Scan Conversion

Projective Rendering Pipeline

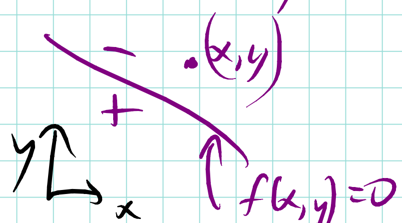
- OCS → **modeling transformation** → WCS → **viewing transformation** → VCS → **projection transformation** → CCS → **/ h** → NDCS → **viewport transformation** → DCS
- 4x4 matrix (modeling and viewing transformations)
- 4x4 matrix (projection transformation)

OCS - object coordinate system
WCS - world coordinate system
VCS - viewing coordinate system
CCS - clipping coordinate system
NDCS - normalized device coordinate system
DCS - device coordinate system

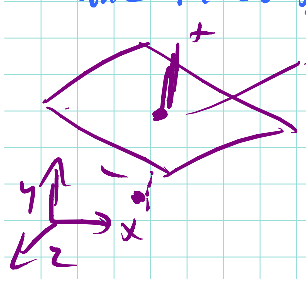
① Implicit

Line in 2D space

$0 = mx + b - y$



Plane in 3D space



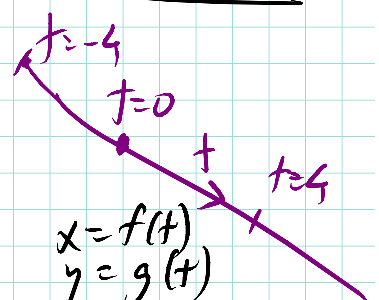
$f(x,y,z)=0$ on the plane

② Explicit

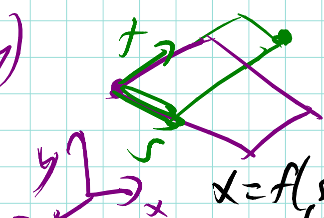
$y = f(x)$
$= mx + b$

$z = f(x,y)$

③ Parametric



$x = f(t)$
$y = g(t)$



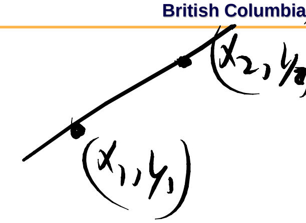
$x = f(s,t)$
$y = g(s,t)$
$z = h(s,t)$

Lines and Curves

Explicit

line $y = mx + b$

$$y = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1) + y_1$$



circle $y = \pm\sqrt{r^2 - x^2}$

plane $z = Ex + Fy + G$

sphere $z = \pm\sqrt{r^2 - x^2 - y^2}$

comes from $x^2 + y^2 + z^2 = r^2$

Lines and Curves

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Parametric

line

$$x(t) = x_0 + t(x_1 - x_0)$$
$$y(t) = y_0 + t(y_1 - y_0)$$
$$t \in [0,1]$$
$$\boxed{P(t) = P_0 + t(P_1 - P_0)}$$
$$P(t) = (1-t)P_0 + t\,P_1$$

Same

"basis functions"

P_0 (x_0, y_0) $t=0$

P_1 (x_1, y_1) $t=1$

$p(t)$ $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$

circle

$$x(\theta) = r\cos(\theta)$$
$$y(\theta) = r\sin(\theta)$$
$$\theta \in [0, 2\pi]$$

r θ

plane $P(s,t) = P_0 + s(P_1 - P_0) + t(P_2 - P_0)$
where P_0, P_1, P_2 are pts on plane

Lines and Curves

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Implicit

$F(x,y) = ax + by + c$

line $\cancel{F(x,y) = (x - x_0)dy - (y - y_0)dx}$

$F(x,y) = 0$ **(x,y) is on line**

$F(x,y) > 0$ **(x,y) is ~~below line~~** is on one side

$F(x,y) < 0$ **(x,y) is ~~above line~~** is on the other

dx dy (x_0, y_0)

circle $F(x,y) = x^2 + y^2 - r^2$

$x^2 + y^2 = r^2$

$0 = x^2 + y^2 - r^2$

$F(x,y) = 0$ **(x,y) is on circle**

$F(x,y) > 0$ **(x,y) is outside**

$F(x,y) < 0$ **(x,y) is inside**

plane: $\boxed{F(x,y,z) = Ax + By + Cz + D = N \cdot P + D}$

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Polygons (really triangles!)

