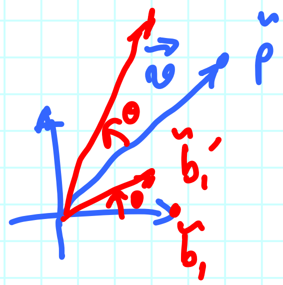
Rotations, Frames

Note Title 2015-01-20

Announcements:
- Office hours from *next* week, will move to Thursdays 3-4.
  Reason: I'm now on a university committee that meets 2-4 on Wednesdays.
- Reminder: Assignment 1 due this Friday
  Don't forget to make the changes for 1c mentioned in the last 2 classes
- Reminder: Quiz 1 is on Jan 30, in class. Scope is everything covered in class till then, and assignment 1

§ Rotations

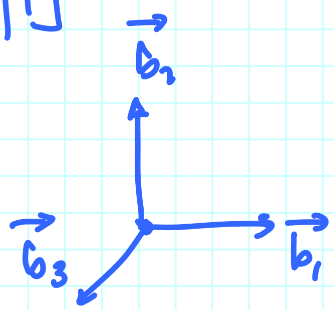
Last class in 2D



$$
\begin{array}{c|c}
 & \begin{array}{cc} 2 & 1 \end{array} \\
\begin{array}{c} 2 \\ 1 \end{array} & \left[\begin{array}{cc|c} \cos\theta & -\sin\theta & \\ \sin\theta & \cos\theta & \\ \hline & & 1 \end{array}\right]
\end{array}
$$

What about in 3D?

★ depends on the "axis of rotation"



What about rotation about $\vec{b}_3$?

$$
Rot(z,\theta) = \left[\begin{array}{ccc|c} \cos\theta & -\sin\theta & & \\ \sin\theta & \cos\theta & & \\ & & 1 & \\ \hline & & & 1 \end{array}\right]
$$

Similarly

$$
Rot(x,\theta) = \left[\begin{array}{ccc|c} 1 & & & \\ & \cos\theta & -\sin\theta & \\ & \sin\theta & \cos\theta & \\ \hline & & & 1 \end{array}\right]
\qquad
Rot(y,\theta) = \left[\begin{array}{ccc|c} \cos\theta & & \sin\theta & \\ & 1 & & \\ -\sin\theta & & \cos\theta & \\ \hline & & & 1 \end{array}\right]
$$

It turns out that, can get <u>any</u> possible rotation matrix as the product of 3 independent rotations.

Question: what is the hallmark of a rotation matrix?

Every row and column has length 1
Every " " " is orthogonal to
Every other " " ".

The matrix is orthogonal*

Think about this: If $\underline{\bar{R}}$ is rotation matrix

then $\underline{\bar{R}}^T \underline{\bar{R}} = I$

* strictly speaking we also want determinant to be +1.

§ Frames

So far: if we have a coordinate frame $\underline{\hat{b}}$

- we can represent every point, vector $\hat{p} = \underline{\hat{b}}\,\bar{p}$
- we can transform it " "

$$\hat{p}' = \underline{\hat{b}}\,\underline{\bar{A}}\,\bar{p}$$

What if we want a new basis $\underline{\hat{a}}$?

That is, want to know the new coordinates of $\hat{p}$ in basis $\underline{\hat{a}}$, knowing coords. in $\underline{\hat{b}}$

$$\hat{p} = \underline{\hat{b}}\,\bar{p} = \underline{\hat{a}}\,\bar{p}' \qquad (1)$$

how to get this?

$$\underline{\hat{a}} = (\vec{a}_1\ \vec{a}_2\ \vec{a}_3\ \hat{a}_0)$$

Express all of these in terms of $\underline{\hat{b}}$

$$(\underline{\hat{b}}\,\bar{a}_1 \ \dots\ \underline{\hat{b}}\,\bar{a}_0)$$

$$\underline{\hat{b}}\left[\bar{a}_1 \,|\, \bar{a}_2 \,|\, \bar{a}_3 \,|\, \bar{a}_0\right]$$

$$\underline{\hat{a}} = \underline{\hat{b}}\,\underline{\bar{A}}$$

Plug this into Eqn (1)

$$\cancel{\underline{\hat{b}}}\,\bar{p} = \cancel{\underline{\hat{b}}}\,\underline{\bar{A}}\,\bar{p}'$$

$$\bar{p} = \underline{\bar{A}}\, \bar{p}'$$

So

$$\boxed{\bar{p}' = \underset{\sim}{\bar{A}}^{-1}\, \bar{p}}$$

Next class: Two views of 4x4 matrices