmatrix} n_x \\ n_y \\ n_z \\ 0 \end{bmatrix}) \cdot (\mathbf{A} \cdot \begin{bmatrix} v_x \\ v_y \\ v_z \\ 0 \end{bmatrix})) = 0$$

- Thus: the normal to any surface has to be transformed by the inverse transpose of the affine transformation (multiplied from the right as a column vector)!



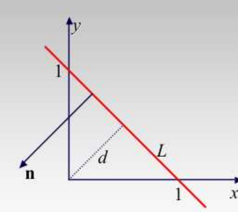
## Transforming Homogeneous Normals

### *Back to 2D example:*

- Before transformation"

$$L = \begin{bmatrix} \frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \end{bmatrix}$$

$$\mathbf{n} = \begin{bmatrix} \frac{-\sqrt{2}}{2} \\ \frac{-\sqrt{2}}{2} \\ 0 \end{bmatrix}$$



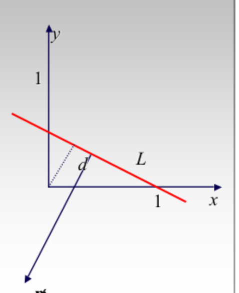



## Transforming Homogeneous Normals

### *Scale by 1/2 in y direction*

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & & \\ & 1/2 & \\ & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = M \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\mathbf{n}' = M^{-T}\mathbf{n} = \begin{bmatrix} 1 & & \\ & 2 & \\ & & 1 \end{bmatrix} \begin{bmatrix} \frac{-\sqrt{2}}{2} \\ \frac{-\sqrt{2}}{2} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{-\sqrt{2}}{2} \\ -\sqrt{2} \\ 0 \end{bmatrix}$$






## Transforming Homogeneous Normals

### *Inverse Transpose of*

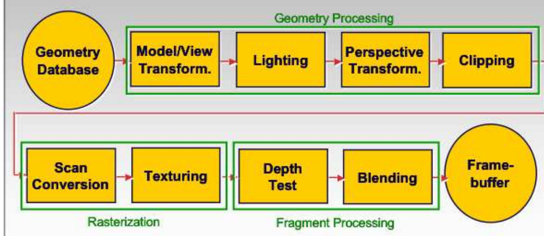
- Rotation by $\alpha$
  - *Rotation by $\alpha$*
- Scale by s
  - *Scale by 1/s*
- Translation by t
  - *Identity matrix!*
- Shear by a along x axis
  - *Shear by -a along y axis*

## OpenGL Transformations, Hierarchical Transformations, Accelerations

***CPSC 314***



## The Rendering Pipeline






## Rendering Geometry in OpenGL

```
glBegin( GL_TRIANGLES );
    glVertex3f( x1, y1, z1 ); // vertex 1 of triangle 1
    glVertex3f( x2, y2, z2 ); // vertex 2 of triangle 1
    glVertex3f( x3, y3, z3 ); // vertex 3 of triangle 1
    glVertex3f( x4, y4, z4 ); // vertex 1 of triangle 2
    glVertex3f( x5, y5, z5 ); // vertex 2 of triangle 2
    glVertex3f( x6, y6, z6 ); // vertex 3 of triangle 2
    …
glEnd();
```



## Rendering Geometry in OpenGL

***Additional attributes***

- glColor3f: RGB color value (0…1 per component)
- glNormal3f: normal vector
- glTexCoord2f: texture coordinate (explained later)

***OpenGL is state machine:***

- Every vertex gets color, normal etc. that corresponds to last specified value



## Rendering Geometry in OpenGL

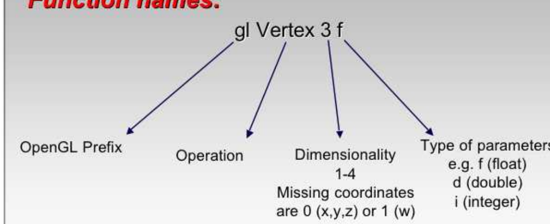
***Example:***

```
glBegin( GL_TRIANGLES );
    glColor3f( 1.0, 0.0, 0.0 );
    glVertex3f( 1.0, 0.0, 0.0 );
    glColor3f( 0.0, 0.0, 1.0 );
    glVertex3f( 0.0, 0.0, 0.0 );
    glVertex3f(1.0, 0.0, 0.0 );
glEnd();
```



