University of British Columbia
CPSC 314 Computer Graphics
Jan-Apr 2016

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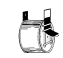
Viewing 2

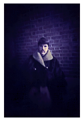
http://www.ugrad.cs.ubc.ca/~cs314/Vjan2016

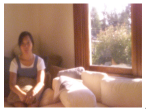
Projections I

Pinhole Camera

- ingredients
  - box, film, hole punch
- result
  - picture

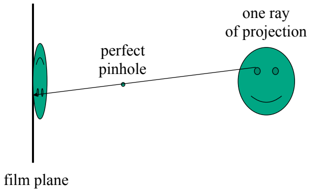

www.kodak.com


www.pinhole.org

www.debevec.org/Pinhole


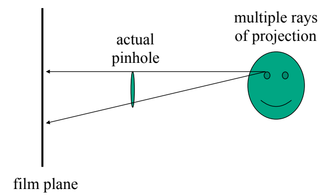
Pinhole Camera

- theoretical perfect pinhole
  - light shining through tiny hole into dark space yields upside-down picture



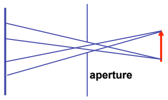
Pinhole Camera

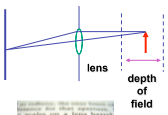
- non-zero sized hole
  - blur: rays hit multiple points on film plane

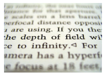


Real Cameras

- pinhole camera has small aperture (lens opening)
  - minimize blur
- problem: hard to get enough light to expose the film
- solution: lens
  - permits larger apertures
  - permits changing distance to film plane without actually moving it
    - cost: limited depth of field where image is in focus



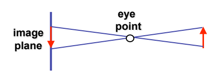




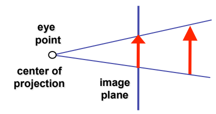
http://en.wikipedia.org/wiki/Image:DOF-ShallowDepthofField.jpg

Graphics Cameras

- real pinhole camera: image inverted

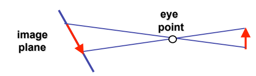


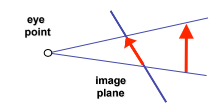
- computer graphics camera: convenient equivalent



General Projection

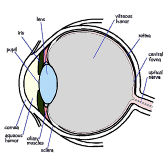
- image plane need not be perpendicular to view plane





Perspective Projection

- our camera must model perspective



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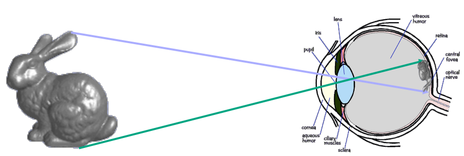


Projective Transformations

- planar geometric projections
  - planar: onto a plane
  - geometric: using straight lines
  - projections: 3D -> 2D
- aka projective mappings
- counterexamples?

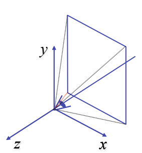
Projective Transformations

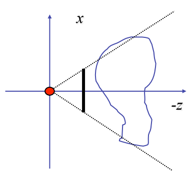
- properties
  - lines mapped to lines and triangles to triangles
  - parallel lines do NOT remain parallel
    - e.g. rails vanishing at infinity

  - affine combinations are NOT preserved
    - e.g. center of a line does not map to center of projected line (perspective foreshortening)

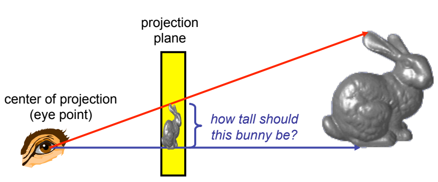
Perspective Projection

- project all geometry
  - through common center of projection (eye point)
  - onto an image plane

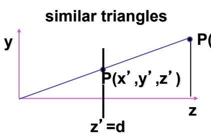




Perspective Projection



Basic Perspective Projection



$$\frac{y'}{d} = \frac{y}{z} \rightarrow y' = \frac{y \cdot d}{z}$$

$$\frac{x'}{d} = \frac{x}{z} \rightarrow x' = \frac{x \cdot d}{z} \quad \textbf{but} \quad z' = d$$

- nonuniform foreshortening
  - not affine

Perspective Projection

- desired result for a point $[x, y, z, 1]^T$ projected onto the view plane:

$$\frac{x'}{d} = \frac{x}{z}, \quad \frac{y'}{d} = \frac{y}{z}$$

$$x' = \frac{x \cdot d}{z} = \frac{x}{z/d}, \quad y' = \frac{y \cdot d}{z} = \frac{y}{z/d}, \quad z' = d$$

- what could a matrix look like to do this?

