University of British Columbia
CPSC 314 Computer Graphics
Jan-Apr 2016

Tamara Munzner

Transformations 4

http://www.ugrad.cs.ubc.ca/~cs314/Vjan2016

Readings for Transformations 1-5

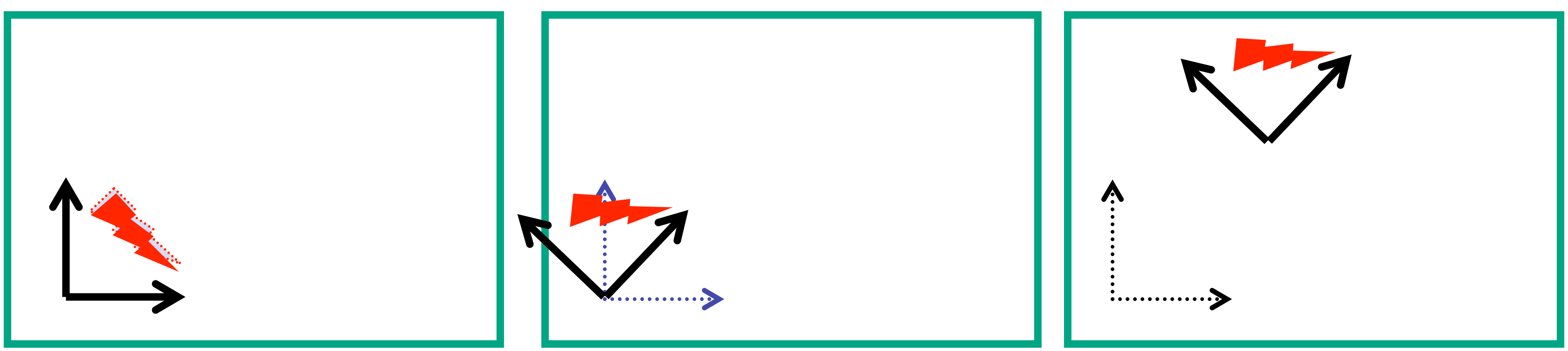
- Shirley/Marschner
  - Ch 6: Transformation Matrices
    - *except* 6.1.6, 6.3.1
  - Sect 12.2 Scene Graphs

- Gortler
  - Ch 2: Linear, Sec 2.5-2.6
  - Ch 3: Affine
  - Ch 4: Respect
  - Ch 5: Frames in Graphics, 5.3-5.4

Correction: Composing Transformations

$$\mathbf{p'} = \mathbf{TRp}$$

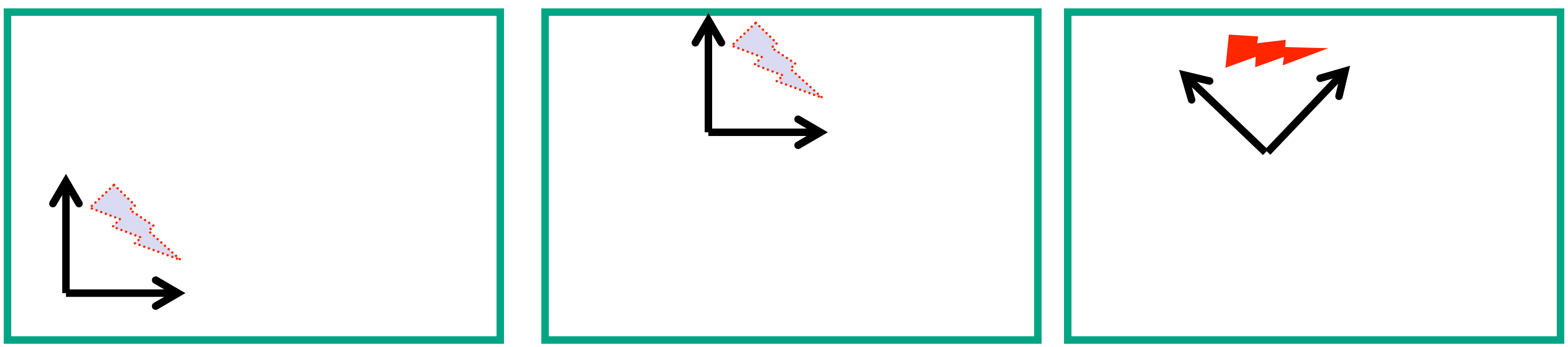
- which direction to read?
  - right to left
    - interpret operations wrt fixed global coordinates
    - moving object
      - draw thing
      - rotate thing by 45 degrees wrt fixed global coords
      - translate it (2, 3) over wrt fixed global coordinates



Correction: Composing Transformations

$$\mathbf{p}' = \mathbf{TRp}$$

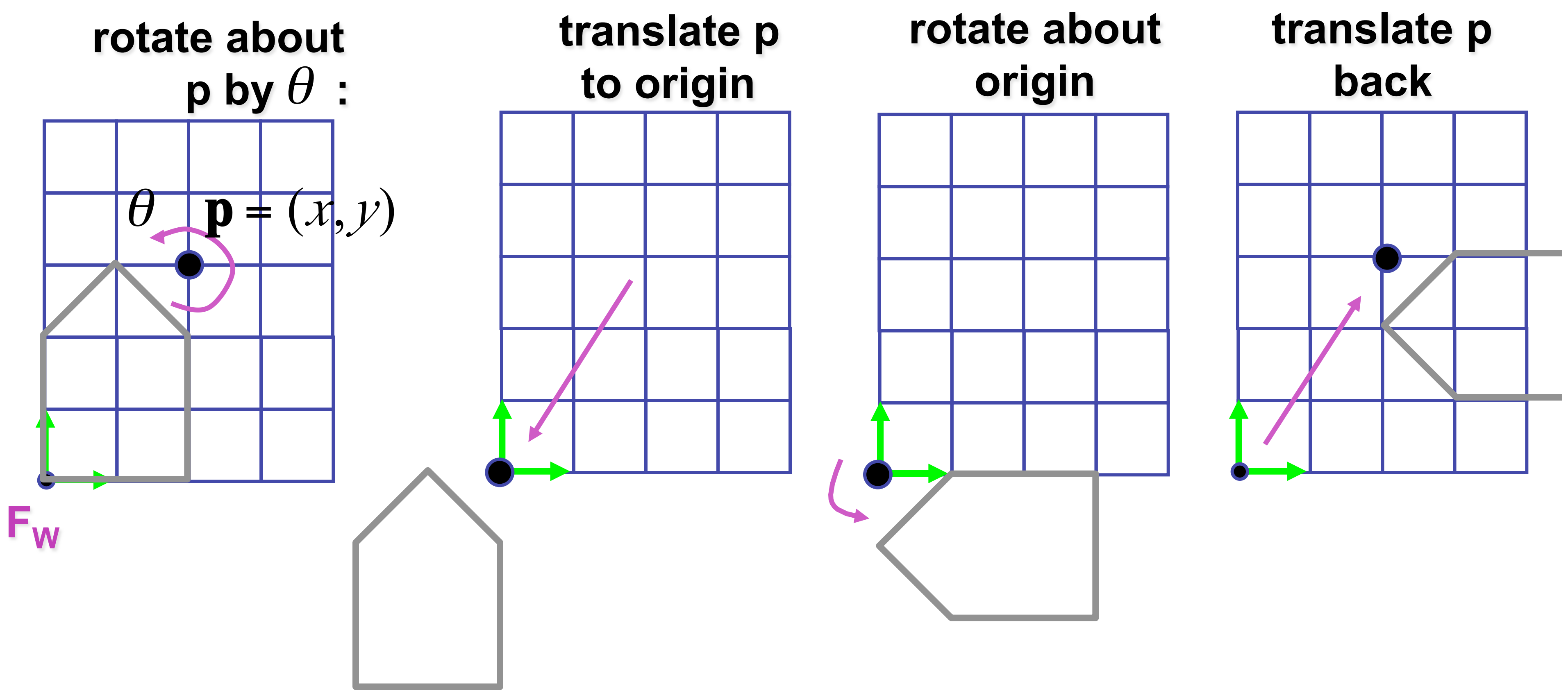
- which direction to read?
  - left to right
    - interpret operations wrt local coordinates
    - changing coordinate system
      - translate coordinate system (2, 3) over
      - rotate coordinate system 45 degrees wrt LOCAL origin
      - draw object in current coordinate system



