1. (4 points) Transformation as a Change of Coordinate Frame

(a) (2 points) Specify the coordinates of point \( P \) with respect to coordinate frames A, B and C.

\[
p_a = \begin{bmatrix} -4 \\ -4 \end{bmatrix} \quad p_b = \begin{bmatrix} 2 \\ -2 \end{bmatrix} \quad p_c = \begin{bmatrix} 0.5 \\ 4.5 \end{bmatrix}
\]

(b) (2 points) Derive a transformation that takes a point from frame C to frame B, i.e., determine \( M_{C \rightarrow B} \), where \( P_B = M_{C \rightarrow B} P_C \). Verify your solution using your answer to part (a).

First the translation between the origins, in column 3. Next, \( i_c \) in terms of \( i_b, j_b \) in column 1 and \( j_c \) in terms of \( i_b, j_b \) in column 2.

\[
M_{C \rightarrow B} = \begin{bmatrix} 1.5 & -0.5 & 3.5 \\ -0.5 & -0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}
\]
2. (2 points) Given a triangle $T = (P_0, P_1, P_2)$ in 2D and a point $P$, write an algorithm to find if the point is inside or outside the triangle.

Let $N_1 = (p_0 - p_1) \times (p - p_1)$
Let $N_2 = (p_0 - p_1) \times (p_2 - p_1)$
If both $N_1$ and $N_2$ point in the same direction, then $P$ and $P_2$ are on the same side of the line $p_0 - p_1$.
Using the same method, check if $P$ is on the same side as $P_1$ with respect to $p_0 - p_2$ and as $P_0$ with respect to $p_1 - p_2$
If all tests are true, $P$ is in the triangle, otherwise it is not.

3. (2 points) What will the following transformations in 3D homogeneous coordinates do to a unit cube centered at the origin (use row vectors)? Sketch your answers.

(a) (1 point)

\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
1 & 0 & 0 & -1
\end{pmatrix}
\]

First translate 1 unit along the x axis, next flip sign of x, y, and z coordinates.

(b) (1 point)

\[
\begin{pmatrix}
0 & -1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

Rotate CW 90° around z axis.
4. (2 points) Write a transformation that mirrors through arbitrary line \(Ax + By + C = 0\) in 2D. (Hint: Break it down to sub problems.)

Assume \(A, B \neq 0\) and using row vectors.

Find new basis vectors: Use the line, and a vector perpendicular to the line. Use two points on the line to find the line vector. \(p_1 = (x_1 = 0, y_1)\) and \(p_2 = (x_2, y_2 = 0)\).

\(x = 0 \rightarrow y = -C/B\) and \(y = 0 \rightarrow x = -C/A\) therefore \(v_1 = p_1 - p_2 = \begin{bmatrix} C/A & -C/B \end{bmatrix}\).

Pick \(v_2\) s.t. \(v_1 \cdot v_2 = 0\) \(\rightarrow v_2 = \begin{bmatrix} C/B & C/A \end{bmatrix}\)

Two ways to continue from here:

Option 1:

Translate: \(T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -C/A & 0 & 1 \end{bmatrix}\)

Rotate: \(\theta = \arctan \frac{-C/B}{-C/A}\) \(R = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\)

Mirror: \(M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\)

Rotate back: \(R^{-1}\)
Translate back: \(T^{-1}\)

**Ending up with** \(M_{ABC} = T R M R^{-1} T^{-1}\)

Option 2:
Describe one frame in terms of the other:
5. (a) (2 points) Given a convex planar polygon $P = P_1, P_2, \ldots, P_n$, describe an algorithm for triangulating the polygon (triangulate = split into triangles) without adding any extra vertices.

Create a triangle fan or a triangle strip.

\[ M_{\text{line\rightarrow world}} = \begin{bmatrix} C/A & -C/B & 0 \\ C/B & C/A & 0 \\ -C/A & 0 & 1 \end{bmatrix} \quad M_{\text{world\rightarrow line}} = M_{\text{line\rightarrow world}}^{-1} \]

Mirror: $M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Ending up with $M_{ABC} = M_{\text{world\rightarrow line}} M M_{\text{line\rightarrow world}}$

(b) (2 points) Will your algorithm work for non-convex polygons? If yes, prove. If not, bring a counter-example. no
6. (6 points) Answer yes/no (no explanation). All the transformations are in 3D.
   (a) (1 point) Does Perspective preserves parallel lines? no
   (b) (1 point) Does Perspective Warp preserves angles? no
   (c) (1 point) Is there an $\alpha$ for which the Perspective Warp becomes a Perspective Projection? yes
   (d) (1 point) Is shear * rotate = rotate * shear? no
   (e) (1 point) Is rotate1 * rotate2 = rotate2 * rotate1? no
   (f) (1 point) Does shear preserve parallel lines? yes

7. (3 points) BONUS: Given a non-convex planar polygon $P = P_1, P_2, \ldots, P_n$, describe an algorithm for splitting the polygon into convex pieces, without adding any extra vertices. Example:

![Polygon](image)

See a possible solution at http://www.thecodeproject.com/csharp/CsPolygonTriangulation.asp