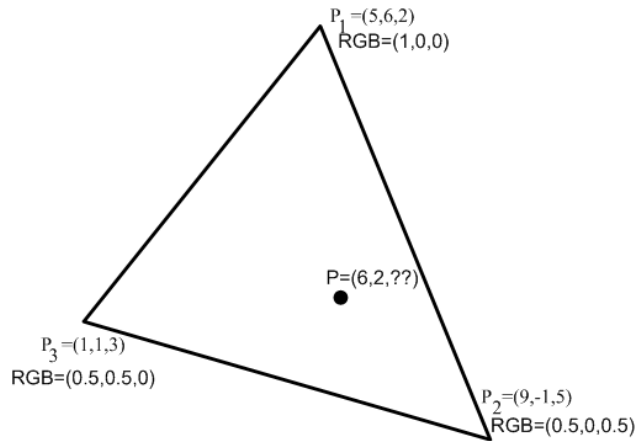


CPSC 314

Assignment 2 Solution

November 13, 2004

1. Scan Conversion and Interpolation



- (a) (2 points) Give the implicit plane equation for the triangle shown above. I.e., $Ax + By + Cz + D = 0$. Show your work.

$$\begin{aligned}
 n &= (P_3 - P_1) \times (P_2 - P_1) \\
 n \cdot P_1 - D &= 0 \\
 n &= \begin{bmatrix} -8 & 16 & 48 \end{bmatrix} \rightarrow A = -8 \quad B = 16 \quad C = 48 \quad D = -152
 \end{aligned}$$

- (b) (1 point) Compute the value of z for point P using your plane equation.

$$n \cdot P = -D \rightarrow Z = \frac{-D - Ax - By}{C} = 3.5$$

- (c) (3 points) Compute the barycentric coordinates for point P . Compute z and r, g, b for point P using the Barycentric coordinates.

Project the triangle onto the plane $z = 1$ and compute using areas.

$$A = \frac{1}{2} \cdot \det \begin{pmatrix} 5 & 6 & 1 \\ 9 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = -24$$

$$\alpha = \frac{1}{2} \cdot \det \begin{pmatrix} 6 & 2 & 1 \\ 9 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix} / A = .375$$

β and γ using the same method: $\beta = .4375$ $\gamma = .1875$

$$z = \alpha P_1^z + \beta P_2^z + \gamma P_3^z = 3.5$$

$$r = \alpha P_1^r + \beta P_2^r + \gamma P_3^r = .6875 \quad g = .0937 \quad b = 0.2188$$

- (d) (3 points) Transform the triangle using the following perspective transformation ($d = .5$ and $\alpha = .1$). Compute the new coordinates for P_1, P_2, P_3 and a new plane equation $Ax + By + Cz + d = 0$. Show your work. M=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{.5}{.5-.1} & \frac{1}{.5} \\ 0 & 0 & \frac{-.1 \cdot .5}{.5-.1} & 0 \end{pmatrix}. \quad (1)$$

$$\begin{aligned} P'_1 &= \begin{bmatrix} 5 & 6 & 2.375 & 4 \end{bmatrix} = \begin{bmatrix} 1.25 & 1.5 & .5938 \end{bmatrix} \\ P'_2 &= \begin{bmatrix} 9 & -1 & 6.125 & 10 \end{bmatrix} = \begin{bmatrix} .9 & -.1 & .6125 \end{bmatrix} \\ P'_3 &= \begin{bmatrix} 1 & 1 & 3.625 & 6 \end{bmatrix} = \begin{bmatrix} .1667 & .1667 & .6042 \end{bmatrix} \end{aligned}$$

Using formula in slides (or by transforming the normal with the inverse of the perspective matrix):

$$A' = A\alpha d \quad B' = B\alpha d \quad C' = D(\alpha - d) \quad D' = (C\alpha d^2 + Dd^2)$$

$$A' = -.4 \quad B' = .8 \quad C' = 60.8 \quad D' = -36.8$$

- (e) (1 point) Compute the value of z for point P using the new plane equation.

Note: You can not use the x, y values of P as is, you must find P' .

$$P' = \begin{bmatrix} 6 & 2 & 4.25 & 7 \end{bmatrix} = \begin{bmatrix} .8571 & .2857 & .6071 \end{bmatrix}$$

Sanity check - $P_3'^z < P'^z < P_2'^z$

Verify P'^z using plane eq. $n' \cdot P' + D = 0$

- (f) (2 points) Compute the barycentric coordinates for point P after transformation.

