

CPSC 314

Assignment 0

Due 4pm, Friday September, 23 2005
(handin box in the CICSR basement)

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Name: _____

Student Number: _____

Theory	/ 5
Question 1	/ 0
Question 2	/ 1
Question 3	/ 1
Question 4	/ 1
Question 5	/ 1
Question 6	/ 1
Question 7 (bonus)	/ 2
Programming Assignment	/ 5
TOTAL	/ 10

- (0 points) Print the plagiarism policy form from the web (<http://www.ugrad.cs.ubc.ca/~cs314/Vsep2005/plag.html>). Sign it and submit with your assignment. **Assignments without the form, will not be checked.**
- (1 point) Vectors

$$a = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} \quad b = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$$

- compute $a \cdot b = (1 * 2) + (3 * 1) + (-1 * 5) = 0$
- compute $a^T b = (1 * 2) + (3 * 1) + (-1 * 5) = 0$
- compute $a \times b$

$$\begin{pmatrix} 3 * 5 - (-1) * 1 \\ -1 * 2 - 1 * 5 \\ 1 * 1 - 3 * 2 \end{pmatrix} = \begin{pmatrix} 16 \\ -7 \\ -5 \end{pmatrix}$$

- compute $b \times a$

$$\begin{pmatrix} -1 * (-1) - 5 * 3 \\ 1 * 5 - (-1) * 2 \\ 3 * 2 - 1 * 1 \end{pmatrix} = \begin{pmatrix} -16 \\ 7 \\ 5 \end{pmatrix} = -a \times b$$

- (1 point) Matrices

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 4 \\ 3 & -1 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 3 & 1 & 6 \\ 1 & 2 & 2 \\ 1 & -1 & 2 \end{pmatrix}$$

- Compute $C = AB$.

$$\begin{pmatrix} 3 * 1 + 2 * 1 + 3 * 1 & 1 * 1 + 2 * 2 + 3 * (-1) & 1 * 6 + 2 * 2 + 3 * 2 \\ 1 * 3 + 0 * 1 + 4 * 1 & 1 * 1 + 0 * 2 + 4 * (-1) & 1 * 6 + 0 * 2 + 4 * 2 \\ 3 * 3 + (-1) * 1 + 2 * 1 & 3 * 1 + (-1) * 2 + 2 * (-1) & 3 * 6 + (-1) * 2 + 2 * 2 \end{pmatrix} \\ = \begin{pmatrix} 8 & 2 & 16 \\ 7 & -3 & 14 \\ 10 & -1 & 20 \end{pmatrix}$$

- Does $AB = BA$? No. $(AB)_{11} = 8$ and $(BA)_{11} = 22$
- Given the vector a from the previous question, compute $c = Aa$.

$$c = \begin{pmatrix} 1 * 1 + 2 * 3 + 3 * (-1) \\ 1 * 1 + 0 * 3 + 4 * (-1) \\ 3 * 1 + (-1) * 3 + 2 * (-1) \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ -2 \end{pmatrix}$$

- Given the vector a from the previous question, compute $d = a^T A$.

$$\begin{aligned} d &= \left(1 * 1 + 3 * 1 + (-1) * 3 \quad 1 * 2 + 3 * 0 + (-1) * (-1) \quad 1 * 3 + 3 * 4 + (-1) * 2 \right) \\ &= \left(1 \quad 3 \quad 13 \right) \end{aligned}$$

4. (1 point) Normals and Planes

T is a triangle in 3D with vertices $P_1 = (1, 0, 0)$, $P_2 = (2, 0, 1)$ and $P_3 = (2, 2, 2)$ (counterclockwise around the normal).

- Compute the normal to T .

$$N = (P_2 - P_1) \times (P_3 - P_1) = \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}$$

- Compute the area of T . $= (\frac{1}{2}) * \|N\| = \sqrt{N_1^2 + N_2^2 + N_3^2} = 1.5$
- Compute the implicit plane equation $Ax + By + Cz + D = 0$ for the plane that T lies in.
 1. Normalize N . So

$$N = \frac{N}{\|N\|} = \begin{pmatrix} -2/3 \\ -1/3 \\ 2/3 \end{pmatrix}$$

2. Place coordinates of N into implicit plane equation $N_1x + N_2y + N_3z + D = 0$
 $\frac{-2}{3}x + \frac{-1}{3}y + \frac{2}{3}z + D = 0$
3. Solve for D by plugging the in the x , y , and z from one of the points in T . So
 $\frac{-2}{3} * 1 + \frac{-1}{3} * 0 + \frac{2}{3} * 0 + D = 0$.
4. The equation is $\frac{-2}{3}x + \frac{-1}{3}y + \frac{2}{3}z + \frac{2}{3} = 0$.

