This exam has 5 questions, for a total of 40 points.
1. Implicit, Explicit, and Parametric Equations

(a) (1 point) Give an implicit equation for a 3D sphere of radius $R$, centred at $(x_c, y_c, z_c)$ and having $F(x, y, z) > 0$ inside the sphere.

(b) (3 points) Develop implicit, explicit, and parametric equations for the 2D line passing through the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$. Ensure that your implicit form would not be problematic to compute for any values of $P_1$ and $P_2$, aside from the degenerate case of $P_1 = P_2$.

(c) (1 point) Suppose that barycentric coordinates are defined for the above triangle such that $P(x, y) = \alpha P_1 + \beta P_2 + \gamma P_3$. In the figure of part (a), label the set or region of all points where $\gamma = 0$, $\gamma = 0.5$, and $\gamma = 1$.

(d) (3 points) Develop an equation to compute $\gamma$ for a given point $P(x, y)$, i.e., develop an expression for $\gamma(x, y)$. Also show how $\gamma(x+1, y)$ be computed efficiently knowing the value of $\gamma(x, y)$?
2. (4 points) Lighting

Sketch the ambient, diffuse, specular, and total illumination for the following scene as a function of $x$. Assume the Phong illumination model, i.e.,

$$I = k_a I_a + k_d I_d (N \cdot L) + k_s I_s (R \cdot V)^n,$$

where $k_a = 0.3$, $k_d = 0.7$, $k_s = 0.7$, $I_a = I_d = I_s = 1.0$, $n = 100$. 

![Light and Eye Diagram](image)

$I(x)$ vs $x$ graph showing the illumination function for different values of $x$. The scene is depicted with a light source and an eye at different positions along the x-axis.
3. Short answer
   (a) (12 points) Define the following terms and give an example of a situation where it
       would be used in practice:

       Nyquist rate

       display list

       MIPmapping

       pixel shader

       1d texture map

       cube map

       texture management unit

       ’baked’ lighting

       procedural texture

       BRDF

       depth complexity

       penumbra
(b) (2 points) Give five different uses for the memory on a graphics card.

(c) (2 points) You are developing a virtual museum application using standard projective-rendering provided by OpenGL. Give four techniques that you can use (or that OpenGL may use) to address the removal of polygons or pixels that do not end up being visible.
4. Texture Mapping and Sampling

(a) (5 points) Suppose that you have a texture map of size $128 \times 32$ pixels which models the surface of exactly one brick that has dimensions of $20 \times 5 \text{cm}$. Suppose that your virtual world is modeled in units of metres, and that $z$ is up in your world. Give the OpenGL code to draw a texture mapped polygon that models a brick wall that is $5m$ wide and $2m$ high and lies in the $xz$ plane. You can assume that the brick texture is loaded as the current texture. You can assume that either GL\_CLAMP or GL\_REPEAT mode is correctly enabled for the texture parameters, but you should describe which of these you would choose.

(b) (3 points) Unweighted area sampling assigns a pixel the “average” colour of what that pixel covers. In practice, it can be approximated by ‘supersampling’, i.e., sampling within a pixel using a finer grid of sample points and then taking the average of these colour samples. Will this eliminate aliasing artifacts? If not, how can it be improved?
5. Miscellaneous

(a) (1 point) Suppose we wish to develop barycentric coordinates for a quadrilateral, i.e., \( P(x, y) = \alpha P_1 + \beta P_2 + \gamma P_3 + \zeta P_4 \), where \( 0 \leq \alpha, \beta, \gamma, \zeta \leq 1 \) for all points inside the polygon. Show that each point in the polygon does or does not have a unique set of barycentric coordinates \((\alpha, \beta, \gamma, \zeta)\).

(b) (1 point) Why is it difficult to correctly model the illumination of skin and marble?

(c) (2 points) Describe some of the challenges you might encounter when using shadow maps to produce shadows.