Using triangle areas (again projecting on $z = 1$):

$$A = \frac{1}{2} \cdot \det \begin{pmatrix} P_1^x & P_1^y & 1 \\ P_2^x & P_2^y & 1 \\ P_3^x & P_3^y & 1 \end{pmatrix}$$

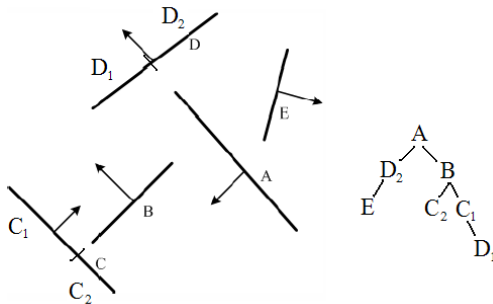
$$\alpha' = \frac{1}{2} \cdot \det \begin{pmatrix} P_1^x & P_1^y & 1 \\ P_2^x & P_2^y & 1 \\ P_3^x & P_3^y & 1 \end{pmatrix} / A = .2143$$

$$\beta' = .625 \quad \gamma' = .1607$$

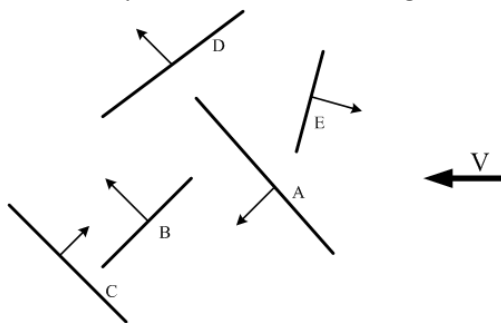
Note that they are not the same as in part c!

2. BSP Trees

- (a) (4 points) Construct the BSP tree for the segments shown below (use alphabetical insertion order for the segments, when possible). Use the convention where the right subtree (child) is located on the side that the normal points to. Show your work.



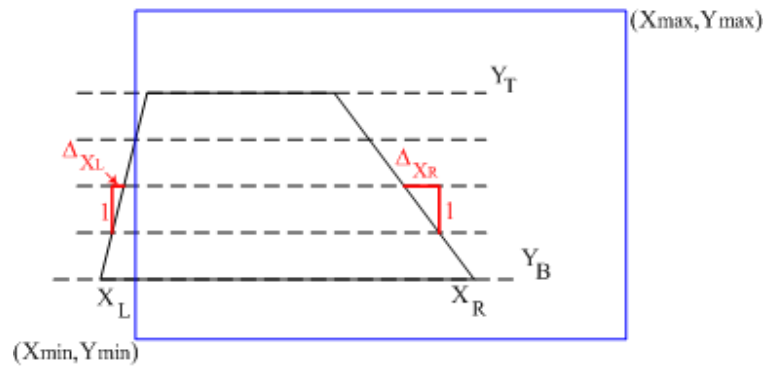
- (b) (3 points) Given the view direction as shown below, describe the complete traversal order of your BSP tree during rendering.



(A+)A(A-)
 (B+)B(B-)A(D2+)D2(D2-)
 (C1-)C1(C1+)BC2AD2E
 C1D1BC2AD2E

3. (8 points) Clipping

Instead of clipping the triangles in a continuous setting as shown in class, it is possible to clip them directly during scan-conversion. Write the pseudo-code for the “scanTrapezoid” function which does clipping as part of the conversion. Hint: use the bounding box of the trapezoid. The arguments passed to “scanTrapezoid” are the parameters of the trapezoid - $X_L, X_R, Y_B, Y_T, \Delta_{X_L}, \Delta_{X_R}$ and the window parameters $X_{min}, X_{max}, Y_{min}, Y_{max}$ as shown in the figure below



Given: $Xl, Xr, Yb, Yt, DXl, DXr, Xmin, Ymin, Xmax, Ymax$
 //x,y are int, others are float

```

y = max(Yb, Ymin) + .5    //clip bottom
yend = min(Yt, Ymax) + .5 //clip top
L = Xl + (y-Yb)*DXl      //adjust L if clipped bottom
R = Xr - (Y-Yb)*DXr     //adjust R if clipped bottom

```

```

while (y <= yend)
  x = max(L, Xmin) + .5
  xend = min(R, Xmax) + .5
  while (x <= xend)
    SetPixel(x,y)
    x = x+1
  end
  y = y+1
  L = L+DXl
  R = R-DXr
end

```