Practice:
Composing Transformations

Transformation Hierarchies

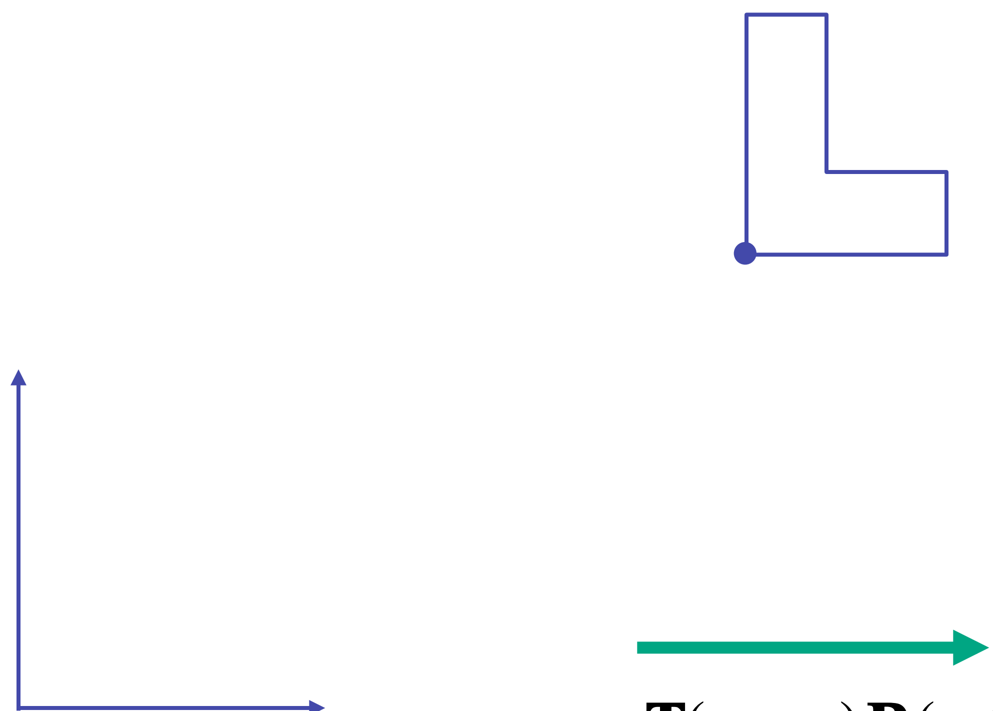
Rotation About a Point: Moving Object



$$\mathbf{T}(x, y, z)\,\mathbf{R}(z, \theta)\,\mathbf{T}(-x, -y, -z)$$

Rotation: Changing Coordinate Systems

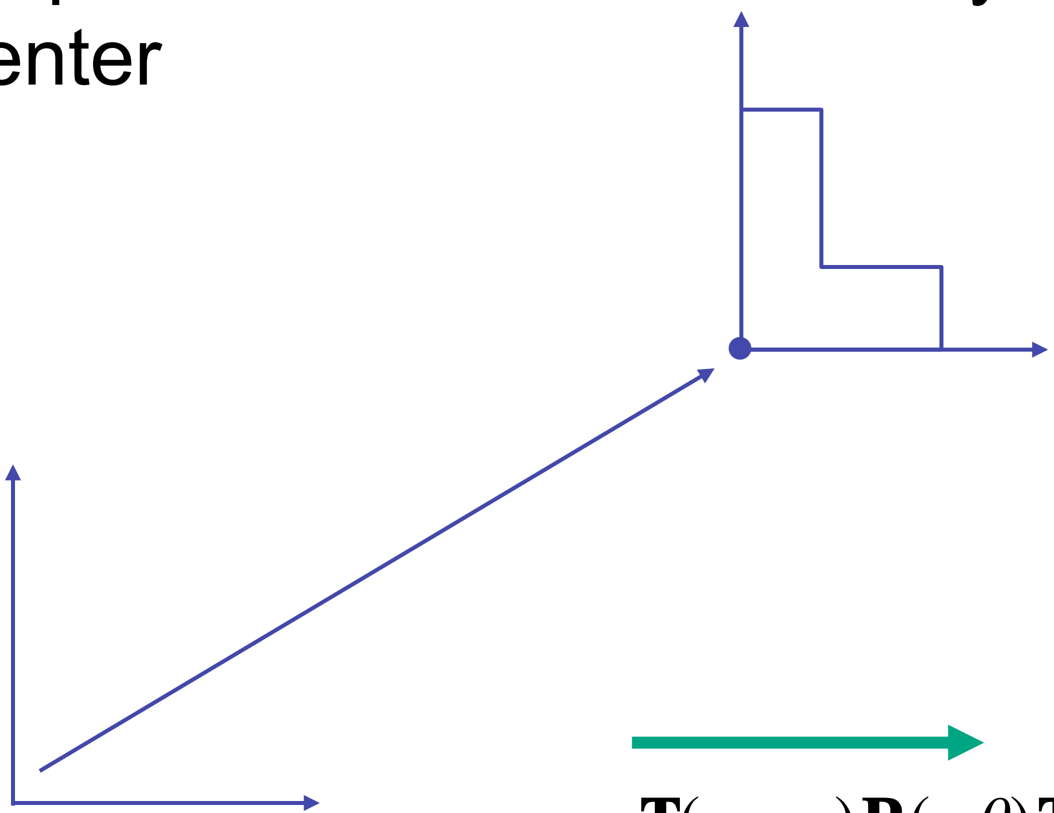
- same example: rotation around arbitrary center