5. (1 point) Segments and Lines

Given two segments in 2D: S_1 from $(0, 1)$ to $(-1, 2)$ and S_2 from $(0, 0)$ to $(2, 2)$,

- Does the point $P = (0, 1)$ lie on S_1 ?

Yes. First we must get the two equations $x(t)$ and $y(t)$ that make up S_1 for $t \in [0, 1]$. These two equations come from the x and y components of the equation $P_1 + t(P_2 - P_1)$, in other words the equations that adjust each component of the vector $(P_2 - P_1)$ positioned at P_1 with the parameter t . When $t = 0$ we are at P_1 , when $t = 1$ we are at P_2 , when $0 < t < 1$ then we are at some position on S_1 between the endpoints. When $t < 0$ or $1 < t$ we are at some position on the line containing P_1 and P_2 outside the bounds of the line segment. So since $P_1 = (0, 1)$ and $P_2 = (-1, 2)$.

$$x(t) = P_{1x} + t(P_{2x} - P_{1x}) = 0 + t(-1 - 0) = -1t$$

$$y(t) = P_{1y} + t(P_{2y} - P_{1y}) = 1 + t(2 - 1) = 1 + t$$

The question asks if $(0, 1)$ is on the segment, but we already know that when $t = 0$, $(x, y) = (0, 1)$, which is one of the the endpoints and therefore part of the segment.

- Do S_1 and S_2 intersect? If yes, compute the intersection, if no, explain.

No. Using the same method to generate equations in the previous question, the two equations that make up S_1 are $x^1(t) = -1t$ and $y^1(t) = 1 + t$ for $t \in [0, 1]$. The two equations that make up S_2 are $x^2(r) = 2r$ and $y^2(r) = 2r$ for $r \in [0, 1]$. S_1 and S_2 intersect when $x^1(t) = x^2(r)$ and $y^1(t) = y^2(r)$ and t and $r \in [0, 1]$. In other words we must solve the two equations:

$$-t = 2r$$

$$t + 1 = 2r$$

or in another form:

$$-1t - 2r = 0$$

$$1t - 2r = -1$$

converting to matrix form:

$$\begin{pmatrix} -1 & -2 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} t \\ r \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

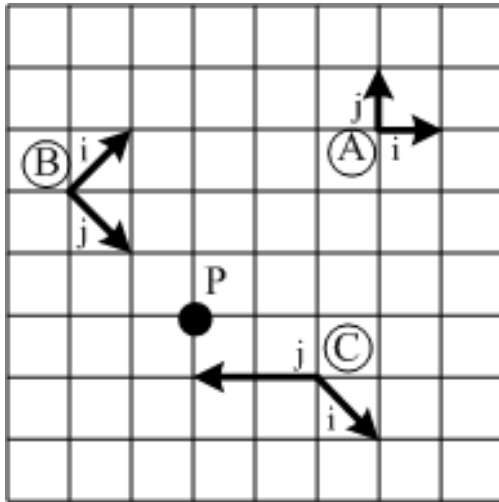
$$\begin{pmatrix} t \\ r \end{pmatrix} = \begin{pmatrix} -0.5 & 0.5 \\ -0.25 & -0.25 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} t \\ r \end{pmatrix} = \begin{pmatrix} -0.5 \\ 0.25 \end{pmatrix}$$

Because $t = -0.5$ is outside the domain $[0, 1]$ we conclude that these line segments do not intersect.

6. (1 point) Frames

Specify the coordinates of point P with respect to coordinate frames A, B and C.



$$P = -3A_i - 3A_j \text{ so in } A (3, -3)$$

$$P = 2B_j \text{ so in } B (0, 2)$$

$$P = -C_i + \frac{1}{2}C_j \text{ so in } C (-1, \frac{1}{2})$$

7. (2 points) BONUS:

Given an arbitrary point P and a triangle (P_1, P_2, P_3) in 2D describe an algorithm to determine if P is inside the triangle.

There are many possibilities. The following is conceptually simple: derive the implicit edge equations for (P_1, P_2) , (P_2, P_3) , (P_3, P_1) . Plug the coordinates of the point into each equation and check that the sign is the same. Recall that the implicit equation for a line in 2D is $Ax + By + C = 0$. The value of the implicit equation tells you where a point is in relation to the line: positive on one side, negative on the other (zero is the special case that the point lies on the line). One can easily see that the intersection of the areas on the positive sides of these lines is the area inside the triangle.