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Ray-Tracing

CPSC 314

Michiel van de Panne

Ray-Tracing

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CAD Raytraced Image of Audi R8C

Raytracing

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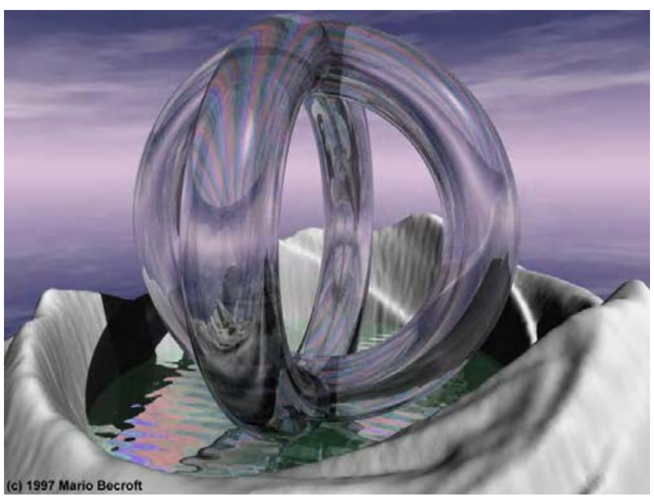

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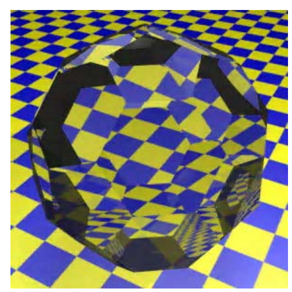

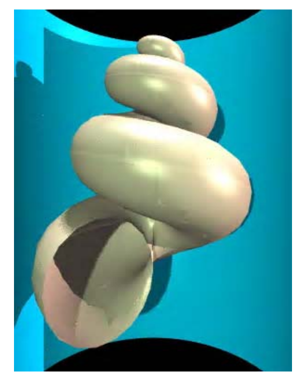

Overview

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So far

- projective rendering (hardware)
- radiosity

Ray-Tracing

- simple algorithm for software rendering
- extremely flexible
- well suited to transparent and specular objects
- global illumination (*)
- partly physics-based: geometric optics