## OpenGL Naming Scheme

***Function names:***



## Matrix Operations in OpenGL

***2 Matrices:***

- Model/view matrix M
- Projective matrix P

***Example:***

```
glMatrixMode( GL_MODELVIEW );
glLoadIdentity(); // M=Id
glRotatef( angle, x, y, z ); // M=Id*R(α)
glTranslatef( x, y, z ); // M= Id*R (α)*T(x,y,z)
glMatrixMode( GL_PROJECTION );
glRotatef( … ); // P= …
```



## Matrix Operations in OpenGL

***Semantics:***

- glMatrixMode sets the matrix that is to be affected by all following transformations (multiplication from the right)
- Transformations that affect a vertex *first* have to be specified *last*
- Whenever primitives are rendered with glBegin(), the vertices are transformed with whatever the current model/view and perspective matrix is
  - *Normals are transformed with the inverse transpose*



## Matrix Operations in OpenGL

***Specifying matrices (replacement)***

- glLoadIdentity()
- glLoadMatrixf( GLfloat *m ) // 16 floats

***Specifying matrices (multiplication)***

- glMultMatrixf( GLfloat *m ) // 16 floats
- glRotatef( GLfloat angle, GLfloat x, Glfloat y, GLfloat z ) // angle and axis
- glScalef( GLfloat x, GLfloat y, GLfloat z )
- glTranslatef( GLfloat x, GLfloat y, GLfloat z )



## Matrix Operations in OpenGL

***Perspective Matrices (details next lecture):***

- glFrustum( left, right, bottom, top, near, far )
  - *Specifies perspective xform (near, far are always positive)*
- glOrtho( left, right, bottom, top, near, far )

***Convenience Functions:***

- gluPerspective( fovy, aspect, near, far )
  - *Another way to do perspective*
- gluLookAt( eyeX, eyeY, eyeZ, centerX, centerY, centerZ, upX, upY, upZ )
  - *Useful for viewing transform*

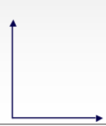


## Interpreting Composite Transformations

***Example for last lecture: Rotation around arbitrary center***

- E.g. rotate 30 degrees around (4,3)
- In OpenGL:

```
glTranslatef( 4, 3 );
glRotatef( 30 );
glTranslatef( -4, -3 );
```



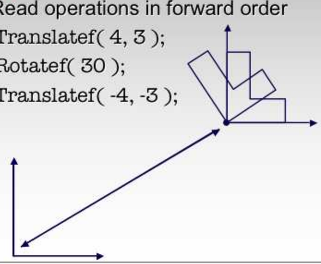


## Interpreting Composite Transformations

***Interpretation 1: moving the coordinate system***

- Read operations in forward order

```
glTranslatef( 4, 3 );
glRotatef( 30 );
glTranslatef( -4, -3 );
```

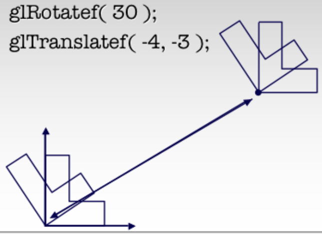


## Interpreting Composite Transformations

***Interpretation 2: moving the object***

- Read operations in reverse order

```
glTranslatef( 4, 3 );
glRotatef( 30 );
glTranslatef( -4, -3 );
```