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Interactive graphics uses Polygons

- Can represent any surface *with arbitrary accuracy*
  - *Splines, mathematical functions, ...*
- simple, regular rendering algorithms
  - *embed well in hardware*

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Polygons

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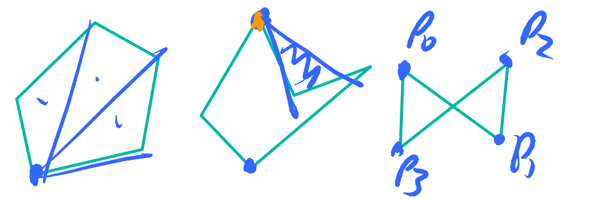
Basic Types



simple convex

simple concave

non-simple (self-intersection)

From Polygons to Triangles

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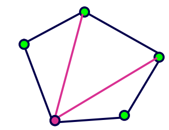
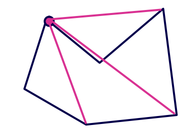
- why? triangles are planar and convex



- simple convex polygons
  - *break into triangles, trivial*
  - `glBegin(GL_POLYGON) ... glEnd()`



- concave or non-simple polygons
  - *break into triangles, more effort*
  - `gluNewTess(), gluTessCallback(), ...`

What is Scan Conversion? (a.k.a. Rasterization)

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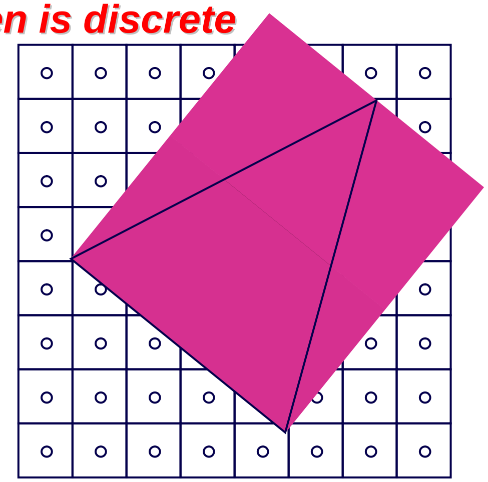
screen is discrete



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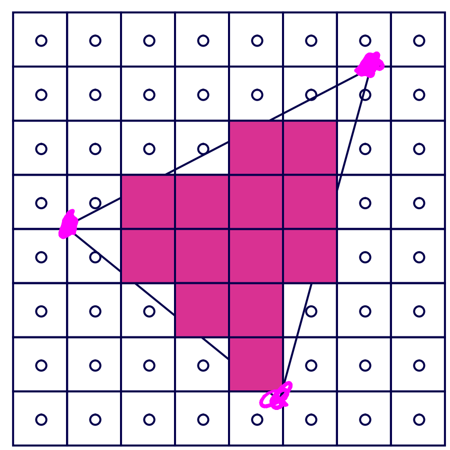
one possible scan conversion



Scan Conversion

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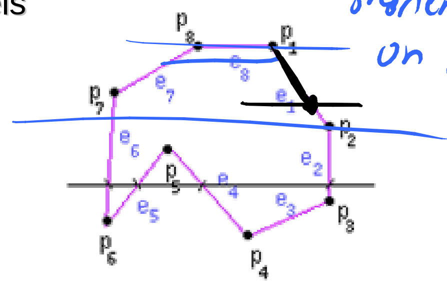
A General Algorithm

