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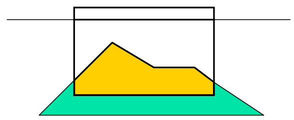
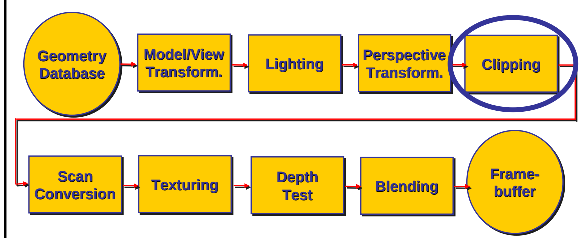
Chapter 9

Clipping



The Rendering Pipeline



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Line/Polygon Clipping (2D)

Problem:
Given a *2D* line/polygon and a window, clip the line/polygon to their regions that are *inside* the window.

Objectives

- Efficiency
- Memory access

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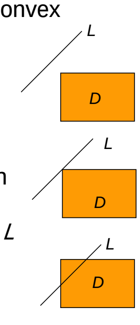
Analytic Solution

- *Intersection* of convex regions is convex
  - Why?
- *L* & *D* are *convex* - intersection is convex
  - single connected segment of *L*
- **Question:** Can boundary of two convex shapes intersect more than twice?
- Clipping - compute intersection of *L* with four boundary segments of window *D*



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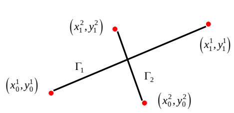
Line-Line Intersection



$$G_1 = \begin{cases} x^1(t) = x_0^1 + (x_1^1 - x_0^1)t \\ y^1(t) = y_0^1 + (y_1^1 - y_0^1)t \end{cases} \quad t \in [0,1] \qquad G_2 = \begin{cases} x^2(r) = x_0^2 + (x_1^2 - x_0^2)r \\ y^2(r) = y_0^2 + (y_1^2 - y_0^2)r \end{cases} \quad r \in [0,1]$$

Intersection: *x* & *y* values equal in both representations - two linear equations in two unknowns (*r*,*t*)

$$x_0^1 + (x_1^1 - x_0^1)t = x_0^2 + (x_1^2 - x_0^2)r$$
$$y_0^1 + (y_1^1 - y_0^1)t = y_0^2 + (y_1^2 - y_0^2)r$$

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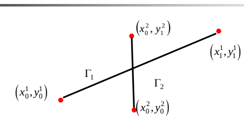
Intersection with vertical/horizontal lines



$$G_1 = \begin{cases} x^1(t) = x_0^1 + (x_1^1 - x_0^1)t \\ y^1(t) = y_0^1 + (y_1^1 - y_0^1)t \end{cases} t \in [0,1] G_2 = \begin{cases} x^2(r) = x_0^2 \\ y^2(r) = y_0^2 + (y_1^2 - y_0^2)r \end{cases} r \in [0,1]$$

Intersection: *x* & *y* values equal in both representations - two linear equations in two unknowns (*r*,*t*)

$$x_0^1 + (x_1^1 - x_0^1)t = x_0^2$$
$$t = \frac{x_0^2 - x_0^1}{x_1^1 - x_0^1}$$
$$y_0^1 + (y_1^1 - y_0^1)t = y_0^2 + (y_1^2 - y_0^2)r$$

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Cohen-Sutherland Algorithm

Purpose:
Fast treatment of line segments that are trivially inside/outside window.

0101	0100	0110
0001	0000	0010
1001	1000	1010

$P = (x,y)$ - point to be classified against window D

Idea: Assign to P a binary code consisting of a bit for each edge of D, using lookup table:

bit	1	0
1	$y < y_{\min}$	$y \geq y_{\min}$
2	$y > y_{\max}$	$y \leq y_{\max}$
3	$x > x_{\max}$	$x \leq x_{\max}$
4	$x < x_{\min}$	$x \geq x_{\min}$

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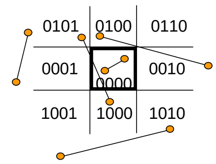
Cohen-Sutherland Algorithm (cont'd)

Given L from (x_0, y_0) to (x_1, y_1) & rectangle D.