$$\mathbf{T}(x, y, z)\,\mathbf{R}(z, \theta)\,\mathbf{T}(-x, -y, -z)$$

Rotation: Changing Coordinate Systems

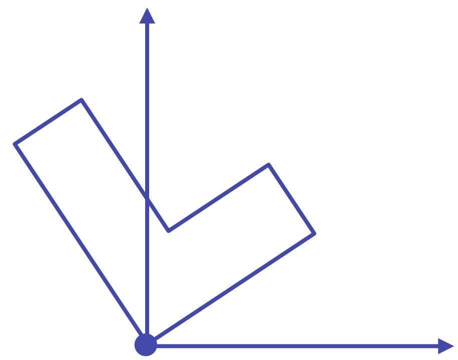
- rotation around arbitrary center
  - step 1: translate coordinate system to rotation center



$$\mathbf{T}(x,y,z)\,\mathbf{R}(z,\theta)\,\mathbf{T}(-x,-y,-z)$$

Rotation: Changing Coordinate Systems

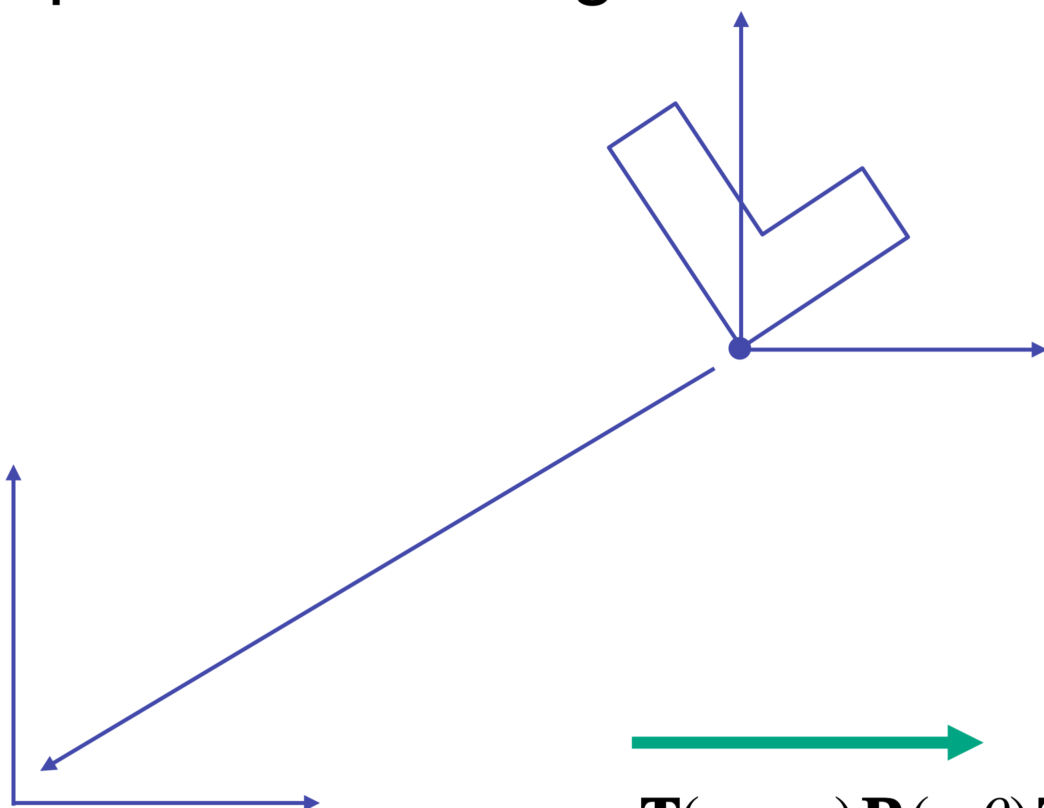
- rotation around arbitrary center
  - step 2: perform rotation



$$\mathbf{T}(x,y,z)\mathbf{R}(z,\theta)\mathbf{T}(-x,-y,-z)$$

Rotation: Changing Coordinate Systems

- rotation around arbitrary center
  - step 3: back to original coordinate system



$$\mathbf{T}(x, y, z)\,\mathbf{R}(z, \theta)\,\mathbf{T}(-x, -y, -z)$$

General Transform Composition

- transformation of geometry into coordinate system where operation becomes simpler
  - typically translate to origin
- perform operation
- transform geometry back to original coordinate system

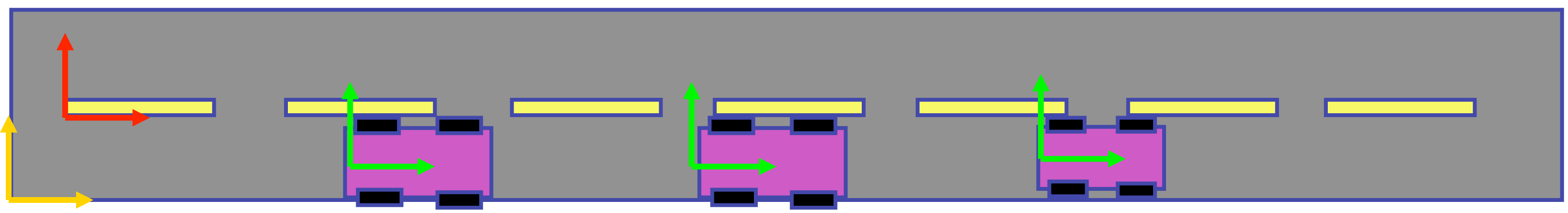
Rotation About an Arbitrary Axis

- axis defined by two points
- translate point to the origin
- rotate to align axis with z-axis (or x or y)
- perform rotation
- undo aligning rotations
- undo translation