- intersect each scanline with all edges
- sort intersections in x
- calculate parity to determine in/out
- fill the 'in' pixels

- ignore horizontal edges
- shorten each edge on one side

1, 2, 3, 4, 5, 6



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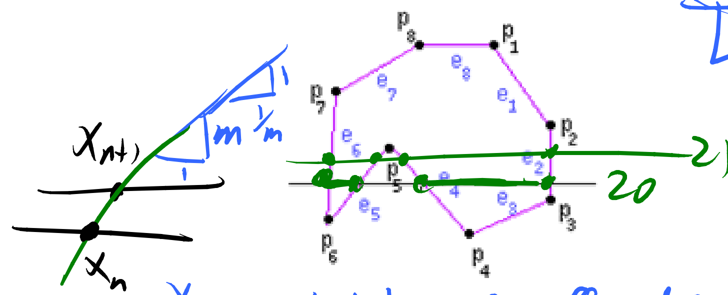
- works for arbitrary polygons
- efficiency improvement:
  - *exploit row-to-row coherence using "edge table"*

for bookkeeping of when edges start/stop



use $x_{n+1} = x_n + 1/m$ for efficient intersection update

Edge Walking

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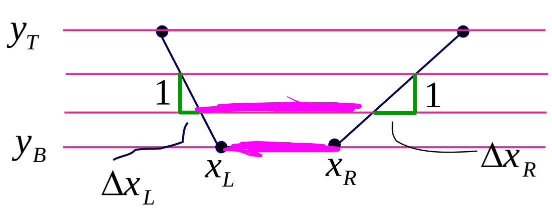
past graphics hardware

- exploit continuous L and R edges on trapezoid

scanTrapezoid($x_L, x_R, y_B, y_T, \Delta x_L, \Delta x_R$)



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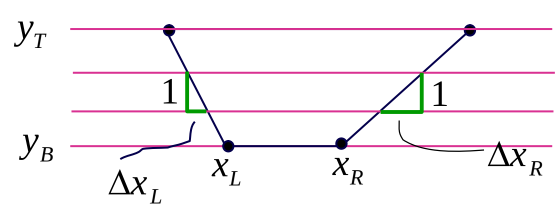
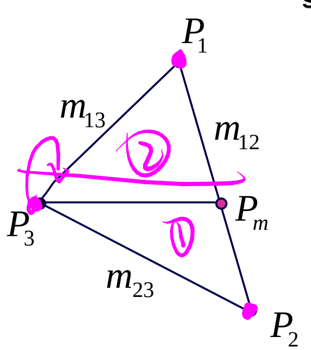
```
for (y=yB; y<=yT; y++) {
  for (x=xL; x<=xR; x++)
    setPixel(x,y);
  xL += DxL;
  xR += DxR;
}
```