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```
raytrace( ray ) {
   find closest intersection
   cast shadow ray, calculate colour_local
R  colour_reflect = raytrace( reflected_ray )
T  colour_refract = raytrace( refracted_ray )
   colour = k1*colour_local +
            k2*colour_reflect +
            k3*colour_refract
   return( colour )
}
```

- "raycasting" : only cast first ray from eye

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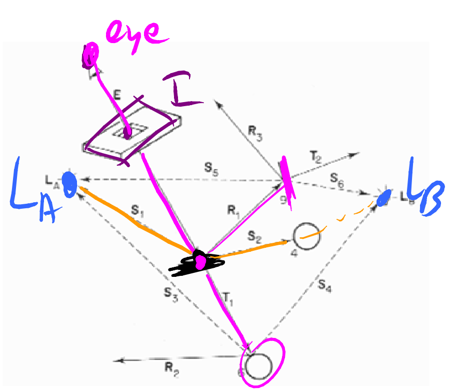

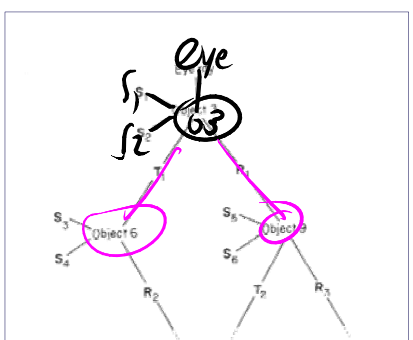

Fig. 12. The ray tree in schematic form.

Figure from Andrew S. Glassner, "An Overview of Ray Tracing" in An Introduction to Ray Tracing, Andrew Glassner, ed., Academic Press Limited, 1989.

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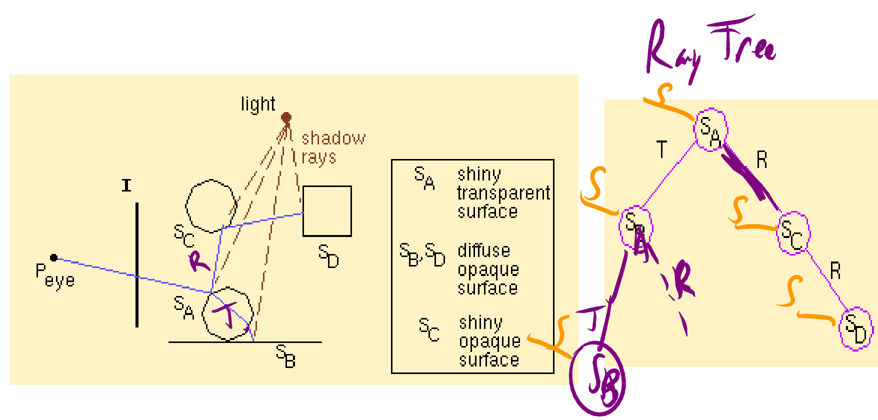

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Ray Termination Criteria:

- **ray hits a diffuse surface**
- **ray exits the scene**
- **threshold on contrib. towards final pixel colour**
- **maximum recursion depth**

Ray-Tracing – Generation of Rays

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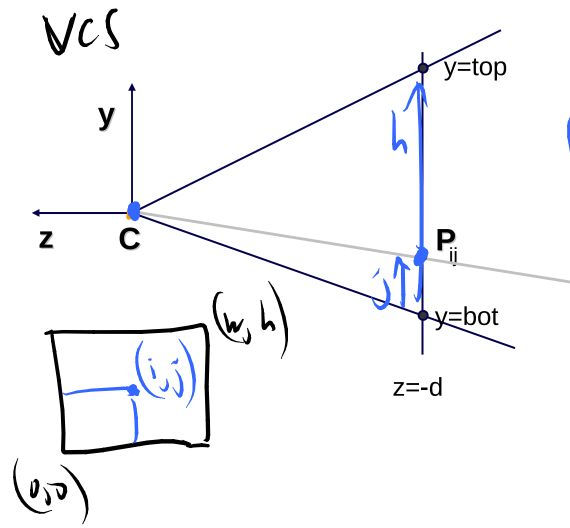
Camera Coordinate System



$C(0,0,0)$

$P_{ij}(x_{ij}, y_{ij}, z_{ij})$

$z_{ij} = -d$

$y_{ij} = bot + \frac{j}{h}(top - bot)$

$x_{ij} = left + \frac{i}{w}(right - left)$

Ray-Tracing – Generation of Rays

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Ray in 3D Space:

$$\mathrm{R}_{i,j}(t) = C + t \cdot (P_{i,j} - C)$$
$$= C + t \cdot v_{i,j}$$

where $t = 0 \ldots \infty$

Task:

- Given an object o, find ray parameter t, such that $\mathbf{R}_{i,j}(t)$ is a point on the object
  - *Such a value for t may not exist*
- Intersection test depends on geometric primitive

Ray Intersections

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Sphere at origin:

- Implicit function:

$$S(x, y, z): x^2 + y^2 + z^2 = r^2$$

- Ray equation:

$$\mathrm{R}_{i,j}(t) = C + t \cdot \mathbf{v}_{i,j} = \begin{pmatrix} c_x \\ c_y \\ c_z \end{pmatrix} + t \cdot \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} c_x + t \cdot v_x \\ c_y + t \cdot v_y \\ c_z + t \cdot v_z \end{pmatrix}$$

Ray Intersections

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To determine intersection:

- Insert ray $\mathbf{R}_{i,j}(t)$ into $S(x,y,z)$:

$$(c_x + t \cdot v_x)^2 + (c_y + t \cdot v_y)^2 + (c_z + t \cdot v_z)^2 = r^2$$

- Solve for t (find roots)
  - *Simple quadratic equation*

Ray Intersections

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Triangles:

$$P = \overset{\alpha}{(1-\beta-\gamma)}\, a + \beta b + \gamma c$$

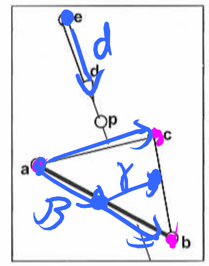

Figure 10.5. The ray hits the plane containing the triangle at point **p**.

ray

point on plane containing triangle

$$\mathbf{e} + t\mathbf{d} = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a}).$$

$$x_e + t x_d = x_a + \beta(x_b - x_a) + \gamma(x_c - x_a),$$
$$y_e + t y_d = y_a + \beta(y_b - y_a) + \gamma(y_c - y_a),$$
$$z_e + t z_d = z_a + \beta(z_b - z_a) + \gamma(z_c - z_a).$$

$$\begin{bmatrix} x_a - x_b & x_a - x_c & x_d \\ y_a - y_b & y_a - y_c & y_d \\ z_a - z_b & z_a - z_c & z_d \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} x_a - x_e \\ y_a - y_e \\ z_a - z_e \end{bmatrix}.$$

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Triangle Intersection (cont.)

Cramer's rule gives us

$$\beta = \frac{j(ei - hf) + k(gf - di) + l(dh - eg)}{M},$$

$$\gamma = \frac{i(ak - jb) + h(jc - al) + g(bl - kc)}{M},$$

$$t = -\frac{f(ak - jb) + e(jc - al) + d(bl - kc)}{M},$$

where

$$M = a(ei - hf) + b(gf - di) + c(dh - eg).$$

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Triangle intersection (cont.): check bounds

Check

$0 \le \beta \le 1$

$0 \le \gamma \le 1$

$0 \le 1 - \beta - \gamma \le 1$

$t > 0$

Ray-Tracing – Geometric Transformations

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Ray Transformation:

- For intersection test, it is only important that ray is in same coordinate system as object representation
- Transform ray into object coordinates $\vec{c}$ $\vec{v}$ or P_i
  - *Transform camera point and ray direction by* *<u>inverse</u> of model/view matrix*
- Shading has to be done in world coordinates (where light sources are given)
  - *Transform object space intersection point to world coordinates*
  - *Thus have to keep both world and object-space ray*