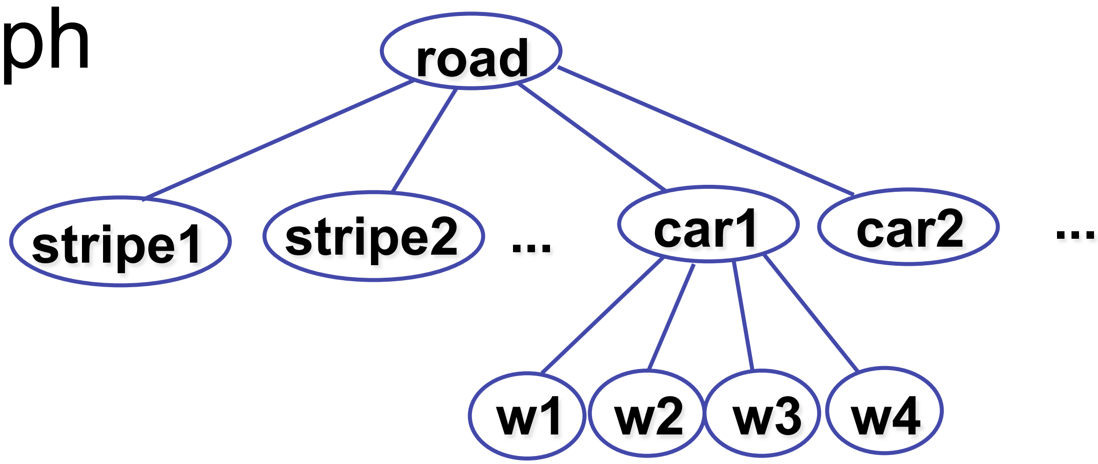
Transformation Hierarchies

Transformation Hierarchies

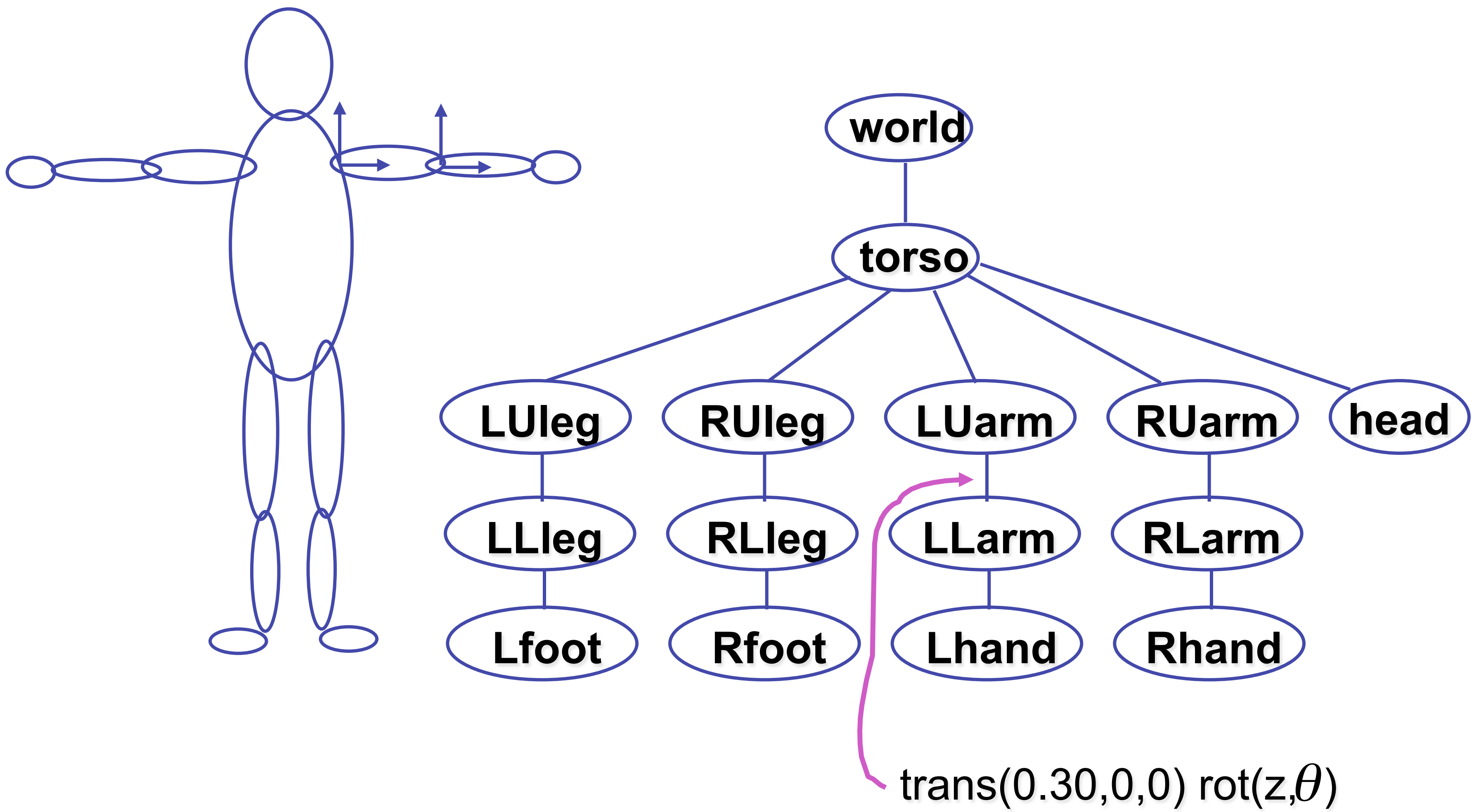
- scene may have a hierarchy of coordinate systems
  - stores matrix at each level with incremental transform from parent's coordinate system