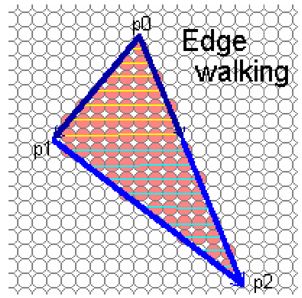
Edge Walking Triangles

- split triangles into two regions
  with continuous left and right edges



scanTrapezoid($x_3, x_m, y_3, y_1, \frac{1}{m_{13}}, \frac{1}{m_{12}}$ **)** 2

scanTrapezoid($x_2, x_2, y_2, y_3, \frac{1}{m_{23}}, \frac{1}{m_{12}}$ **)** 1



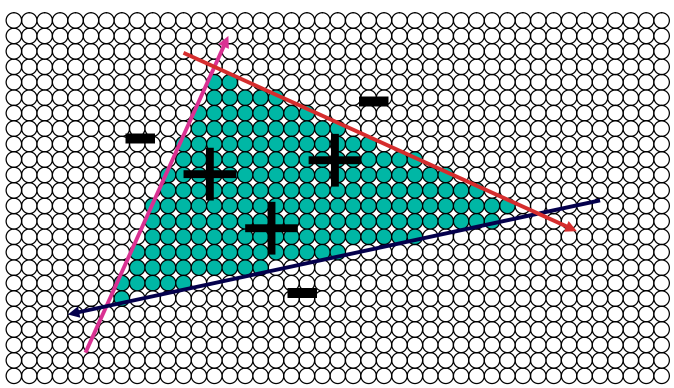
Edge Walking Triangles

Issues

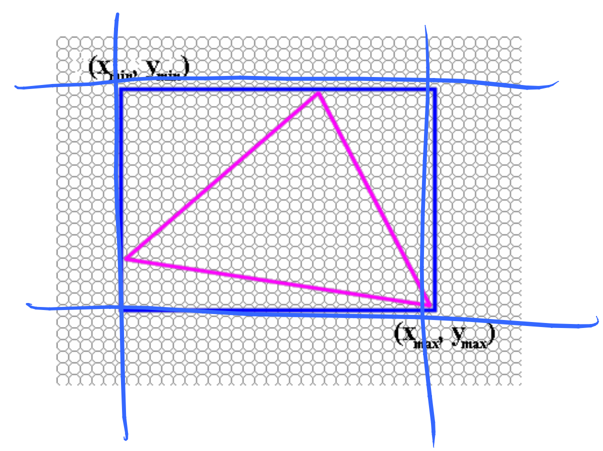
- many applications have small triangles
  - *setup cost is non-trivial*
- clipping triangles produces non-triangles

Modern Rasterization

Define a triangle as follows:



Using Edge Equations



Computing Edge Equations

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Computing A,B,C from $(x_1, y_1), (x_2, y_2)$

$Ax_1 + By_1 + C = 0$

$Ax_2 + By_2 + C = 0$

$Ax_3 + By_3 + C = 1$

- ~~two~~ three equations, three unknowns
- ~~solve for A & B in terms of C~~

Computing Edge Equations

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$$\begin{bmatrix} x_0 & y_0 \\ x_1 & y_1 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = -C \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \end{bmatrix} = \frac{-C}{x_0 y_1 - x_1 y_0} \begin{bmatrix} y_1 - y_0 \\ x_1 - x_0 \end{bmatrix}$$

- choose $C = x_0 y_1 - x_1 y_0$ for convenience
- Then $A = y_0 - y_1$ and $B = x_1 - x_0$

Edge Equations

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- So…we can find edge equation from two verts.
- Given $\mathbf{P_0}, \mathbf{P_1}, \mathbf{P_2}$, what are our three edges?

  How do we make sure the half-spaces defined by the edge equations all share the same sign on the interior of the triangle?
- A: Be consistent (Ex: $[\mathbf{P_0 P_1}], [\mathbf{P_1 P_2}], [\mathbf{P_2 P_0}]$)

  How do we make sure that sign is positive?
- A: Test, and flip if needed ($A = -A$, $B = -B$, $C = -C$)

Edge Equations: Code

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Basic structure of code:

- Setup: compute edge equations, bounding box
- (Outer loop) For each scanline in bounding box...
- (Inner loop) …check each pixel on scanline, evaluating edge equations and drawing the pixel if all three are positive

Edge Equations: Code

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```
findBoundingBox(&xmin, &xmax, &ymin, &ymax);
setupEdges (&a0,&b0,&c0,&a1,&b1,&c1,&a2,&b2,&c2);

for (int y = yMin; y <= yMax; y++) {
      for (int x = xMin; x <= xMax; x++) {
            float e0 = a0*x + b0*y + c0;
            float e1 = a1*x + b1*y + c1;
            float e2 = a2*x + b2*y + c2;
            if (e0 > 0 && e1 > 0 && e2 > 0)
                        Image[x][y] = TriangleColor;
      }
}
```


Edge Equations: Code

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```
  // more efficient inner loop
for (int y = yMin; y <= yMax; y++) {
      float e0 = a0*xMin + b0*y + c0;
      float e1 = a1*xMin + b1*y + c1;
      float e2 = a2*xMin + b2*y + c2;
      for (int x = xMin; x <= xMax; x++) {
            if (e0 > 0 && e1 > 0 && e2 > 0)
                  Image[x][y] = TriangleColor;
            e0 += a0;   e1+= a1;    e2 += a2;
      }
}
```


Triangle Rasterization Issues

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Exactly which pixels should be lit?