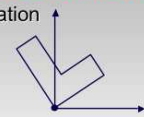




## Compositing of Affine Transformations

***Example: Rotation around arbitrary center***

- Step 2: perform rotation

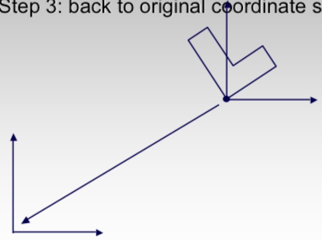




## Compositing of Affine Transformations

***Example: Rotation around arbitrary center***

- Step 3: back to original coordinate system

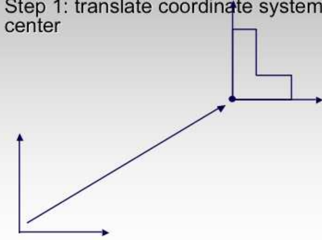




## Compositing of Affine Transformations

***Example: Rotation around arbitrary center***

- Step 1: translate coordinate system to rotation center

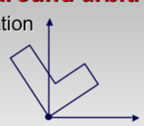




## Compositing of Affine Transformations

***Example: Rotation around arbitrary center***

- Step 2: perform rotation

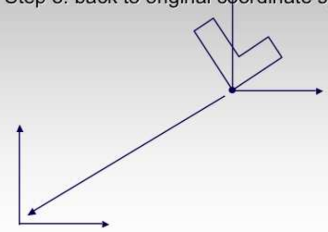




## Compositing of Affine Transformations

***Example: Rotation around arbitrary center***

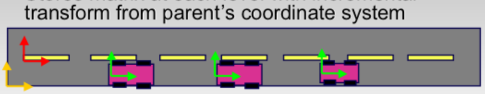
- Step 3: back to original coordinate system



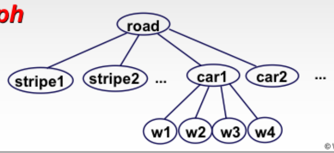
## Transformation Hierarchies

***Scene may have a hierarchy of coordinate systems***

- Stores matrix at each level with incremental transform from parent's coordinate system

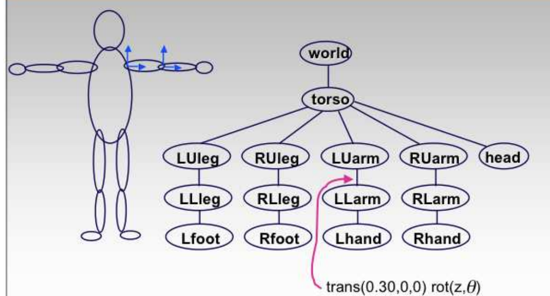


***Scene graph***





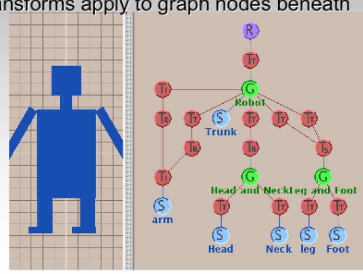

## Transformation Hierarchy Example 1






## Transformation Hierarchies

- Hierarchies don't fall apart when changed
- transforms apply to graph nodes beneath






## Brown Applets

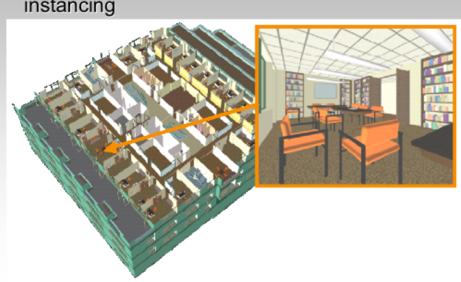
`http://www.cs.brown.edu/exploratories/freeSoftware/catalogs/scenegraphs.html`