- scene graph

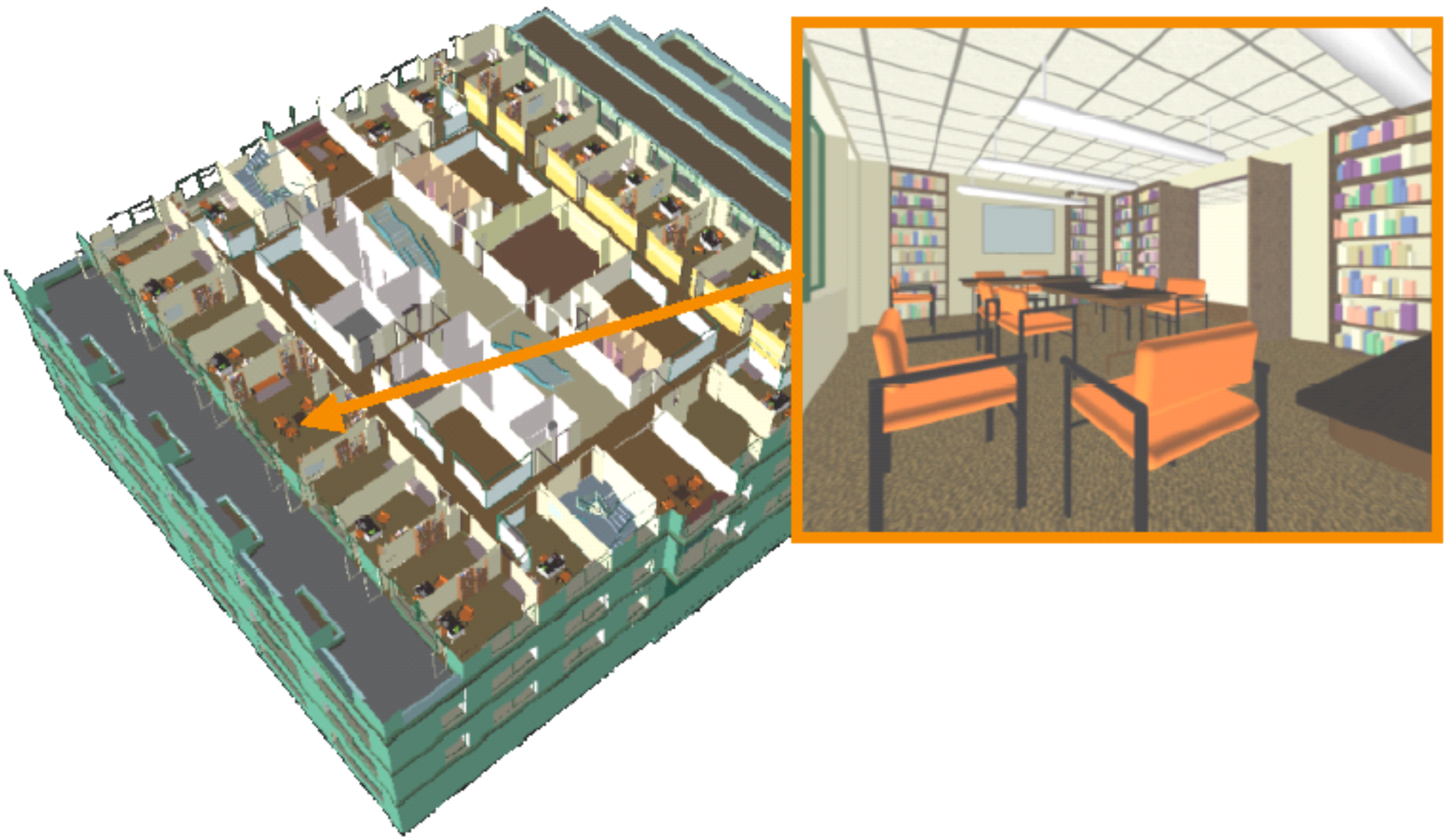


Transformation Hierarchy Example 1



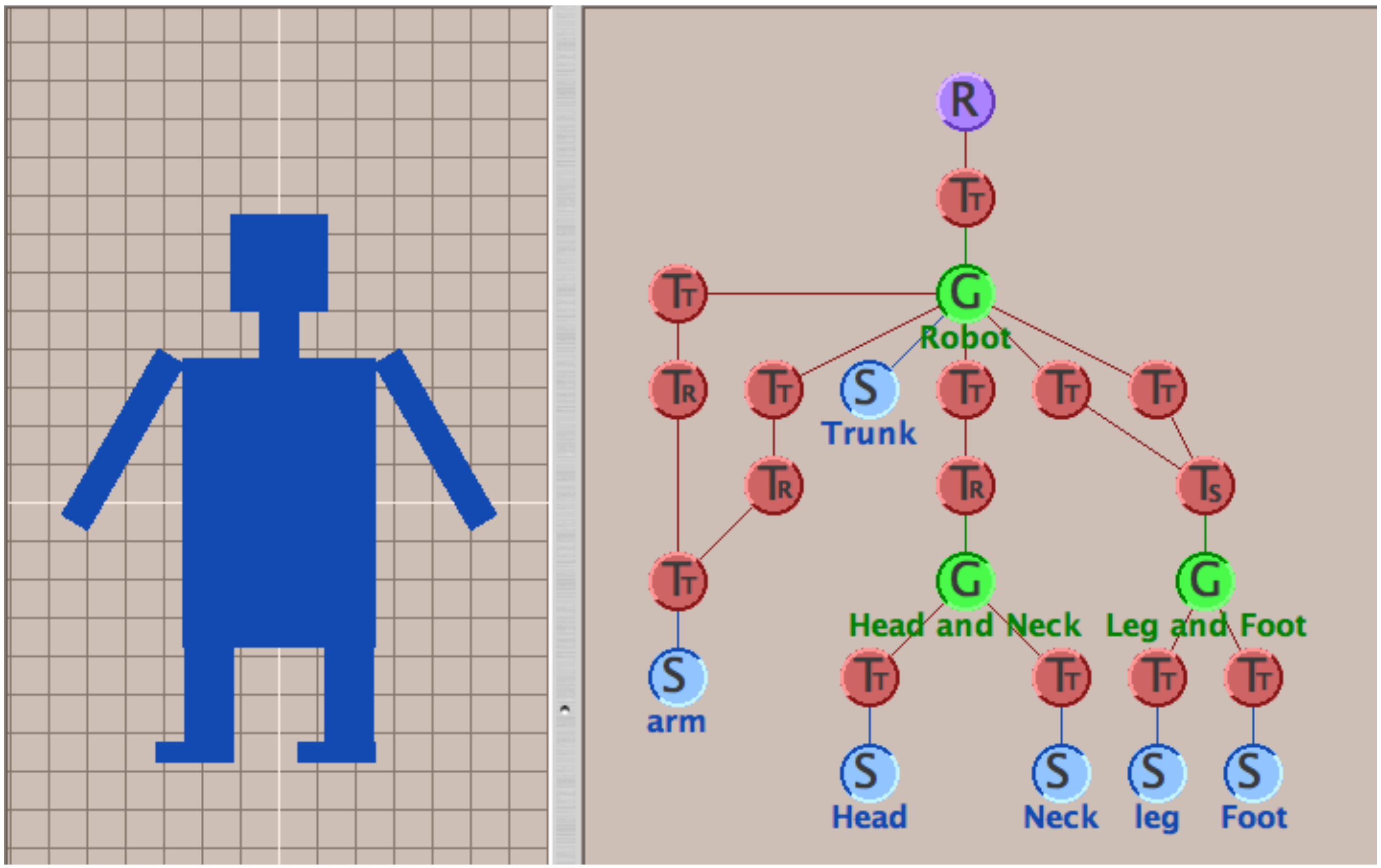
Transformation Hierarchy Example 2

- draw same 3D data with different transformations: instancing



Transformation Hierarchies Demo

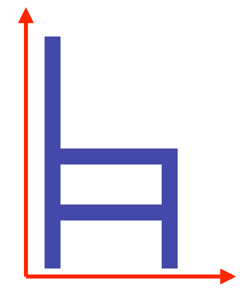
- transforms apply to graph nodes beneath