A: Those pixels inside the triangle edges

What about pixels exactly on the edge?

- Draw them: order of triangles matters (it shouldn't)
- Don't draw them: gaps possible between triangles

We need a consistent (if arbitrary) rule

- Example: draw pixels on left or top edge, but not on right or bottom edge

Triangle Rasterization Issues

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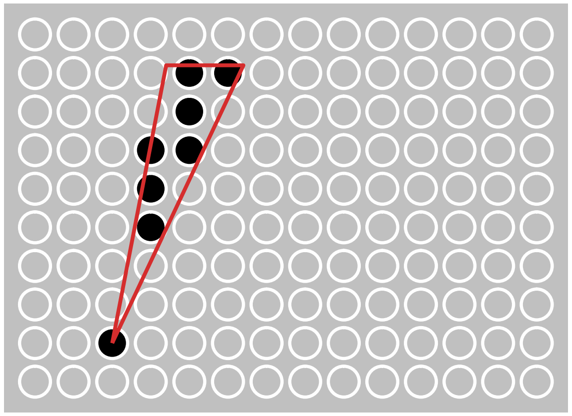
Sliver



Triangle Rasterization Issues

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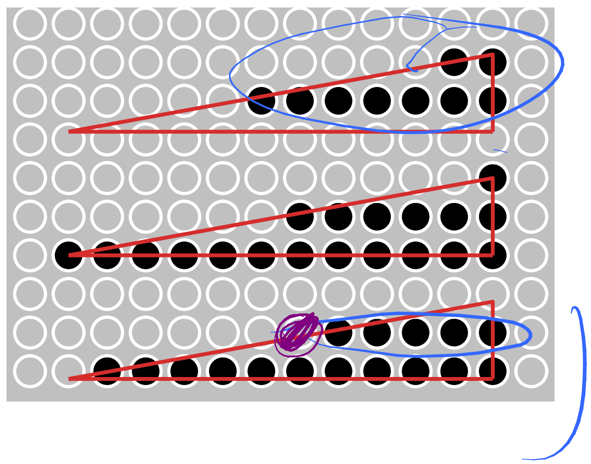
Moving Slivers



Triangle Rasterization Issues

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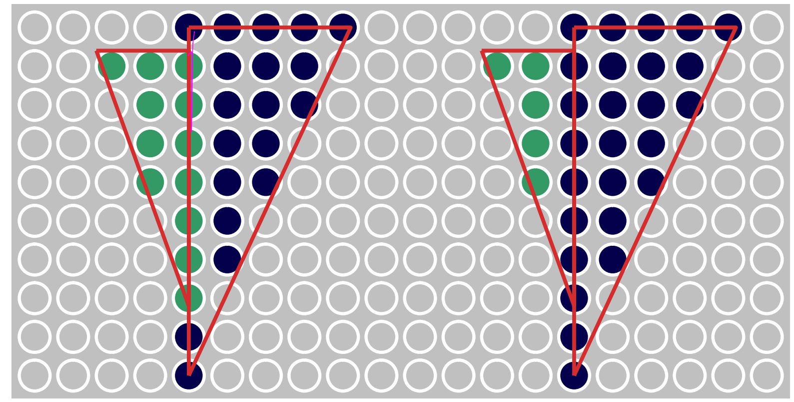
Shared Edge Ordering