Simple Perspective Projection Matrix

$$\begin{bmatrix} \frac{x}{z/d} \\ \frac{y}{z/d} \\ d \end{bmatrix}$$

Simple Perspective Projection Matrix

$$\begin{bmatrix} \frac{x}{z/d} \\ \frac{y}{z/d} \\ d \end{bmatrix}$$ is homogenized version of $$\begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

where w = z/d

Simple Perspective Projection Matrix

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where w = z/d

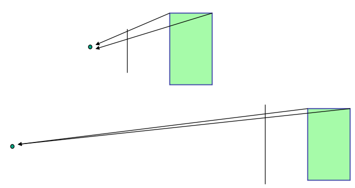
$$\begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Perspective Projection

- expressible with 4x4 homogeneous matrix
  - use previously untouched bottom row
- perspective projection is irreversible
  - many 3D points can be mapped to same (x, y, d) on the projection plane
  - no way to retrieve the unique z values

Moving COP to Infinity

- as COP moves away, lines approach parallel
  - when COP at infinity, orthographic view



Orthographic Camera Projection

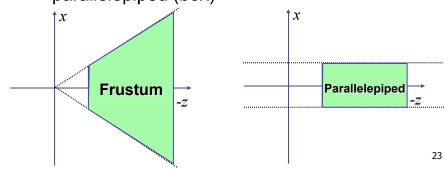
- camera's back plane parallel to lens
- infinite focal length
- no perspective convergence
- just throw away z values

$$\begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

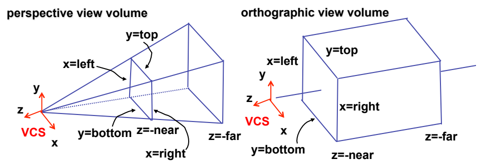
Perspective to Orthographic

- transformation of space
  - center of projection moves to infinity
  - view volume transformed
    - from frustum (truncated pyramid) to parallelepiped (box)



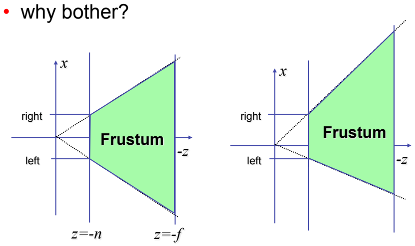
View Volumes

- specifies field-of-view, used for clipping
- restricts domain of z stored for visibility test



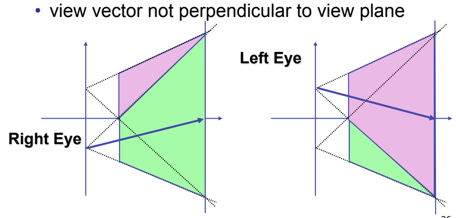
Asymmetric Frusta

- our formulation allows asymmetry
  - why bother?



Asymmetric Frusta

- our formulation allows asymmetry
  - why bother? binocular stereo
    - view vector not perpendicular to view plane

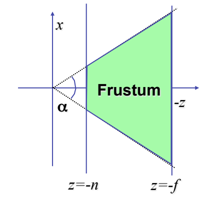


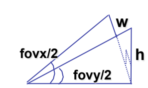
Simpler Formulation

- left, right, bottom, top, near, far
  - nonintuitive
  - often overkill
- look through window center
  - symmetric frustum
- constraints
  - left = -right, bottom = -top

Field-of-View Formulation

- FOV in one direction + aspect ratio (w/h)
  - determines FOV in other direction
  - also set near, far (reasonably intuitive)





THREE.PerspectiveCamera (fovy,aspect,near,far);

Demos

- frustum
  - http://webglfundamentals.org/webgl/frustum-diagram.html
  - http://www.ugrad.cs.ubc.ca/~cs314/Vsep2014/webGL/view-frustum.html
- orthographic vs projection cameras
  - http://threejs.org/examples/#canvas_camera_orthographic2
  - http://threejs.org/examples/#webgl_camera
  - https://www.script-tutorials.com/webgl-with-three-js-lesson-9/