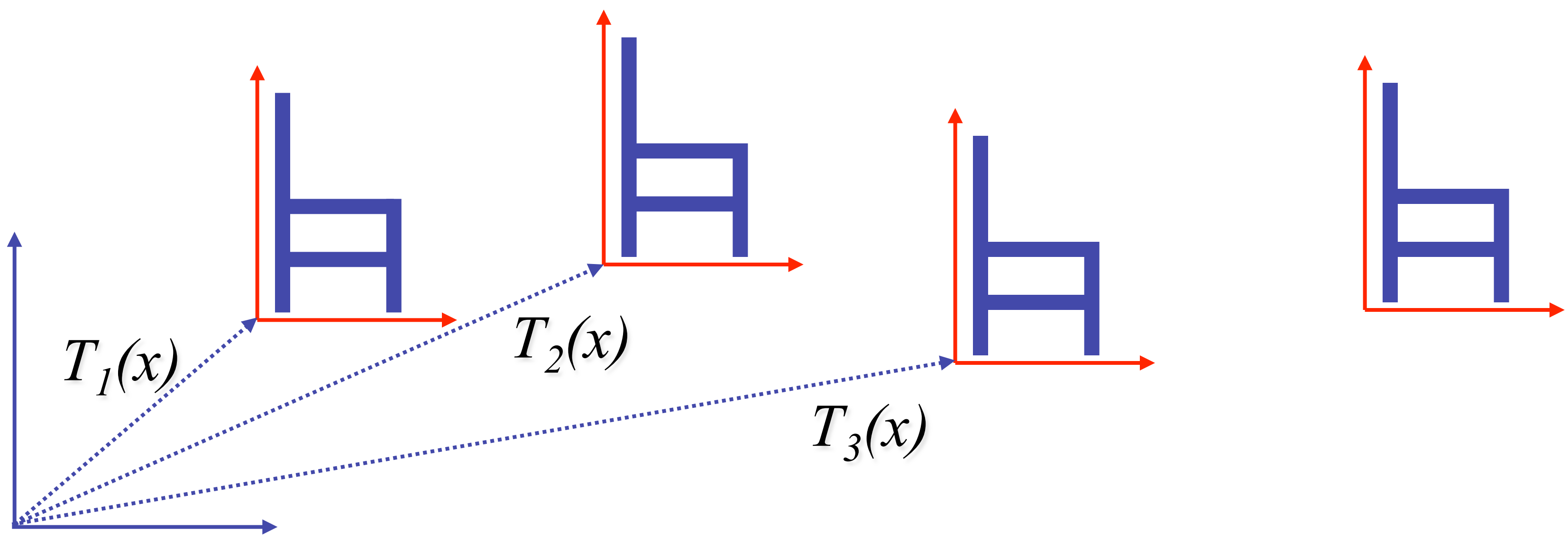


`http://www.cs.brown.edu/exploratories/freeSoftware/catalogs/scenegraphs.html`

Matrix Stacks

- challenge of avoiding unnecessary computation
  - using inverse to return to origin
  - computing incremental T_1 -> T_2

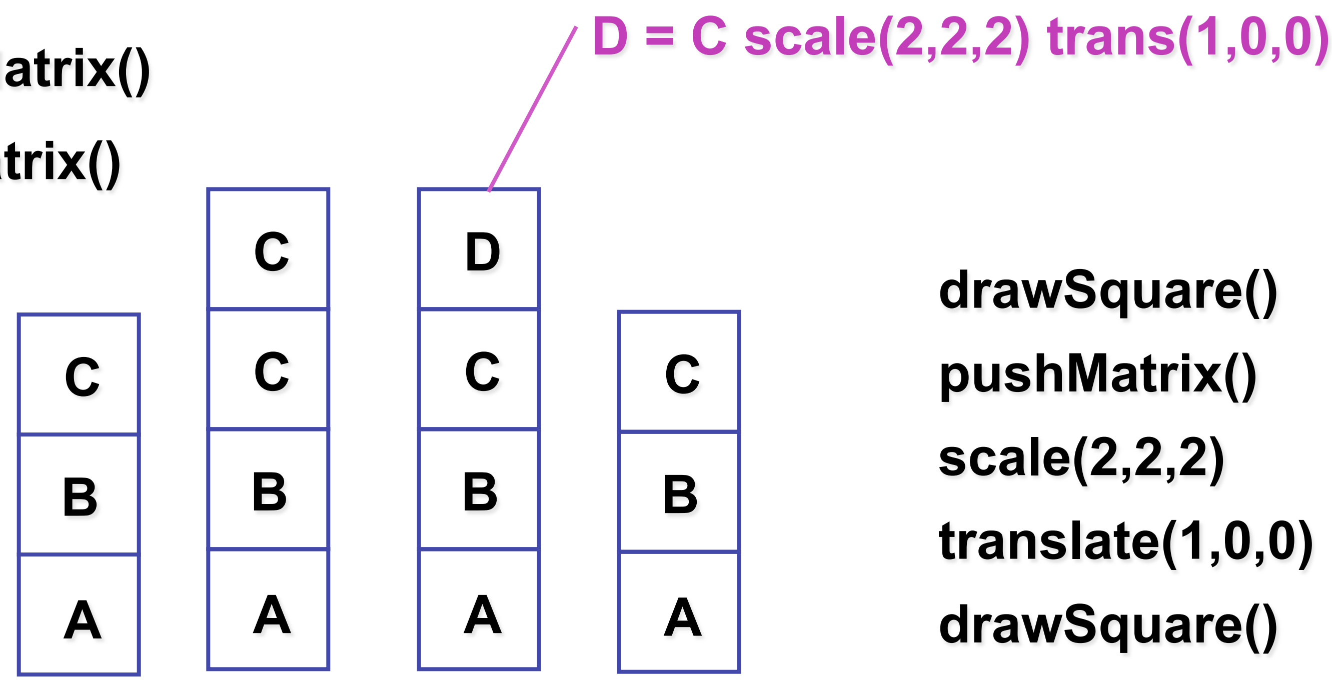




Matrix Stacks

pushMatrix()

popMatrix()



drawSquare()

pushMatrix()

scale(2,2,2)

translate(1,0,0)