- Have a look later



## Transformation Hierarchy Example 2

- Draw same 3D data with different transformations: instancing

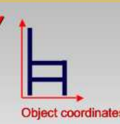


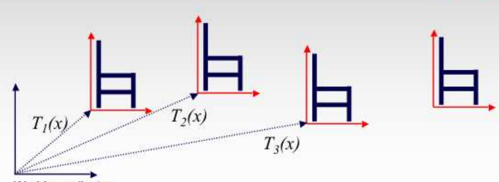


## Matrix Stacks

***Challenge of avoiding unnecessary computation***

- Using inverse to return to origin
- Computing incremental $T_1$ -> $T_2$

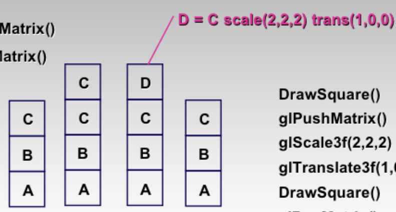


## Matrix Stacks

glPushMatrix()

glPopMatrix()




DrawSquare()
glPushMatrix()
glScale3f(2,2,2)
glTranslate3f(1,0,0)
DrawSquare()
glPopMatrix()



## Modularization

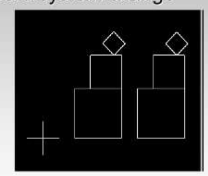
### *Drawing a scaled square*

- Push/pop ensures no coord system change

```
void drawBlock(float k) {
  glPushMatrix();

  glScalef(k,k,k);
  glBegin(GL_LINE_LOOP);
  glVertex3f(0,0,0);
  glVertex3f(1,0,0);
  glVertex3f(1,1,0);
  glVertex3f(0,1,0);
  glEnd();

  glPopMatrix();
}
```





## Matrix Stacks

### *Advantages*

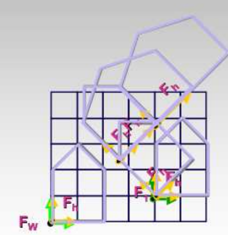
- No need to compute inverse matrices all the time
- Modularize changes to pipeline state
- Avoids incremental changes to coordinate systems
  - *Accumulation of numerical errors*

### *Practical issues*

- In graphics hardware, depth of matrix stacks is limited
  - *(typically 16 for model/view and about 4 for projective matrix)*



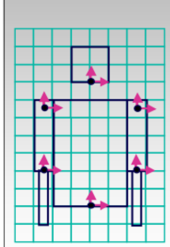
## Transformation Hierarchy Example 3

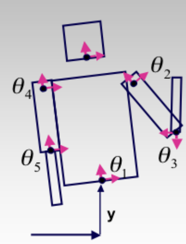



```
glLoadIdentity();
glTranslatef(4,1,0);
glPushMatrix();
glRotatef(45,0,0,1);
glTranslatef(0,2,0);
glScalef(2,1,1);
glTranslate(1,0,0);
glPopMatrix();
```



## Transformation Hierarchy Example 4






```
glTranslate3f(x,y,0);
glRotatef(θ1,0,0,1);
DrawBody();
glPushMatrix();
   glTranslate3f(0,7,0);
   DrawHead();
glPopMatrix();
glPushMatrix();
   glTranslate(2.5,5.5,0);
   glRotatef(θ2,0,0,1);
   DrawUArm();
   glTranslate(0,-3.5,0);
   glRotatef(θ3,0,0,1);
   DrawLArm();
glPopMatrix();
... (draw other arm)
```



## Hierarchical Modeling

### *Advantages*

- Define object once, instantiate multiple copies
- Transformation parameters often good control knobs
- Maintain structural constraints if well-designed

### *Limitations*

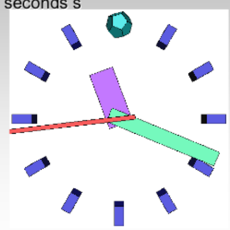
- Expressivity: not always the best controls
- Can't do closed kinematic chains
  - *Keep hand on hip*

## Single Parameter: simple

***Parameters as functions of other params***

- Clock: control all hands with seconds s

m = s/60, h=m/60,
theta_s = (2 pi s) / 60,
theta_m = (2 pi m) / 60,
theta_h = (2 pi h) / 60