If bitwise **and** of the codes of (x_0, y_0) and (x_1, y_1) is not zero, or the bitwise **or** is zero,

then L can be trivially handled (it is either totally outside or totally inside D).



Why?

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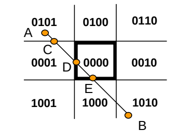
Cohen-Sutherland Algorithm (cont'd)

$\textbf{C-S-Clip}(P_0 = (x_0, y_0), P_1 = (x_1, y_1), x_{\min}, x_{\max}, y_{\min}, y_{\max})$
$C_0 \Leftarrow \text{code}(P_0)$; $C_1 \Leftarrow \text{code}(P_1)$;
if $((C_0$ and $C_1) \mathrel{!=} 0)$ then return;
if $((C_0$ or $C_1) == 0)$ then draw(P_0, P_1);
else if (OutsideWindow(P_0)) then
begin
$Edge \Leftarrow$ Window boundary of leftmost non-zero bit of C_0;
$P_2 \Leftarrow \overline{P_0, P_1} \cap Edge$;
$\textbf{C-S-Clip}(P_2, P_1, x_{\min}, x_{\max}, y_{\min}, y_{\max})$;
end
else
$Edge \Leftarrow$ Window boundary of leftmost non-zero bit of C_1;
$P_2 \Leftarrow \overline{P_0, P_1} \cap Edge$;
$\textbf{C-S-Clip}(P_0, P_2, x_{\min}, x_{\max}, y_{\min}, y_{\max})$;
end



AB → CB → DB → DE

bit	1	0
1	$y < y_{\min}$	$y \geq y_{\min}$
2	$y > y_{\max}$	$y \leq y_{\max}$
3	$x > x_{\max}$	$x \leq x_{\max}$
4	$x < x_{\min}$	$x \geq x_{\min}$

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3D clipping

- Determine portion of line inside axis-aligned parallelpiped (viewing frustum in NDC)
- Simple extension to 2D algorithms
- After perspective transform
  - means that clipping volume always the same
    - xmin=ymin= -1, xmax=ymax= 1 in OpenGL
  - boundary lines become boundary planes
    - but bit-codes still work the same way
    - additional front and back clipping plane
      - zmin = -1, zmax = 1 in OpenGL

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Triangle Clipping

- How does intersection of rectangle & triangle looks like?
  - How many sides?
- How to expand clipping to triangles?
  - Hint: it is convex
  - Will develop on the board...

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Cohen-Sutherland Algorithm for convex polygons

$\textbf{C-S-Clip}(\text{poly} = P_0, \ldots, P_n, x_{\min}, x_{\max}, y_{\min}, y_{\max})$
for i = 1 to n $C_i \Leftarrow \text{code}(P_i)$;
if $((C_0$ and C_1 and...and $C_n) \mathrel{!=} 0)$ then return;
if $((C_0$ or C_1 or...or $C_n) == 0)$ then draw(poly);
else
for i = 1 to n if (OutsideWindow(P_i)) then
begin
$Edge \Leftarrow$ Window boundary of leftmost non-zero bit of C_i;
$P_{i-1,i} \Leftarrow \overline{P_{i-1}, P_i} \cap Edge$;
$P_{i,i+1} \Leftarrow \overline{P_i, P_{i+1}} \cap Edge$;
$\textbf{C-S-Clip}(P_0, \ldots, P_{i-1}, P_{i-1,i}, P_{i,i+1} P_{i+1}, \ldots, P_n, x_{\min}, x_{\max}, y_{\min}, y_{\max})$;
end

bit	1	0
1	$y < y_{\min}$	$y \geq y_{\min}$
2	$y > y_{\max}$	$y \leq y_{\max}$
3	$x > x_{\max}$	$x \leq x_{\max}$
4	$x < x_{\min}$	$x \geq x_{\min}$

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