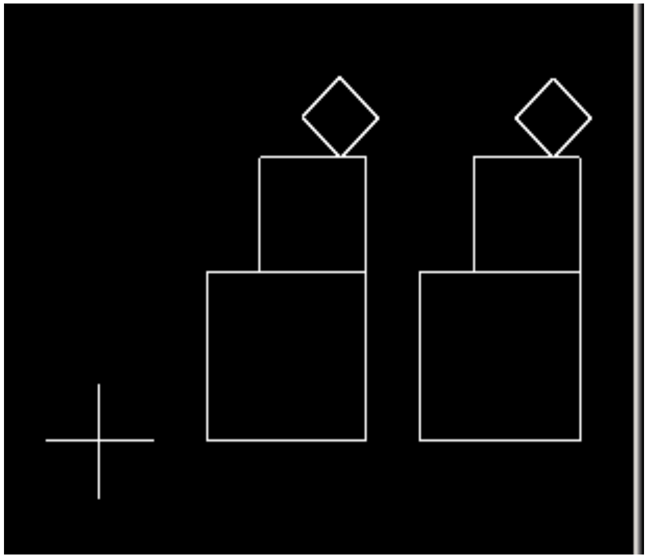
drawSquare()

popMatrix()

Modularization

- drawing a scaled square
  - push/pop ensures no coord system change

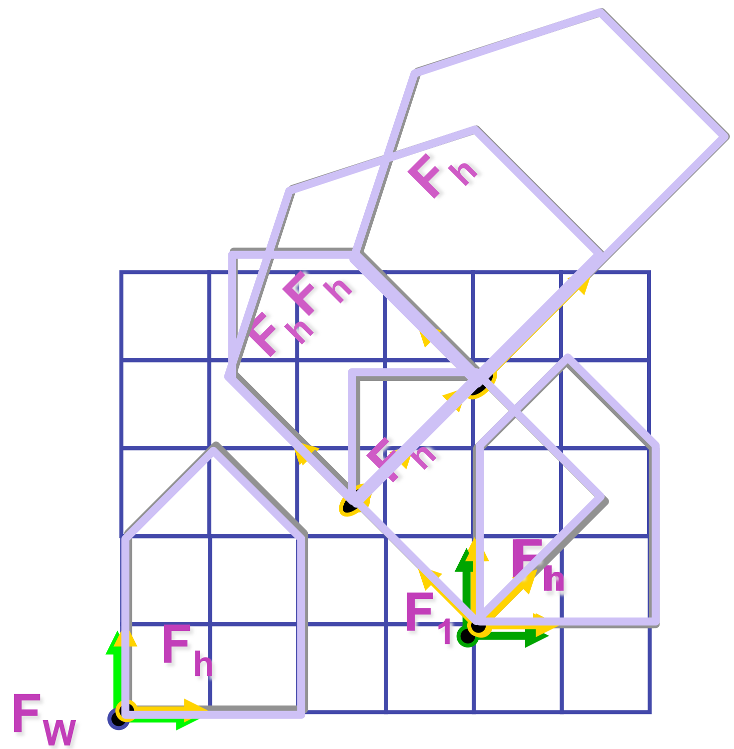
```
void drawBlock(float k) {
  pushMatrix();
  scale(k,k,k);
  drawBox();
  popMatrix();
}
```



Matrix Stacks

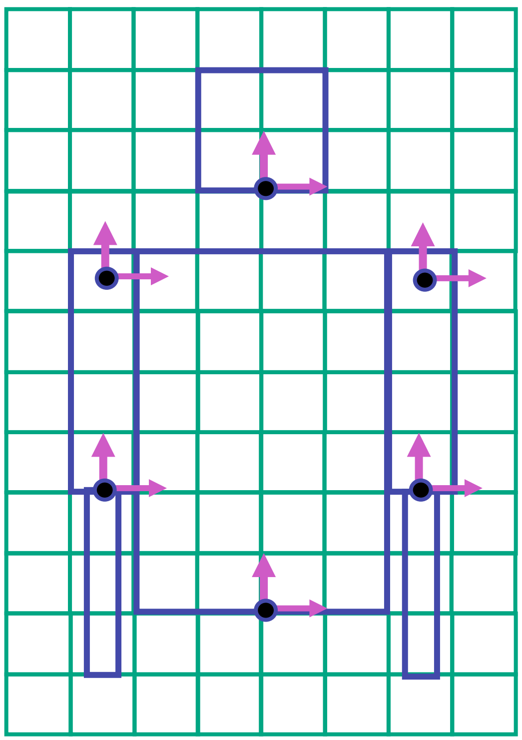
- advantages
  - no need to compute inverse matrices all the time
  - modularize changes to pipeline state
  - avoids incremental changes to coordinate systems
    - accumulation of numerical errors

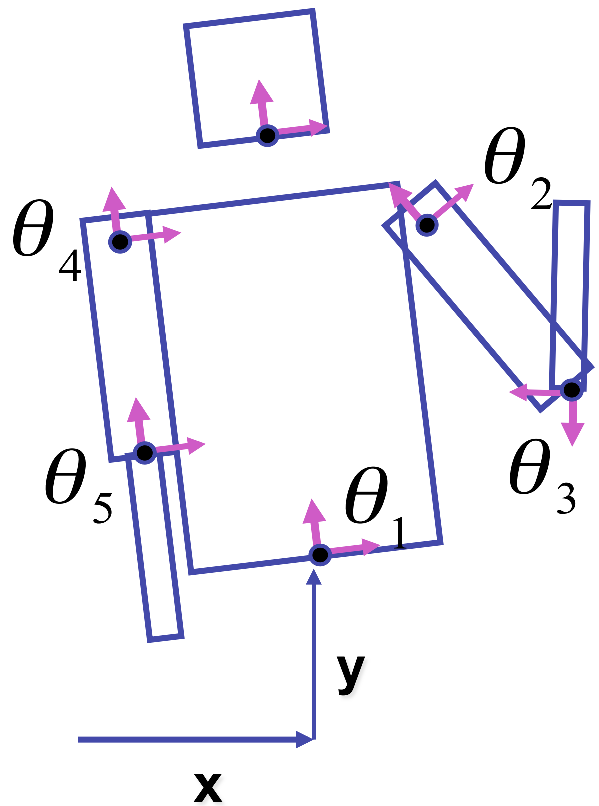
Transformation Hierarchy Example 3




```
loadIdentity();
translate(4,1,0);
pushMatrix();
rotate(45,0,0,1);
translate(0,2,0);
scale(2,1,1);
translate(1,0,0);
popMatrix();
```

