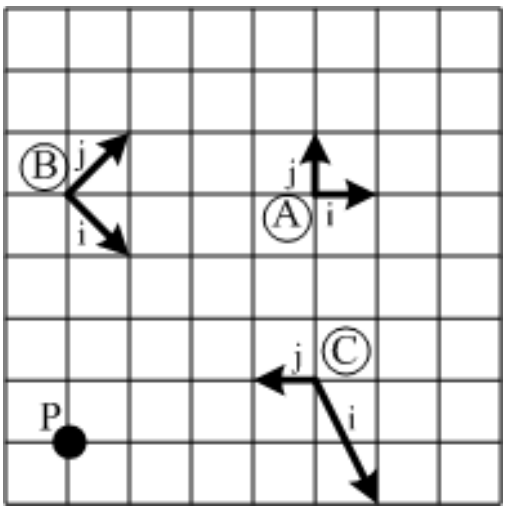
CPSC 314
Assignment 1 Solutions

October 17, 2004

1. (4 points) Transformation as a Change of Coordinate Frame



(a) (2 points) Specify the coordinates of point P with respect to coordinate frames A, B and C.

$$p_a = \begin{bmatrix} -4 \\ -4 \end{bmatrix} \quad p_b = \begin{bmatrix} 2 \\ -2 \end{bmatrix} \quad p_c = \begin{bmatrix} 0.5 \\ 4.5 \end{bmatrix}$$

(b) (2 points) Derive a transformation that takes a point from frame C to frame B, i.e., determine $M_{C\to B}$, where $P_B = M_{C\to B}P_C$. Verify your solution using your answer to part (a).

First the translation between the origins, in column 3. Next, i_c in terms of i_b, j_b in column 1 and j_c in terms of i_b, j_b in column 2.

$$M_{C\to B} = \begin{bmatrix} 1.5 & -0.5 & 3.5 \\ -0.5 & -0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$$

2. (2 points) Given a triangle $T = (P_0, P_1, P_2)$ in 2D and a point P, write an algorithm to find if the point is inside or outside the triangle.

   Let $N_1 = (p_0 - p_1) \times (p - p_1)$
   Let $N_2 = (p_0 - p_1) \times (p_2 - p_1)$
   If both N_1 and N_2 point in the same direction, then p and p_2 are on the same side of the line $p_0 - p_1$.
   Using the same method, check if p is on the same side as p_1 with respect to $p_0 - p_2$ and as p_0 with respect to $p_1 - p2$
   If all tests are true, p is in the triangle, otherwise it is not.

3. (2 points) What will the following transformations in 3D homogeneous coordinates do to a unit cube centered at the origin (use row vectors)? Sketch your answers.

   (a) (1 point)

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

   First translate 1 unit along the x axis, next flip sign of x, y, and z coordinates.

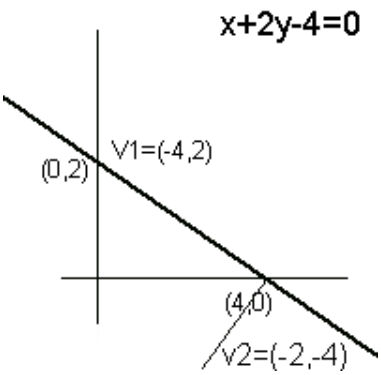
   (b) (1 point)

$$\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

   Rotate CW 90° around z axis.

4. (2 points) Write a transformation that mirrors through arbitrary line Ax + By + C = 0 in 2D. (Hint: Break it down to sub problems.)

Assume $A, B \neq 0$ and using row vectors.



Find new basis vectors: Use the line, and a vector perpendicular to the line.
Use two points on the line to find the line vector. $p_1 = (x_1 = 0, y_1)$ and $p_2 = (x_2, y_2 = 0)$.
$x = 0 \rightarrow y = -C/B$ and $y = 0 \rightarrow x = -C/A$ therefore $v_1 = p_1 - p_2 = \begin{bmatrix} C/A & -C/B \end{bmatrix}$.
Pick v_2 s.t. $v_1 \cdot v_2 = 0 \rightarrow v_2 = \begin{bmatrix} C/B & C/A \end{bmatrix}$

Two ways to continue from here:

Option 1:

Translate: $T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -C/A & 0 & 1 \end{bmatrix}$

Rotate: $\theta = arctan\frac{-C/B}{-C/A}$ $R = \begin{bmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Mirror: $M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Rotate back: R^{-1}
Translate back: T^{-1}

Ending up with $M_{ABC} = TRMR^{-1}T^{-1}$

Option 2:
Describe one frame in terms of the other:

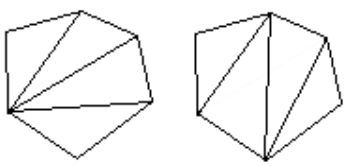
$$M_{line\to world} = \begin{bmatrix} C/A & -C/B & 0 \\ C/B & C/A & 0 \\ -C/A & 0 & 1 \end{bmatrix} \quad M_{world\to line} = M^{-1}_{line\to world}$$

Mirror: $M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Ending up with $M_{ABC} = M_{world\to line} M M_{line\to world}$

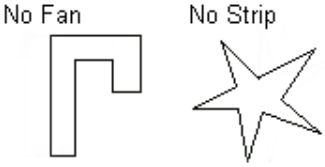
5. (a) (2 points) Given a convex planar polygon $P = P_1, P_2, \ldots, P_n$, describe an algorithm for triangulating the polygon (triangulate = split into triangles) without adding any extra vertices.

Create a triangle fan or a triangle strip.



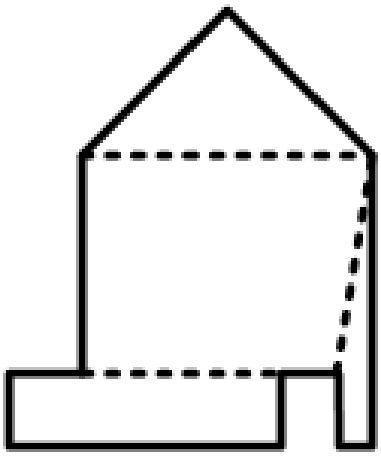
Fan - Add edges a_1, a_j for $j = 3, ..., n-1$
Strip - Add edges $(a_1, a_3), (a_3, a_n), (a_n, a_4), ...$

(b) (2 points) Will your algorithm work for non-convex polygons? If yes, prove. If not, bring a counter-example. no



6. (6 points) Answer yes/no (no explanation). All the transformations are in 3D.
   (a) (1 point) Does Perspective preserves parallel lines? no
   (b) (1 point) Does Perspective Warp preserves angles? no
   (c) (1 point) Is there an α for which the Perspective Warp becomes a Perspective Projection? yes
   (d) (1 point) Is shear * rotate = rotate * shear? no
   (e) (1 point) Is rotate1 * rotate2 = rotate2 * rotate1? no
   (f) (1 point) Does shear preserve parallel lines? yes

7. (3 points) BONUS: Given a non-convex planar polygon $P = P_1, P_2, \ldots, P_n$, describe an algorithm for splitting the polygon into convex pieces, without adding any extra vertices. Example:



See a possible solution at http://www.thecodeproject.com/csharp/CsPolygonTriangulation.asp