Interpolation During Scan Conversion

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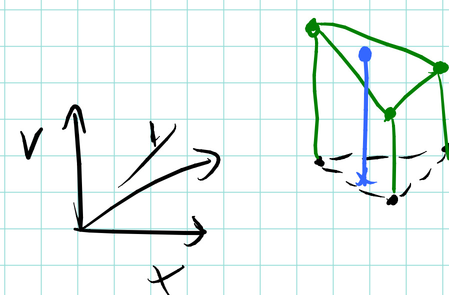
- interpolate between vertices: (demo)
  - *z*
  - *r,g,b*   *colour components*
  - *u,v*   *texture coordinates*
  - N_x, N_y, N_z *surface normals*

"v"

- three equivalent ways of viewing this (for triangles)
  1. *bilinear interpolation*
  2. *plane equation*
  3. *barycentric coordinates*



1. Bilinear Interpolation

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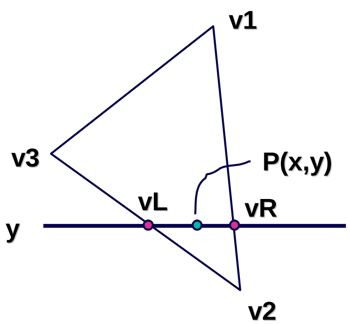
- interpolate quantity along LH and RH edges, as a function of y
  - *then interpolate quantity as a function of x*



2. Plane Equation

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- $v = A'x + B'y + C'$

Plane equation (3D): $Ax + By + Cv + D = 0$

$\Rightarrow$ solve for v:

$$v = \frac{-Ax - By - D}{C}$$

Building Plane Equations

$Ax + By + Cz + D = 0$

$\langle A, B, C\rangle \cdot \langle x, y, z\rangle + D = 0$

$\vec{N} \cdot \vec{P} + D = 0$

(1) compute A, B, C by computing a normal

e.g. $\vec{N} = (P_3 - P_1) \times (P_2 - P_1)$

(2) compute $D = -N \cdot P$ for any point P on triangle

eg. $D = -N \cdot P_1$

3. Barycentric Coordinates

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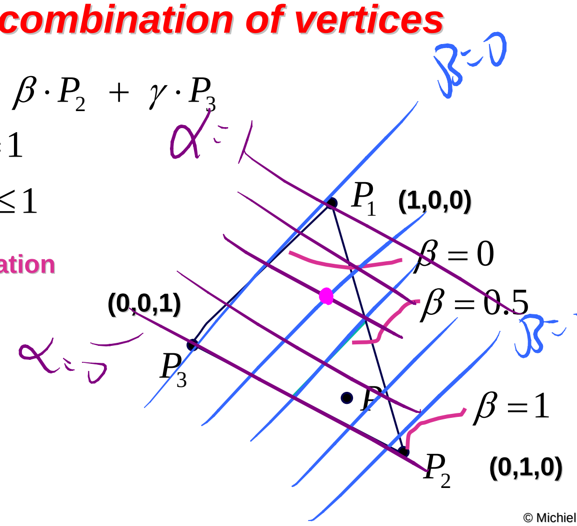
•weighted combination of vertices

$$\begin{cases} P = \alpha \cdot P_1 + \beta \cdot P_2 + \gamma \cdot P_3 \\ \alpha + \beta + \gamma = 1 \\ 0 \le \alpha, \beta, \gamma \le 1 \end{cases}$$

"convex combination of points"



Barycentric Coordinates

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- once computed, use to interpolate any # of parameters from their vertex values

$$z = \alpha \cdot z_1 + \beta \cdot z_2 + \gamma \cdot z_3$$
$$r = \alpha \cdot r_1 + \beta \cdot r_2 + \gamma \cdot r_3$$
$$g = \alpha \cdot g_1 + \beta \cdot g_2 + \gamma \cdot g_3$$

etc.

Computing Barycentric Coords

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