## Single Parameter: complex

***Mechanisms not easily expressible with affine transforms***

http://www.flying-pig.co.uk



## Display Lists

***Concept:***

- If multiple copies of an object are required, it can be compiled into a display list:

```
glNewList( listId, GL_COMPILE );
    glBegin( …);
    … // geometry goes here
glEndList();
// render two copies of geometry offset by 1 in z-direction:
glCallList( listId );
glTranslatef( 0.0, 0.0, 1.0 );
glCallList( listId) ;
```



## Display Lists

***Advantages:***

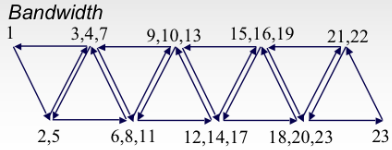
- More efficient than individual function calls for every vertex/attribute
- Can be cached on the graphics board (bandwidth!)
- Display lists exist across multiple frames
  - *Represent static objects in an interactive application*



## Shared Vertices

***Triangle Meshes***

- Multiple triangles share vertices
- If individual triangles are sent to graphics board, every vertex is sent and transformed multiple times!
  - *Computational expense*
  - *Bandwidth*

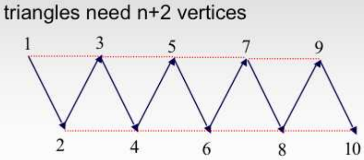





## Triangle Strips

***Idea:***

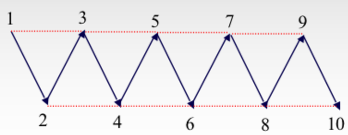
- Encode neighboring triangles that share vertices
- Use an encoding that requires only a constant-sized part of the whole geometry to determine a single triangle
- N triangles need n+2 vertices



## Triangle Strips

### *Orientation:*

- Strip starts with a counter-clockwise triangle
- Then alternates between clockwise and counter-clockwise

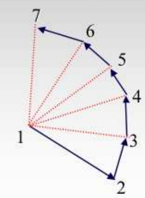





## Triangle Fans

### *Similar concept:*

- All triangles share on center vertex
- All other vertices are specified in CCW order






## Triangle Strips and Fans

### *Transformations:*

- n+2 for n triangles
- Only requires 3 vertices to be stored according to simple access scheme
- Ideal for pipeline (local knowledge)

### *Generation*

- E.g. from directed edge data structure
- Optimize for longest strips/fans

Strippification by Dana Sharon

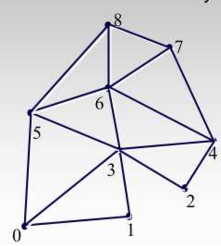


## Vertex Arrays

### *Concept:*

- Store array of vertex data for meshes with aribtrary connectivity (topology)

```
GLfloat *points[3*nvertices];
GLfloat *colors[3*nvertices];
Glint *tris[numtris]=
    {0,1,3,   3,2,4, …};
glVertexPointer( …, points );
glColorPointer( …,colors );
glDrawElements(
    GL_TRIANGLES,…,tris );
```






## Vertex Arrays

### *Benefits:*

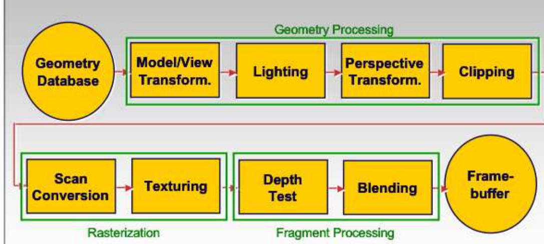
- Ideally, vertex array fits into memory on graphics chip
- Then all vertices are transformed exactly once

### *In practice:*

- Graphics memory may not be sufficient to hold model
- Then either:
  - *Cache only parts of the vertex array on board (may lead to cache trashing!)*
  - *Transform everything in software and just send results for individual triangles (bandwidth problem: multiple transfers of same vertex!)*



## The Rendering Pipeline



## Coming Up…

***Thursday, Sep 20:***

- Perspective transformations

***Tuesday, Sep 25:***

- Lighting