Transformation Hierarchy Example 4



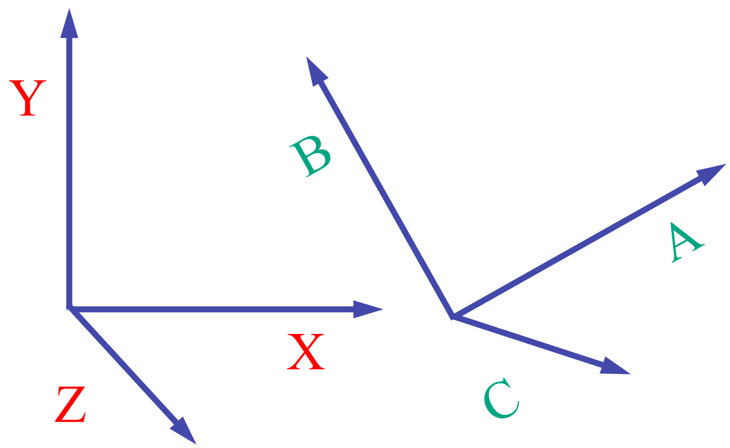


translate(x,y,0);
rotate(θ_1,0,0,1);
DrawBody();
pushMatrix();
  translate(0,7,0);
  DrawHead();
popMatrix();
pushMatrix();
  translate(2.5,5.5,0);
  rotate(θ_2,0,0,1);
  DrawUArm();
  translate(0,-3.5,0);
  rotate(θ_3,0,0,1);
  DrawLArm();
popMatrix();
... (draw other arm)

Hierarchical Modelling

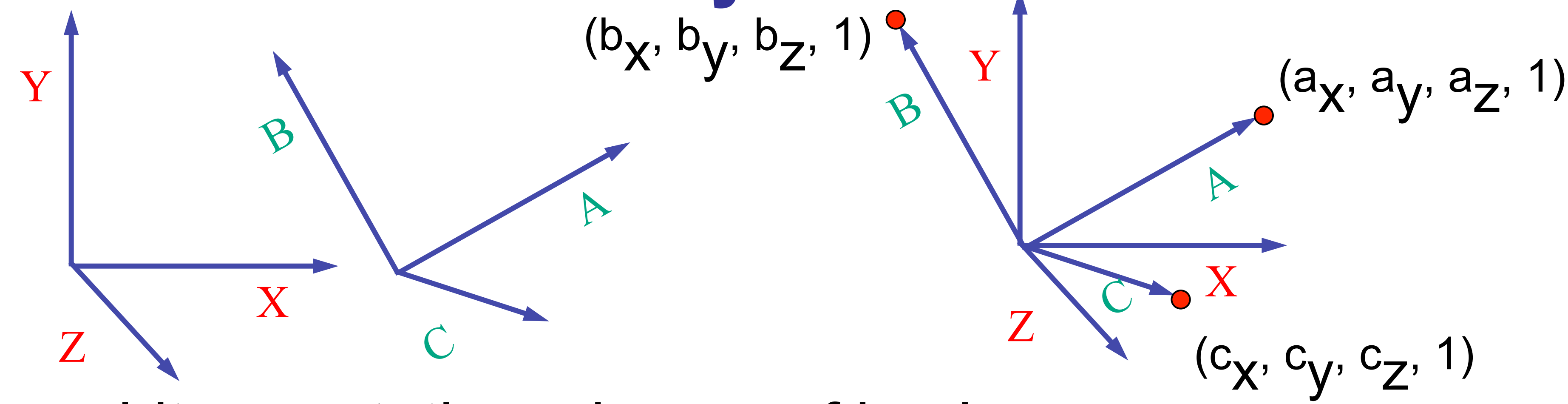
- advantages
  - define object once, instantiate multiple copies
  - transformation parameters often good control knobs
  - maintain structural constraints if well-designed
- limitations
  - expressivity: not always the best controls
  - can’t do closed kinematic chains
    - keep hand on hip
  - can’t do other constraints
    - collision detection
      - self-intersection
      - walk through walls

Arbitrary Rotation



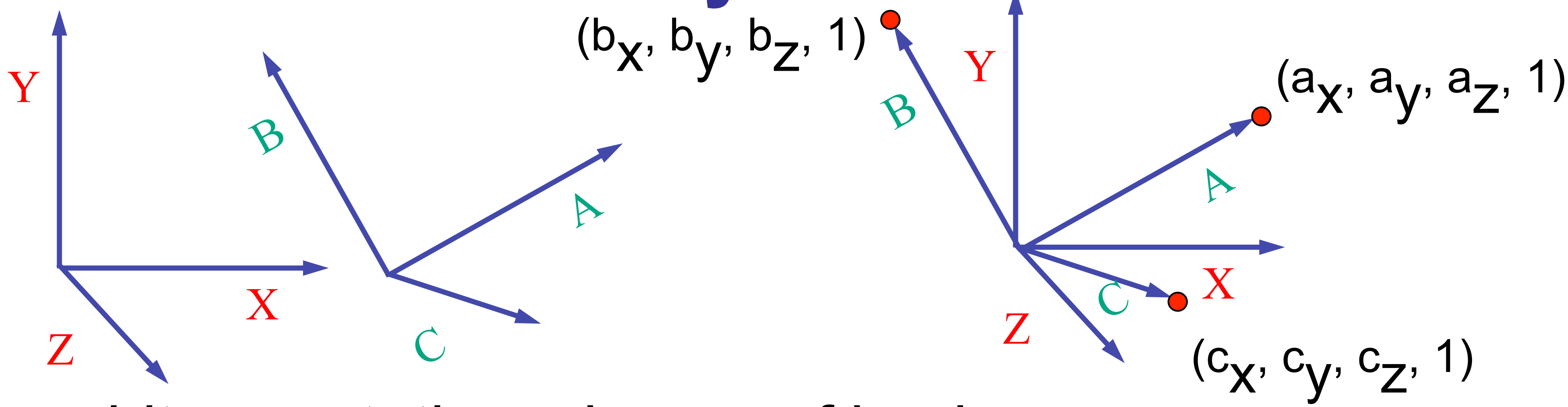
- arbitrary rotation: change of basis
  - given two orthonormal coordinate systems *XYZ* and *ABC*
    - *A*'s location in the XYZ coordinate system is $(a_x, a_y, a_z, 1)$, ...

Arbitrary Rotation



- arbitrary rotation: change of basis
  - given two orthonormal coordinate systems *XYZ* and *ABC*
    - *A*'s location in the XYZ coordinate system is $(a_x, a_y, a_z, 1)$, ...

Arbitrary Rotation



- arbitrary rotation: change of basis
  - given two orthonormal coordinate systems *XYZ* and *ABC*
    - *A*'s location in the XYZ coordinate system is $(a_x, a_y, a_z, 1)$, ...
- transformation from one to the other is matrix R whose columns are *A,B,C:*

$$R(X) = \begin{bmatrix} a_x & b_x & c_x & 0 \\ a_y & b_y & c_y & 0 \\ a_z & b_z & c_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = (a_x, a_y, a_z, 1) = A$$